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On Interval Valued Intuitionistic (S, T)-fuzzyH_v-ideals

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ABSTRACT

Atanassov introduced the concept of the interval valued intuitionistic fuzzy sets. By using this we introduce the notion of interval valued intuitionistic **-H_v** fuzzy -ideals of an **H_v** -ring with respect to a *t*-norm *T* and an *s*-norm *S.* Also some of their characteristic properties are described. The homomorphic image and the inverse image are investigated.

Keywords: **H_v** -ideal, interval valued intuitionistic (*S*, *T*)-fuzzy **H_v** -ideal, interval valued intuitionistic (*S*, *T*)-fuzzy ideal

I. INTRODUCTION

The concept of hyperstructure was introduced in 1934 by Marty [1]. Hyperstructures have many applications to several branches of pure and applied sciences. Vougiouklis [2] introduced the notion of H_v-structures, and Davvaz [3] surveyed the theory of H_v-structures. After the introduction of fuzzy sets by Zadeh [4], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [5] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [6, 7].

In [8] Biswas applied the concept of intuitoinistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. In [9] Kim et al. introduced the notion of fuzzy subquasigroups of a quasigroup. In [10] Kim and Jun introduced the concept of fuzzy ideals of a semigroup. Zhan et al. [11] introduced the notion of an interval valued intuitionistic (S,T) -fuzzy H_v -submodule of an H_v-module. This paper continues this line of research for fuzzy H_v-ideal of H_v-ring. In this paper, we introduce the notion of interval valued intuitionistic (S,T) -fuzzy H_v -ideals of an H_v -ring and describe the characteristic properties. We give the homomorphic image and the inverse image.

The paper is organized as follows: in section 2 some fundamental definitions on H_v-structures and fuzzy sets are explored, in section 3 we define interval valued intuitionistic (S, T)-fuzzy H_v-ideals and establish some useful properties.

II. METHODS AND MATERIAL

1. Basic Definitions

We first give some basic definitions for proving the further results.

Definition 1.1 [12] Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X .The complement of μ, denoted by μ^c, is the fuzzy set in X given by

$$\mu^c(x) = 1 - \mu(x) \quad \forall x \in X .$$

Definition 1.2 [12] Let f be a mapping from a set X to a set Y. Let μ be a fuzzy set in X and λ be a fuzzy set in Y . Then the inverse image $f^{-1}(\lambda)$ of λ is a fuzzy set in X defined by

$$f^{-1}(\lambda)(x) = \lambda(f(x)) \quad \forall x \in X .$$

The image $f(\mu)$ of μ is the fuzzy set in Y defined by

$$f(\mu)(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)} \mu(x), & f^{-1}(y)\neq\phi\\ 0, & otherwise\end{cases}$$

For all $y \in Y$.

Definition 1.3 [12] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A=\{(x,\mu_A(x),\lambda_A(x)): x\in X\}$, where the functions $\mu_A: X\to[0,1]$ and $\lambda_A: X\to[0,1]$ denote the degree of membership and degree of non membership of each element $x\in X$ to the set A respectively and $0\le\mu_A(x)+\lambda_A(x)\le 1$ for all $x\in X$. We shall use the symbol $A=\{\mu_A,\lambda_A\}$ for the intuitionistic fuzzy set $A=\{(x,\mu_A(x),\lambda_A(x)): x\in X\}$.

Definition1.4 [12] Let $A=\{\mu_A,\lambda_A\}$ and $B=\{\mu_B,\lambda_B\}$ be intuitionistic fuzzy sets in X. Then

(1) $A\subseteq B \Leftrightarrow \mu_A(x)\le\mu_B(x)$ *and* $\lambda_A(x)\le\lambda_B(x)$,

(2) $A^c=\{(x,\lambda_A(x),\mu_A(x)): x\in X\}$,

(3) $A\cap B=\left\{\begin{array}{l}(x,\min\{\mu_A(x),\mu_B(x)\},\\ \max\{\lambda_A(x),\lambda_B(x)\}): x\in X\end{array}\right\}$,

(4) $A\cup B=\left\{\begin{array}{l}(x,\max\{\mu_A(x),\mu_B(x)\},\\ \min\{\lambda_A(x),\lambda_B(x)\}): x\in X\end{array}\right\}$,

(5) $\Box A=\left\{(x,\mu_A(x),\mu_A^c(x)): x\in X\right\}$,

(6) $\Diamond A=\left\{(x,\lambda_A^c(x),\lambda_A(x)): x\in X\right\}$.

Definition 1.5 [13]Let G be a non–empty set and $*: G\times G\to\wp^*(G)$ be a hyperoperation, where $\wp^*(G)$ is the set of all the non-empty subsets of G. Where $A*B=\bigcup\limits_{a\in A,\, b\in B} a*b,\ \forall A,B\subseteq G$.

The $*$ is called weak commutativeif

$x*y\cap y*x\neq\phi,\quad \forall x,y\in G$.

The $*$ is called weak associativeif

$(x*y)*z\cap x*(y*z)\neq\phi,\quad \forall x,y,z\in G$.

A hyperstructure $(G,*)$ is called an H_v–group if

(i) $*$ is weak associative.

(ii) $a*G=G*a=G,\quad \forall a\in G$ (Reproduction axiom).

Definition 1.6 [14] Let G be a hypergroup (or H_v-group) and let μ be a fuzzy subset of G. Then μ is said to be a fuzzy subhypergroup (or fuzzy H_v-subgroup) of G if the following axioms hold:

$(i)\ \min\{\mu(x),\mu(y)\}\le\inf\limits_{\alpha\in x*y}\{\mu(\alpha)\},\quad \forall x,y\in G$ *(ii)* For all $x,a\in G$ there exists $y\in G$ such that $x\in a*y$ and $\min\{\mu(a),\mu(x)\}\le\{\mu(y)\}$.

Definition 1.7 [15] Let G be a hypergroup (or H_v-group). An intuitionistic fuzzy set $A=\{\mu_A,\lambda_A\}$ of G is called intuitionistic fuzzy subhypergroup (or intuitionistic fuzzy H_v-subgroup) of G if the following axioms hold:

$(i)\ \min\{\mu_A(x),\mu_A(y)\}\le\inf\limits_{\alpha\in x*y}\{\mu_A(\alpha)\},\quad \forall x,y\in G.$

(ii) For all $x,a\in G$ there exists $y\in G$ such that $x\in a*y$ and $\min\{\mu_A(a),\mu_A(x)\}\le\{\mu_A(y)\}$.

$(iii)\ \sup\limits_{\alpha\in x*y}\{\lambda_A(\alpha)\}\le\max\{\lambda_A(x),\lambda_A(y)\},\quad \forall x,y\in G.$

(iv) For all $x,a\in G$ there exists $y\in G$ such that $x\in a*y$ and $\{\lambda_A(y)\}\le\max\{\lambda_A(a),\lambda_A(x)\}$.

Definition 1.8 [13] An H_v–ring is a system $(R,+,\cdot)$ with two hyperoperations satisfying the ring-like axioms:

(i) $(R,+)$ is an H_v-group, that is,

$((x+y)+z)\cap(x+(y+z))\neq\phi\quad \forall x,y\in R,$

$a+R=R+a=R\quad \forall a\in R;$

(ii) $(R,\cdot)$ is an H_v-semigroup;

(iii) $(\cdot)$ is weak distributive with respect to $(+)$, that is, for all $x,y,z\in R$,

$(x\cdot(y+z))\cap(x\cdot y+x\cdot z)\neq\phi,$

$((x+y)\cdot z)\cap(x\cdot z+y\cdot z)\neq\phi.$

Definition 1.9 [16] Let R be an H_v-ring. A nonempty subset I of R is called a left (resp., right) H_v-ideal if the following axioms hold:

(i) $(I,+)$ is an H_v-subgroup of $(R,+)$,

(ii) $R\cdot I\subseteq I$ (resp., $I\cdot R\subseteq I$).

Definition 1.10 [16] Let $(R,+,\cdot)$ be an H_v–ring and μ a fuzzy subset of R. Then μ is said to be a left (resp., right) fuzzy H_v-ideal of R if the following axioms hold:

(1) $\min\{\mu(x),\mu(y)\}\le\inf\{\mu(z): z\in x+y\}$ $\forall x,y\in R,$

(2) For all $x, a \in R$ there exists $y \in R$ such that $x \in a + y$ and $\min\{\mu(a), \mu(x)\} \le \mu(y)$,

(3) For all $x, a \in R$ there exists $z \in R$ such that $x \in z + a$ and $\min\{\mu(a), \mu(x)\} \le \mu(z)$,

(4) $\mu(y) \le \inf\{\mu(z) : z \in x \cdot y\}$ [respectively $\mu(x) \le \inf\{\mu(z) : z \in x \cdot y\} \quad \forall x, y \in R$].

Definition 1.11 [16] An intuitionistic fuzzy set $A = \{\mu_A, \lambda_A\}$ in R is called a left (resp., right) intuitionistic fuzzy H$_v$-ideal of R if

(1) $\min\{\mu_A(x), \mu_A(y)\} \le \inf\{\mu_A(z) : z \in x + y\}$
$\max\{\lambda_A(x), \mu_A(y)\} \ge \sup\{\lambda_A(z) : z \in x + y\}$
$\forall x, y \in R$;

(2) For all $x, a \in R$ there exists $y \in R$ such that $x \in a + y$ and $\min\{\mu_A(a), \mu_A(x)\} \le \mu_A(y)$ and $\max\{\lambda_A(a), \lambda_A(x)\} \ge \lambda_A(y)$;

(3) For all $x, a \in R$ there exists $z \in R$ such that $x \in z + a$ and $\min\{\mu_A(a), \mu_A(x)\} \le \mu_A(z)$ and $\max\{\lambda_A(a), \lambda_A(x)\} \ge \lambda_A(z)$;

(4) $\mu_A(y) \le \inf\{\mu_A(z) : z \in x \cdot y\}$ [respectively $\mu_A(x) \le \inf\{\mu_A(z) : z \in x \cdot y\} \quad \forall x, y \in R$] and $\lambda_A(y) \ge \sup\{\lambda_A(z) : z \in x \cdot y\}$ [respectively $\lambda_A(x) \ge \sup\{\lambda_A(z) : z \in x \cdot y\} \quad \forall x, y \in R$].

Definition 1.12 [17] By a t-norm T, we mean a function $T : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions:

$(i)\, T(x,1) = x,$

$(ii)\, T(x,y) \le T(x,z) \quad if\ y \le z,$

$(iii)\, T(x,y) = T(y,x),$

$(iv)\, T(x, T(y,z)) = T(T(x,y), z)$

For all $x, y, z \in [0,1]$.

Definition 1.13 [17] By a s-norm S, we mean a function $S : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions:

$(i)\, S(x,0) = x,$

$(ii)\, S(x,y) \le S(x,z) \quad if\ y \le z,$

$(iii)\, S(x,y) = S(y,x),$

$(iv)\, S(x, S(y,z)) = S(S(x,y), z)$

For all $x, y, z \in [0,1]$.

It is clear that $T(\alpha, \beta) \le \min\{\alpha, \beta\} \le \max\{\alpha, \beta\} \le S(\alpha, \beta)$ For all $\alpha, \beta \in [0,1]$.

By an interval number $\tilde{a}$ we mean an interval $[a^-, a^+]$ where $0 \le a^- \le a^+ \le 1$. The set of all interval numbers is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a \in [0,1]$.

For the interval numbers $\tilde{a}_i = [a_i^-, a_i^+] \in D[0,1], i \in I$, we define

$$\max\{\tilde{a}_i, \tilde{b}_i\} = [\max(a_i^-, b_i^-), \max(a_i^+, b_i^+)],$$

$$\min\{\tilde{a}_i, \tilde{b}_i\} = [\min(a_i^-, b_i^-), \min(a_i^+, b_i^+)],$$

$$\inf \tilde{a}_i = \left[\wedge_{i\in I} a_i^-, \wedge_{i\in I} a_i^+\right], \sup \tilde{a}_i = \left[\vee_{i\in I} a_i^-, \vee_{i\in I} a_i^+\right]$$

and put

$(1)\, \tilde{a}_1 \le \tilde{a}_2 \Leftrightarrow a_1^- \le a_2^- \text{ and } a_1^+ \le a_2^+,$

$(2)\, \tilde{a}_1 = \tilde{a}_2 \Leftrightarrow a_1^- = a_2^- \text{ and } a_1^+ = a_2^+,$

$(3)\, \tilde{a}_1 < \tilde{a}_2 \Leftrightarrow \tilde{a}_1 \le \tilde{a}_2 \text{ and } \tilde{a}_1 \ne \tilde{a}_2,$

$(4)\, k\tilde{a} = [ka^-, ka^+], \text{whenever } 0 \le k \le 1.$

It is clear that $(D[0,1], \le, \vee, \wedge)$ is a complete lattice with $0 = [0,0]$ as least element and $1 = [1,1]$ as greatest element.

By an interval valued fuzzy set F on X we mean the set $F = \{(x, [\mu_F^-(x), \mu_F^+(x)]) : x \in X\}$. Where μ_F^- and μ_F^+ are fuzzy subsets of X such that $\mu_F^-(x) \le \mu_F^+(x)$ for all $x \in X$. Put $\tilde{\mu}_F(x) = [\mu_F^-(x), \mu_F^+(x)]$. Then $F = \{(x, \tilde{\mu}_F(x)) : x \in X\}$, where $\tilde{\mu}_F : X \to D[0,1]$.

If A, B are two interval valued fuzzy subsets of X, then we define

$A \subseteq B$ if and only if for all $x \in X$, $\mu_A^-(x) \le \mu_B^-(x)$ and $\mu_A^+(x) \le \mu_B^+(x)$,

$A = B$ if and only if for all $x \in X$, $\mu_A^-(x) = \mu_B^-(x)$ and $\mu_A^+(x) = \mu_B^+(x)$.

Also, the union, intersection and complement are defined as follows: let A; B be two interval valued fuzzy subsets of X, then

$$A\cup B=\left\{\left(x,\left[\max\left\{\mu_A^-(x),\mu_B^-(x)\right\},\max\left\{\mu_A^+(x),\mu_B^+(x)\right\}\right]\right):x\in X\right\},$$
$$A\cap B=\left\{\left(x,\left[\min\left\{\mu_A^-(x),\mu_B^-(x)\right\},\min\left\{\mu_A^+(x),\mu_B^+(x)\right\}\right]\right):x\in X\right\},$$
$$A^c=\left\{\left(x,\left[\left\{1-\mu_A^-(x),1-\mu_A^+(x)\right\}\right]\right):x\in X\right\}.$$

According to Atanassov an interval valued intuitionistic fuzzy set on X is defined as an object of the form $A=\left\{\left(x,\tilde{\mu}_A(x),\tilde{\lambda}_A(x)\right):x\in X\right\}$, where $\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x)$ are interval valued fuzzy sets on X such that $0\le\sup\tilde{\mu}_A(x)+\sup\tilde{\lambda}_A(x)\le 1$ for all $x\in X$.

For the sake of simplicity, in the following such interval valued intuitionistic fuzzy sets will be denoted by $A=\left(\tilde{\mu}_A,\tilde{\lambda}_A\right)$.

2.Interval Valued Intuitionistic (S,T)-Fuzzy H_v-Ideals

In what follows, let R denote an H_v-ring unless otherwise specified.

Definition 2.1 An interval valued intuitionistic fuzzy set $A=\left(\tilde{\mu}_A,\tilde{\lambda}_A\right)$ of R is called an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of Rif the following conditions hold:

$(1)\,T\left(\tilde{\mu}_A(x),\tilde{\mu}_A(y)\right)\le\inf\limits_{\alpha\in x+y}\tilde{\mu}_A(\alpha)$ and

$S\left(\tilde{\lambda}_A(x),\tilde{\lambda}_A(y)\right)\ge\sup\limits_{\alpha\in x+y}\tilde{\lambda}_A(\alpha),\forall x,y\in R,$

$(2)\,\forall x,a\in R$ there exists $y\in R$ such that

$x\in a+y, T\left(\tilde{\mu}_A(x),\tilde{\mu}_A(a)\right)\le\tilde{\mu}_A(y)$ and

$S\left(\tilde{\lambda}_A(x),\tilde{\lambda}_A(a)\right)\ge\tilde{\lambda}_A(y),$

$(3)\,\forall x,a\in R$ there exists $z\in R$ such that

$x\in z+a, T\left(\tilde{\mu}_A(x),\tilde{\mu}_A(a)\right)\le\tilde{\mu}_A(z)$ and

$S\left(\tilde{\lambda}_A(x),\tilde{\lambda}_A(a)\right)\ge\tilde{\lambda}_A(z),$

$(4)\,\tilde{\mu}_A(x)\le\inf\limits_{\alpha\in r\cdot x}\tilde{\mu}_A(\alpha)$ and

$\tilde{\lambda}_A(x)\ge\sup\limits_{\alpha\in r\cdot x}\tilde{\lambda}_A(\alpha),\forall x,r\in R.$

With any interval valued intuitionistic fuzzy set $A=\left(\tilde{\mu}_A,\tilde{\lambda}_A\right)$ of R there are connected two levels:

$U\left(\tilde{\mu}_A;[t,s]\right)=\left\{x\in R:\tilde{\mu}_A(x)\ge[t,s]\right\}$, and

$L\left(\tilde{\lambda}_A;[t,s]\right)=\left\{x\in R:\tilde{\lambda}_A(x)\le[t,s]\right\}.$

Theorem 2.2 Let T (resp. S)be an idempotent interval t-norm (resp. s-norm). Then $A=\left(\tilde{\mu}_A,\tilde{\lambda}_A\right)$ is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of Rif and only if for all $t,s\in[0,1],t\le s, U\left(\tilde{\mu}_A;[t,s]\right)$ and $L\left(\tilde{\lambda}_A;[t,s]\right)$ are H_v-ideals of R.

Proof Let $A=\left(\tilde{\mu}_A,\tilde{\lambda}_A\right)$ be an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R. Then for every $x,y\in U\left(\tilde{\mu}_A;[t,s]\right)$ we have $\tilde{\mu}_A(x)\ge[t,s]$ and $\tilde{\mu}_A(y)\ge[t,s]$. Hence $T\left(\tilde{\mu}_A(x),\tilde{\mu}_A(y)\right)\ge T\left([t,s],[t,s]\right)=[t,s]$, and so $\inf\limits_{\alpha\in x+y}\tilde{\mu}_A(\alpha)\ge[t,s]$. Therefore $\alpha\in U\left(\tilde{\mu}_A;[t,s]\right)$ for every $\alpha\in x+y$, so $x+y\subseteq U\left(\tilde{\mu}_A;[t,s]\right)$. Thus, for every $a\in U\left(\tilde{\mu}_A;[t,s]\right)$, we have $a+U\left(\tilde{\mu}_A;[t,s]\right)\subseteq U\left(\tilde{\mu}_A;[t,s]\right)$. On the other hand, for $x,a\in U\left(\tilde{\mu}_A;[t,s]\right)$ there exists $y\in R$ such that $x\in a+y$ and $T\left(\tilde{\mu}_A(x),\tilde{\mu}_A(a)\right)\le\tilde{\mu}_A(y)$. But $T\left(\tilde{\mu}_A(x),\tilde{\mu}_A(a)\right)\ge[t,s]$ for all $x,a\in U\left(\tilde{\mu}_A;[t,s]\right)$ so $\tilde{\mu}_A(y)\ge[t,s]$ that is, $y\in U\left(\tilde{\mu}_A;[t,s]\right)$, whence $U\left(\tilde{\mu}_A;[t,s]\right)\subseteq a+U\left(\tilde{\mu}_A;[t,s]\right)$, and, in consequence, $U\left(\tilde{\mu}_A;[t,s]\right)=a+U\left(\tilde{\mu}_A;[t,s]\right)$. Similarly, we can prove that $U\left(\tilde{\mu}_A;[t,s]\right)=U\left(\tilde{\mu}_A;[t,s]\right)+a$. That is, $a+U\left(\tilde{\mu}_A;[t,s]\right)=U\left(\tilde{\mu}_A;[t,s]\right)=U\left(\tilde{\mu}_A;[t,s]\right)+a.$ This proves that $\left(U\left(\tilde{\mu}_A;[t,s]\right),+\right)$ is an H_v-subgroup of $(R,+)$.

If $r\in R$ and $x\in U\left(\tilde{\mu}_A;[t,s]\right)$ then $\tilde{\mu}_A(x)\ge[t,s]$, which means that $\inf\limits_{\alpha\in r\cdot x}\tilde{\mu}_A(\alpha)\ge[t,s]$. So, $\alpha\in U\left(\tilde{\mu}_A;[t,s]\right)$ for every $\alpha\in r\cdot x$. Therefore,

$r\cdot x\subseteq U(\tilde{\mu}_A;[t,s])$, i.e. $r\cdot U(\tilde{\mu}_A;[t,s])\subseteq U(\tilde{\mu}_A;[t,s])$. This proves that $U(\tilde{\mu}_A;[t,s])$ is an H_v-ideal of R. Similarly, we can show that $L(\tilde{\lambda}_A;[t,s])$ is an H_v-ideal of R.

Conversely, assume that for every $[t,s]\in D[0,1]$ any non-empty $U(\tilde{\mu}_A;[t,s])$ is an H_v-ideal of R. If $[t_0,s_0]=T(\tilde{\mu}_A(x),\tilde{\mu}_A(y))$ for some $x,y\in R,$ then $x,y\in U(\tilde{\mu}_A;[t_0,s_0])$, and so $x+y\subseteq U(\tilde{\mu}_A;[t_0,s_0])$. Therefore $\alpha\in U(\tilde{\mu}_A;[t_0,s_0])$ for every $\alpha\in x+y,$ and so $\inf_{\alpha\in x+y}\tilde{\mu}_A(\alpha)\geq T(\tilde{\mu}_A(x),\tilde{\mu}_A(y))$. Now, if $[t_1,s_1]=T(\tilde{\mu}_A(a),\tilde{\mu}_A(x))$ for some $a,x\in R,$ then $a+x\in U(\tilde{\mu}_A;[t_1,s_1])$, so there exists $y\in U(\tilde{\mu}_A;[t_1,s_1])$ such that $x\in a+y$. But for $y\in U(\tilde{\mu}_A;[t_1,s_1])$ we have $\tilde{\mu}_A(y)\geq[t_1,s_1],$ whence $\tilde{\mu}_A(y)\geq T(\tilde{\mu}_A(a),\tilde{\mu}_A(x))$.

Similarly, we can show that for $a,x\in R,$ there exists $z\in R$ such that $x\in z+a$ and $\tilde{\mu}_A(z)\geq T(\tilde{\mu}_A(a),\tilde{\mu}_A(x))$. If $[t_2,s_2]=\tilde{\mu}_A(x)$ for some $x\in R,$ then $x\in U(\tilde{\mu}_A;[t_2,s_2])$, and so $r\cdot x\in U(\tilde{\mu}_A;[t_2,s_2])$ for every $x\in R.$ Therefore for every $\alpha\in r\cdot x,$ we have $\alpha\in U(\tilde{\mu}_A;[t_2,s_2])$, consequently $\inf_{\alpha\in r\cdot x}\tilde{\mu}_A(\alpha)\geq[t_2,s_2]=\tilde{\mu}_A(x)$.

This proves that $\tilde{\mu}_A$ is an interval valued T-fuzzy H_v-ideal of R.

Similarly, we can show that $\tilde{\lambda}_A$ is an interval valued S-fuzzy H_v-ideal of R. Therefore, $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R.

Definition 2.3 Let $f:X\to Y$ be a mapping and $A=(\tilde{\mu}_A,\tilde{\lambda}_A), B=(\tilde{\mu}_B,\tilde{\lambda}_B)$ an interval valued intuitionistic sets X and $Y,$ respectively. Then the image $f[A]=(f(\tilde{\mu}_A),f(\tilde{\lambda}_A))$ of A is the interval valued intuitionistic fuzzy set of Y defined by

$$f(\tilde{\mu}_A)(y)=\begin{cases}\sup_{z\in f^{-1}(y)}\tilde{\mu}_A(z), & f^{-1}(y)\neq\phi\\ [0,0], & f^{-1}(y)=\phi\end{cases}$$ and

$$f(\tilde{\lambda}_A)(y)=\begin{cases}\inf_{z\in f^{-1}(y)}\tilde{\lambda}_A(z), & f^{-1}(y)\neq\phi\\ [1,1], & f^{-1}(y)=\phi\end{cases}$$

for all $y\in Y.$

The inverse image $f^{-1}(B)$ of B is an interval valued intuitionistic fuzzy set defined by $f^{-1}(\tilde{\mu}_B)(x)=\tilde{\mu}_{f^{-1}(B)}(x)=\tilde{\mu}_B(f(x))$, $f^{-1}(\tilde{\lambda}_B)(x)=\tilde{\lambda}_{f^{-1}(B)}(x)=\tilde{\lambda}_B(f(x))$ for all $x\in X$.

Definition 2.4 [18] Let R and S be two H_v-rings. A mapping $f:R\to S$ is called an H_v-homomorphism or weak homomorphism if for all $x,y,r\in R$ the following relations hold: $f(x+y)\cap(f(x)+f(y))\neq\phi$ and $f(r\cdot x)\cap r\cdot f(x)\neq\phi.$

f is called an inclusion homomorphism if $f(x+y)\subseteq f(x)+f(y)$ and $f(r\cdot x)\subseteq r\cdot f(x)$ for all $x,y,r\in R.$ Finally, f is called a strong homomorphism if for all $x,y,r\in R$ we have $f(x+y)=f(x)+f(y)$ and $f(r\cdot x)=r\cdot f(x)$.

Lemma 2.5 [18] Let R_1 and R_2 be two H_v-ringsand $f:R_1\to R_2$ a strong epimorphism. If S is an H_v-ideal of R_2, then $f^{-1}(S)$ is an H_v-ideal of R_1.

Theorem 2.6 Let R_1 and R_2 be two H_v-rings, f a strong epimorphism from R_1 into R_2 and T (resp. S) an idempotent interval t-norm (resp. s-norm).

(i) If $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R_1, then the image $f[A]$of A is an interval intuitionistic (S,T)-fuzzy H_v-ideal of R_2.

(ii) If $B=(\tilde{\mu}_B,\tilde{\lambda}_B)$ is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R_2, then the inverse image

$f^{-1}(B)$ of B is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R_1.

Proof (i) Let $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ be an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R_1. By Theorem 2.2, $U(\tilde{\mu}_A;[t,s])$ and $L(\tilde{\lambda}_A;[t,s])$ are H_v-ideals of R_1 for every $[t,s]\in D[0,1]$. Therefore, by Lemma 3.5, $f(U(\tilde{\mu}_A;[t,s]))$ and $f(L(\tilde{\lambda}_A;[t,s]))$ are H_v-ideals of R_2. But

$U(f(\tilde{\mu}_A);[t,s])=f(U(\tilde{\mu}_A;[t,s]))$ and

$L(f(\tilde{\lambda}_A);[t,s])=f(L(\tilde{\lambda}_A;[t,s]))$, so

$U(f(\tilde{\mu}_A);[t,s])$ and $L(f(\tilde{\lambda}_A);[t,s])$ are H_v-ideals of R_2. Therefore $f[A]$ is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R_2.

(ii) For any $x,y\in R$ and $\alpha\in x+y$, we have

$$\tilde{\mu}_{f^{-1}(B)}(\alpha)=\tilde{\mu}_B(f(\alpha))\geq T(\tilde{\mu}_B(f(x)),\tilde{\mu}_B(f(y)))=T(\tilde{\mu}_{f^{-1}(B)}(x),\tilde{\mu}_{f^{-1}(B)}(y)).$$

Therefore

$\inf_{\alpha\in x+y}\tilde{\mu}_{f^{-1}(B)}(\alpha)\geq T(\tilde{\mu}_{f^{-1}(B)}(x),\tilde{\mu}_{f^{-1}(B)}(y))$. For $x,a\in R_2$ there exists $y\in R_2$ such that $x\in a+y$.

Thus $f(x)\in f(a)+f(y)$ and

$$T(\tilde{\mu}_{f^{-1}(B)}(x),\tilde{\mu}_{f^{-1}(B)}(a))=T(\tilde{\mu}_B(f(x)),\tilde{\mu}_B(f(y)))\leq\tilde{\mu}_B(f(y))=\tilde{\mu}_{f^{-1}(B)}(y).$$

In the same manner, we can show that for $x,a\in R_2$ there exists $z\in R_2$ such that $x\in z+a$ and

$$T(\tilde{\mu}_{f^{-1}(B)}(x),\tilde{\mu}_{f^{-1}(B)}(a))\leq\tilde{\mu}_{f^{-1}(B)}(z).$$

It is not difficult to see that, for all $x\in R_2, r\in R$ and $\alpha\in r\cdot x$, we have

$$\tilde{\mu}_{f^{-1}(B)}(\alpha)=\tilde{\mu}_B(f(\alpha))\geq\tilde{\mu}_B(f(x))=\tilde{\mu}_{f^{-1}(B)}(x),$$

whence $\inf_{\alpha\in r\cdot x}\tilde{\mu}_{f^{-1}(B)}(\alpha)\geq\tilde{\mu}_{f^{-1}(B)}(x)$.

This completes the proof that $\tilde{\mu}_{f^{-1}(B)}$ is an interval valued T-ideal of R_1.

Similarly, we can prove $\tilde{\lambda}_{f^{-1}(B)}$ is an interval valued S-fuzzy H_v-ideal of R_1. Therefore $f^{-1}(B)$ is an interval valued intuitionistic (S,T)-fuzzy H_v-ideal of R_1.

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