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Level Sets of Fuzzy HX Ideal of a HX Ring

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ABSTRACT

In this paper, we introduce the concept of level sets of fuzzy HX left ideal and fuzzy HX right ideal of a HX ring . Also we have discussed the properties of fuzzy HX righ t(left) ideal under homomorphism and anti homomorphism by establishing the relationship among them.
Keywords: Fuzzy HX left ideal, fuzzy HX right ideal, fuzzy HX ideal, homomorphism, anti homomorphism.

I. INTRODUCTION

In 1965, Zadeh [10] introduced the concept of fuzzy subset . Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague. In 1967, Rosenfeld [9] defined the idea of fuzzy subgroups and gave some of its properties. In 1988, Professor Li Hong Xing [4] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1,2] gave the structures of HX ring on a class of ring.

II. METHODS AND MATERIAL

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, $\cdot$) is a ring, e is the additive identity element of R and xy, we mean x.y.

2.1 Definition

Let R be a ring. Let μ be a fuzzy subset defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\Re$ is called a fuzzy HX right ideal on $\Re$ or a fuzzy right ideal induced by μ if the following conditions are satisfied. For all A,B $\in \Re$,

i. $\lambda^\mu(A - B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,
ii. $\lambda^\mu(AB) \geq \lambda^\mu(A)$
iii.

where $\lambda^\mu(A) = \max\{\mu(x) / \text{ for all } x \in A \subseteq R\}$.

2.2 Definition

Let R be a ring. Let μ be a fuzzy subset defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\Re$ is called a fuzzy HX left ideal on $\Re$ or a fuzzy left ideal induced by μ if the following conditions are satisfied. For all A,B $\in \Re$,

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where $\lambda^\mu(A) = \max\{\mu(x) / \text{ for all } x \in A \subseteq R\}$.

2.3 Definition

Let R be a ring. Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\Re$ is called a fuzzy HX ideal on $\Re$ or a fuzzy ideal induced by μ if it is both fuzzy HX right ideal and fuzzy HX left ideal on $\Re$. That is, For all A, B $\in \Re$,

i. $\lambda^\mu(A - B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,
ii. $\lambda^\mu(AB) \geq \max\{\lambda^\mu(A), \lambda^\mu(B)\}$

where $\lambda^\mu(A) = \max\{\mu(x) / \text{ for all } x \in A \subseteq R\}$.

3. Level Subsets of Fuzzy Hx Ideal

In this section, we introduce the idea of level subsets of a fuzzy HX ideal of a HX ring. We also discuss the relation between a given fuzzy HX ideals of a HX ring and its level HX ideals and investigate the conditions under which a given HX ring has a properly inclusive chain of HX ideals.

3.1 Definition

Let λ^{μ} be a fuzzy HX ideal of a HX ring $\Re$. For any $t \in [0,1]$, we define the set $U(\lambda^{\mu}; t) = \{ A \in \Re / \lambda^{\mu}(A) \geq t \}$ is called an upper level subset or a level subset of λ^{μ}.

3.2 Theorem

Let λ^{μ} be a fuzzy HX right ideal of a HX ring $\Re$ and $U(\lambda^{\mu}; t)$ is non-empty, then for $t \in [0,1]$, $U(\lambda^{\mu}; t)$ is a HX right ideal of $\Re$.

Proof

Let λ^{μ} be a fuzzy HX right ideal of a HX ring $\Re$.

For any $A, B \in U(\lambda^{\mu}; t)$, we have, $\lambda^{\mu}(A) \geq t$ and $\lambda^{\mu}(B) \geq t$.Now, $\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$\geq \min\{t, t\} = t$, for some $t \in [0,1]$.

$\lambda^{\mu}(A - B) \geq t$.

For any $A \in U(\lambda^{\mu}; t)$ and $B \in \Re$, we have, $\lambda^{\mu}(A) \geq t$.

Now, $\lambda^{\mu}(AB) \geq \lambda^{\mu}(A) \geq t$.

$\lambda^{\mu}(AB) \geq t$

Hence, $A - B$, $AB \in U(\lambda^{\mu}; t)$.

Hence, $U(\lambda^{\mu}; t)$ is a HX right ideal of a HX ring $\Re$.

3.3 Theorem

Let λ^{μ} be a fuzzy HX left ideal of a HX ring $\Re$ and $U(\lambda^{\mu}; t)$ is non-empty, then for $t \in [0,1]$, $U(\lambda^{\mu}; t)$ is a HX left ideal of $\Re$.

Proof

Let λ^{μ} be a fuzzy HX left ideal of a HX ring $\Re$.

For any $A,B \in U(\lambda^{\mu}; t)$,we have, $\lambda^{\mu}(A) \geq t$ and $\lambda^{\mu}(B) \geq t$.Now, $\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$\geq \min\{t, t\} = t$, for some $t \in [0,1]$.

$\lambda^{\mu}(A - B) \geq t$.

For any $A \in U(\lambda^{\mu}; t)$ and $B \in \Re$, we have, $\lambda^{\mu}(A) \geq t$.

Now, $\lambda^{\mu}(BA) \geq \lambda^{\mu}(A) \geq t$.

$\lambda^{\mu}(BA) \geq t$

Hence, $A - B$, $BA \in U(\lambda^{\mu}; t)$.

Hence, $U(\lambda^{\mu}; t)$ is a HX left ideal of a HX ring $\Re$.

3.4 Theorem

Let λ^{μ} be a fuzzy HX ideal of a HX ring $\Re$ and $U(\lambda^{\mu}; t)$ is non-empty, then for $t \in [0,1]$, $U(\lambda^{\mu}; t)$ is a HX ideal of $\Re$.

Proof

It is clear.

3.5 Theorem

Let $\Re$ be a HX ring and λ^{μ} be a fuzzy subset of $\Re$ such that $U(\lambda^{\mu}; t)$ is a HX right ideal of $\Re$ for all $t \in [0,1]$, then λ^{μ} is a fuzzy HX right ideal of $\Re$.

Proof

Let $A, B \in \Re$ and let $t_2 < t_1$.

Let $A \in U(\lambda^{\mu}; t_1) \Rightarrow \lambda^{\mu}(A) \geq t_1$

and $B \in U(\lambda^{\mu}; t_2) \Rightarrow \lambda^{\mu}(B) \geq t_2$.

Suppose $U(\lambda^{\mu}; t_1)$, $U(\lambda^{\mu}; t_2) \in \Re$ and

$A, B \in U(\lambda^{\mu}; t_2)$,As $U(\lambda^{\mu}; t_2)$ is a HX right ideal of $\Re$,

$A - B \in U(\lambda^{\mu}; t_2)$ .Then, $\lambda^{\mu}(A - B) \geq t_2$

$= \min\{t_1, t_2\}$

$= \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$.

For any $A \in U(\lambda^{\mu}; t_2)$ and $B \in \Re$, $AB \in U(\lambda^{\mu}; t_2)$. Then, $\lambda^{\mu}(AB) \geq t_2$

$= \lambda^{\mu}(A)$.

$\lambda^{\mu}(AB) \geq \lambda^{\mu}(A)$.

Hence, λ^{μ} is a fuzzy HX right ideal of $\Re$.

3.6 Theorem

Let $\Re$ be a HX ring and λ^{μ} be a fuzzy subset of $\Re$ such that $U(\lambda^{\mu}; t)$ is a HX left ideal of $\Re$ for all $t \in [0,1]$,then λ^{μ} is a fuzzy HX left ideal of $\Re$.

Proof

Let $A, B \in \Re$ and let $t_2 < t_1$.

Let $A \in U(\lambda^{\mu}; t_1) \Rightarrow \lambda^{\mu}(A) \geq t_1$

and $B \in U(\lambda^{\mu}; t_2) \Rightarrow \lambda^{\mu}(B) \geq t_2$.

Suppose $U(\lambda^{\mu}; t_1)$, $U(\lambda^{\mu}; t_2) \in \Re$ and $A, B \in U(\lambda^{\mu}; t_2)$, As $U(\lambda^{\mu}; t_2)$ is a HX left ideal of $\Re$, $A - B \in U(\lambda^{\mu}; t_2)$ .Then, $\lambda^{\mu}(A - B) \geq t_2$

$= \min\{t_1, t_2\}$

$= \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$.

For any $A \in U(\lambda^{\mu}; t_2)$ and $B \in \Re$, $BA \in U(\lambda^{\mu}; t_2)$. Then, $\lambda^{\mu}(BA) \geq t_2 = \lambda^{\mu}(A)$.

$\lambda^{\mu}(BA) \geq \lambda^{\mu}(A)$.

Hence, λ^{μ} is a fuzzy HX left ideal of $\Re$.

3.7 Theorem

Let $\Re$ be a HX ring and λ^{μ} be a fuzzy subset of $\Re$ such that $U(\lambda^{\mu}; t)$ is a HX ideal of $\Re$ for all $t \in [0,1]$ then λ^{μ} is a fuzzy HX ideal of $\Re$.

Proof

It is clear.

3.8 Theorem

A fuzzy subset λ^{μ} of $\Re$ is a fuzzy HX ideal of a HX ring $\Re$ if and only if the level HX subsets $U(\lambda^{\mu}; t)$, $t \in$ Image λ^{μ}, are HX ideals of $\Re$.

Proof

It is clear.

3.9 Theorem

Let λ^{μ} be a fuzzy HX ideal of a HX ring $\Re$. If two level HX ideals, $U(\lambda^{\mu}; t_1)$, $U(\lambda^{\mu}; t_2)$ with $t_1 < t_2$ of λ^{μ} are equal if and only if there is no A in $\Re$ such that $t_1 \leq \lambda^{\mu}(A) < t_2$.

Proof

It is clear.

3.10 Theorem

Any HX ideal H of a HX ring $\Re$ can be realized as a level HX ideal of some fuzzy HX ideal of $\Re$.

Proof

It is clear.

3.11 Remark

As a consequence of the **Theorem 3.9 and 3.10**, the level HX ideals of a fuzzy HX ideal λ^{μ} of a HX ring $\Re$ form a chain. Since $\lambda^{\mu}(Q) \geq \lambda^{\mu}(A)$ for all A in $\Re$ and therefore $U(\lambda^{\mu}; t_0)$, where $\lambda^{\mu}(Q) = t_0$ is the smallest and we have the chain :$\{Q\} = U(\lambda^{\mu}; t_0) \subset U(\lambda^{\mu}; t_1) \subset U(\lambda^{\mu}; t_2) \subset \ldots \subset U(\lambda^{\mu}; t_n) = \Re$, where $t_0 > t_1 > t_2 > \ldots\ldots > t_n$.

4. Homomorphism and Anti Homomorphism of Level Subsets of Fuzzy Hx Ideal

In this section, we introduce the concept of homomorphism and anti homomorphism of level subsets of a fuzzy HX ideal and discuss some of its properties. Throughout this section, $t \in [0,1]$.

4.1 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^{μ} be a fuzzy HX right ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the image of a level HX right ideal $U(\lambda^{\mu}; t)$ of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$ is a level HX right ideal $U(f(\lambda^{\mu}); t)$ of a fuzzy HX right ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

Proof

Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.Let λ^{μ} be a fuzzy HX right ideal of a HX ring $\Re_1$. Clearly, $f(\lambda^{\mu})$ is a fuzzy HX right ideal of a HX ring $\Re_2$.Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$. Let $U(\lambda^{\mu}; t)$ is a level HX right ideal of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$. Choose $t \in [0,1]$ in such a way that $X, Y \in U(\lambda^{\mu}; t)$ and hence $X-Y \in U(\lambda^{\mu}; t)$.Then, $\lambda^{\mu}(X) \geq t$ and $\lambda^{\mu}(Y) \geq t$ and $\lambda^{\mu}(X-Y) \geq t$.For this $t \in [0,1]$, let $X \in U(\lambda^{\mu}; t)$ and $Y \in \Re_1$ then $XY \in U(\lambda^{\mu}; t)$, as $U(\lambda^{\mu}; t)$ is a level HX right ideal of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$.Then, $\lambda^{\mu}(X) \geq t$ and $\lambda^{\mu}(XY) \geq t$. We have to prove that $U(f(\lambda^{\mu}); t)$ is a level HX right ideal of a fuzzy HX right ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$.Let $X, Y \in U(\lambda^{\mu}; t)$ and hence $X-Y \in U(\lambda^{\mu}; t)$. For $f(X), f(Y) \in U(f(\lambda^{\mu}); t)$,

i. $(f(\lambda^{\mu}))(f(X)-f(Y)) = (f(\lambda^{\mu}))(f(X-Y))$,

$= \lambda^{\mu}(X-Y)$

$\geq t$

$(f(\lambda^{\mu}))(f(X)-f(Y)) \geq t.$

$(f(X)-f(Y)) \in U(f(\lambda^{\mu}); t).$

let $X \in U(\lambda^{\mu}; t)$ and $Y \in \Re_1$ then $XY \in U(\lambda^{\mu}; t)$, as U $(\lambda^{\mu}; t)$ is a level HX right ideal of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$.For, $f(X) \in U(f(\lambda^{\mu}); t)$ and $f(Y) \in \Re_2$,

i. $(f(\lambda^{\mu}))(f(X) f(Y)) \geq (f(\lambda^{\mu}))f(X)$

$\geq t$

$(f(\lambda^{\mu}))(f(X)(f(Y)) \geq t.$

$(f(X) f(Y)) \in U(f(\lambda^{\mu}); t).$

Hence, $U(f(\lambda^{\mu}); t)$ is a level HX right ideal of a fuzzy HX right ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

4.2 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^{μ} be a fuzzy HX left ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the image of a level HX left ideal $U(\lambda^{\mu}; t)$ of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$ is a level HX left ideal $U(f(\lambda^{\mu}); t)$ of a fuzzy HX left ideal f (λ^{μ}) of a HX ring $\Re_2$.

Proof

Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.Let λ^{μ} be a fuzzy HX left ideal of a HX ring $\Re_1$. Clearly,f (λ^{μ}) is a fuzzy HX left ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$. Let U $(\lambda^{\mu}; t)$ is a level HX left ideal of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$.Choose $t \in [0,1]$ in such a way that $X, Y \in U(\lambda^{\mu}; t)$ and hence $X-Y \in U(\lambda^{\mu}; t)$.Then, $\lambda^{\mu}(X) \geq t$ and $\lambda^{\mu}(Y) \geq t$ and $\lambda^{\mu}(X-Y) \geq t$.For this $t \in [0,1]$, let $X \in U(\lambda^{\mu}; t)$ and $Y \in \Re_1$ then $YX \in U(\lambda^{\mu}; t)$, as U $(\lambda^{\mu}; t)$ is a level HX left of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$.Then, $\lambda^{\mu}(X) \geq t$ and $\lambda^{\mu}(YX) \geq t$. We have to prove that U (f (λ^{μ}) ; t) is a level HX left ideal of a fuzzy HX left ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$.Let $X, Y \in U(\lambda^{\mu}; t)$ and hence $X-Y \in U(\lambda^{\mu}; t)$. For $f(X), f(Y) \in U(f(\lambda^{\mu}); t)$,

i. $(f(\lambda^{\mu}))(f(X)-f(Y)) = (f(\lambda^{\mu}))(f(X-Y)),$

$= \lambda^{\mu}(X-Y)$

$\geq t$

$(f(\lambda^{\mu}))(f(X)-f(Y)) \geq t.$

$(f(X)-f(Y)) \in U(f(\lambda^{\mu}); t).$

Let $X \in U(\lambda^{\mu}; t)$ and $Y \in \Re_1$ then $YX \in U(\lambda^{\mu}; t)$, as U $(\lambda^{\mu}; t)$ is a level HX left ideal of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$.For, $f(X) \in U(f(\lambda^{\mu}); t)$ and $f(Y) \in \Re_2$,

ii. $(f(\lambda^{\mu}))(f(Y) f(X)) \geq (f(\lambda^{\mu}))f(X)$

$\geq t$

$(f(\lambda^{\mu}))(f(Y)(f(X)) \geq t.$

$(f(Y) f(X)) \in U(f(\lambda^{\mu}); t).$

Hence, $U(f(\lambda^{\mu}); t)$ is a level HX right ideal of a fuzzy HX left ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$.

4.3 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^{μ} be a fuzzy HX ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the image of a level HX ideal $U(\lambda^{\mu}; t)$ of a fuzzy HX ideal λ^{μ} of a HX ring $\Re_1$ is a level HX ideal $U(f(\lambda^{\mu}); t)$ of a fuzzy HX ideal f (λ^{μ}) of a HX ring $\Re_2$.

Proof

It is clear.

4.4 Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^{α} be a fuzzy HX right ideal on $\Re_2$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $U(\eta^{\alpha}; t)$ be a level HX right ideal of a fuzzy HX right ideal η^{α} of a HX ring $\Re_2$ then $U(f^{-1}(\eta^{\alpha}); t)$ is a level HX right ideal of a fuzzy HX right ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Proof

Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.Let η^{α} be a fuzzy HX right ideal of a HX ring $\Re_2$.Clearly, $f^{-1}(\eta^{\alpha})$ is a fuzzy HX right ideal

of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$. Let $U(\eta^\alpha;\ t)$ be a level HX right ideal of a fuzzy HX right ideal η^α of the HX ring $\Re_2$.Let X ,Y $\in \Re_1$ then f(X) ,f(Y) $\in \Re_2$.Choose t $\in$ [0 , 1] in such a way that f(X), f(Y)$\in U(\eta^\alpha;\ t)$ and hence ,f(X)–f(Y) $\in U(\eta^\alpha;\ t)$.Then, $\eta^\alpha(f(X)) \geq t$, $\eta^\alpha(f(Y)) \geq t$ and $\eta^\alpha(f(X)-f(Y)) \geq t$ .For this t $\in$ [0 , 1], f(X) $\in U(\eta^\alpha;\ t)$ and f(Y)$\in \Re_2$ then f(X)f(Y) $\in U(\eta^\alpha;\ t)$, as $U(\eta^\alpha;\ t)$ be a level HX right ideal of a fuzzy HX right ideal η^α of the HX ring $\Re_2$.Then, $\eta^\alpha(f(X)) \geq t$,$\eta^\alpha(f(X)f(Y)) \geq t$.
We have to prove that $U(f^{-1}(\eta^\alpha);\ t)$ is a level HX right ideal of a fuzzy HX right ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$. Now, Let X , Y $\in U(f^{-1}(\eta^\alpha);\ t)$.

i.
$$\begin{aligned}(f^{-1}(\eta^\alpha))(X-Y) &= \eta^\alpha(f(X-Y))\\ &= \eta^\alpha(f(X)-f(Y))\\ &\geq t\\ (f^{-1}(\eta^\alpha))(X-Y) &\geq t\\ X-Y &\in U(f^{-1}(\eta^\alpha);\ t).\end{aligned}$$

Let f(X) $\in U(\eta^\alpha;\ t)$ and f(Y)$\in \Re_2$ then f(X)f(Y)$\in U(\eta^\alpha;\ t)$, as $U(\eta^\alpha;\ t)$ be a level HX right ideal of a fuzzy HX right ideal η^α of the HX ring $\Re_2$.

ii.
$$\begin{aligned}(f^{-1}(\eta^\alpha))(XY) &\geq (f^{-1}(\eta^\alpha))(X)\\ &= \eta^\alpha(f(X))\\ &\geq t\\ (f^{-1}(\eta^\alpha))(XY) &\geq t\\ XY &\in U(f^{-1}(\eta^\alpha);\ t).\end{aligned}$$

Hence, $U(f^{-1}(\eta^\alpha);\ t)$ is a level HX right ideal of a fuzzy HX right ideal of a HX ring $\Re_1$.

4.5 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^α be a fuzzy HX left ideal on $\Re_2$. If f : $\Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $U(\eta^\alpha;\ t)$ be a level HX left ideal of a fuzzy HX left ideal η^α of a HX ring $\Re_2$ then

$U(f^{-1}(\eta^\alpha);\ t)$ is a level HX left ideal of a fuzzy HX left ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$.

Proof

Let R_1 and R_2 be any two rings and f : $\Re_1 \rightarrow \Re_2$ be a homomorphism.Let η^α be a fuzzy HX left ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta^\alpha)$ is a fuzzy HX left ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$. Let $U(\eta^\alpha;\ t)$ be a level HX left ideal of a fuzzy HX left ideal η^α of the HX ring $\Re_2$.Let X ,Y $\in \Re_1$ then f(X) ,f(Y) $\in \Re_2$.Choose t $\in$ [0 , 1] in such a way that f(X), f(Y)$\in U(\eta^\alpha;\ t)$ and hence ,f(X)–f(Y) $\in U(\eta^\alpha;\ t)$. Then, $\eta^\alpha(f(X)) \geq t$, $\eta^\alpha(f(Y)) \geq t$ and $\eta^\alpha(f(X)-f(Y)) \geq t$ .For this t $\in$ [0 , 1], f(X) $\in U(\eta^\alpha;\ t)$ and f(Y)$\in \Re_2$ then f(X)f(Y) $\in U(\eta^\alpha;\ t)$, as $U(\eta^\alpha;\ t)$ be a level HX left ideal of a fuzzy HX left ideal η^α of the HX ring $\Re_2$. Then, $\eta^\alpha(f(X)) \geq t$, $\eta^\alpha(f(X)f(Y)) \geq t$ .We have to prove that $U(f^{-1}(\eta^\alpha);\ t)$ is a level HX left ideal of a fuzzy HX left ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$. Now,

Let X , Y $\in U(f^{-1}(\eta^\alpha);\ t)$.

i.
$$\begin{aligned}(f^{-1}(\eta^\alpha))(X-Y) &= \eta^\alpha(f(X-Y))\\ &= \eta^\alpha(f(X)-f(Y))\\ &\geq t\\ (f^{-1}(\eta^\alpha))(X-Y) &\geq t\\ X-Y &\in U(f^{-1}(\eta^\alpha);\ t).\end{aligned}$$

Let f(X) $\in U(\eta^\alpha;\ t)$ and f(Y)$\in \Re_2$ then f(Y)f(X)$\in U(\eta^\alpha;\ t)$, as $U(\eta^\alpha;\ t)$ be a level HX left ideal of a fuzzy HX left ideal η^α of the HX ring $\Re_2$.

ii.
$$\begin{aligned}(f^{-1}(\eta^\alpha))(YX) &\geq (f^{-1}(\eta^\alpha))(X)\\ &= \eta^\alpha(f(X))\\ &\geq t\\ (f^{-1}(\eta^\alpha))(YX) &\geq t\\ YX &\in U(f^{-1}(\eta^\alpha);\ t).\end{aligned}$$

Hence, $U(f^{-1}(\eta^\alpha);\ t)$ is a level HX left ideal of a fuzzy HX left ideal of a HX ring $\Re_1$.

4.6 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^α be a fuzzy HX ideal on $\Re_2$. If f : $\Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $U(\eta^\alpha;\ t)$ be a level HX ideal of a fuzzy HX ideal η^α of a HX ring $\Re_2$ then $U(f^{-1}(\eta^\alpha);\ t)$ is a level HX ideal of a fuzzy HX ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$.

Proof

It is clear.

4.7 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^μ be a fuzzy HX right ideal on $\Re_1$. If f : $\Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the image of a level HX right ideal $U(\lambda^\mu;\ t)$

of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$ is a level HX left ideal U(f(λ^{μ}); t) of a fuzzy HX left ideal f (λ^{μ}) of a HX ring $\Re_2$.

Proof

Let R_1 and R_2 be any two rings and f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism. Let λ^{μ} be a fuzzy HX right ideal of a HX ring $\Re_1$. Clearly, f (λ^{μ}) is a fuzzy HX left ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$. Let U (λ^{μ} ; t) is a level HX right ideal of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$. Choose t $\in$ [0,1] in such a way that X, Y $\in$ U(λ^{μ} ; t) and hence Y-X $\in$ U(λ^{μ} ; t).Then, λ^{μ}(X) $\geq$ t and λ^{μ}(Y) $\geq$ t and λ^{μ}(Y–X) $\geq$ t.For this t $\in$ [0,1], let X $\in$ U(λ^{μ} ; t) and Y $\in \Re_1$ then XY $\in$ U(λ^{μ} ; t) , as U (λ^{μ} ; t) is a level HX right ideal of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$.Then, λ^{μ}(X) $\geq$ t and λ^{μ}(XY) $\geq$ t. We have to prove that U (f (λ^{μ}) ; t) is a level HX left ideal of a fuzzy HX left ideal f(λ^{μ}) of a HX ring $\Re_2$. Let X, Y $\in$ U(λ^{μ} ; t) and hence X–Y $\in$ U(λ^{μ} ; t).

For, f(X) , f(Y) $\in$ U(f(λ^{μ}) ; t),

i. (f(λ^{μ}))(f(X)–f(Y))= (f(λ^{μ}))(f(Y–X)),
= λ^{μ} (Y–X)
$\geq$ t

(f(λ^{μ})) (f(X)–f(Y))$\geq$ t .

f(X) –f(Y) $\in$ U(f(λ^{μ}); t).

Let X $\in$ U(λ^{μ} ; t) and Y $\in \Re_1$ then XY $\in$ U(λ^{μ} ; t) , as U (λ^{μ} ; t) is a level HX right ideal of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$. For, f(X) $\in$ U(f(λ^{μ}) ; t) and f(Y) $\in \Re_2$,

ii. (f(λ^{μ}))(f(Y) f(X))$\geq$ (f(λ^{μ}))f(X)
$\geq$ t

(f(λ^{μ}))(f(Y)(f(X))$\geq$ t.

f(Y) f(X) $\in$ U(f(λ^{μ}); t).

Hence, U(f(λ^{μ}) ; t) is a level HX left ideal of a fuzzy HX right ideal f(λ^{μ}) of a HX ring $\Re_2$.

4.8 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^{μ} be a fuzzy HX left ideal on $\Re_1$. If f : $\Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the image of a level HX left ideal U(λ^{μ} ; t) of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$ is a level HX right ideal U(f(λ^{μ}); t) of a fuzzy HX right ideal f (λ^{μ}) of a HX ring $\Re_2$.

Proof

Let R_1 and R_2 be any two rings and f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism. Let λ^{μ} be a fuzzy HX left ideal of a HX ring $\Re_1$. Clearly, f (λ^{μ}) is a fuzzy HX right ideal of a HX ring $\Re_2$.Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$. Let U (λ^{μ} ; t) is a level HX left ideal of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$. Choose t $\in$ [0,1] in such a way that X, Y $\in$ U(λ^{μ} ; t) and hence Y–X $\in$ U(λ^{μ} ; t).Then, λ^{μ}(X) $\geq$ t and λ^{μ}(Y) $\geq$ t and λ^{μ} (Y–X) $\geq$ t. For this t $\in$ [0,1], let X $\in$ U(λ^{μ} ; t) and Y $\in \Re_1$ then YX $\in$ U(λ^{μ} ; t) , as U (λ^{μ} ; t) is a level HX left of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$.

Then, λ^{μ}(X) $\geq$ t and λ^{μ}(YX) $\geq$ t. We have to prove that U (f (λ^{μ}) ; t) is a level HX right ideal of a fuzzy HX right ideal f(λ^{μ}) of a HX ring $\Re_2$.Let X, Y $\in$ U(λ^{μ} ; t) and hence X–Y $\in$ U(λ^{μ} ; t). For, f(X) , f(Y) $\in$ U(f(λ^{μ}) ; t),

i. (f(λ^{μ}))(f(X)–f(Y))= (f(λ^{μ}))(f(Y–X)),
= λ^{μ} (Y–X)
$\geq$ t
(f(λ^{μ})) (f(X)–f(Y))$\geq$ t .
f(X) –f(Y) $\in$ U(f(λ^{μ}); t).

Let X $\in$ U(λ^{μ} ; t) and Y $\in \Re_1$ then YX $\in$ U(λ^{μ} ; t) , as U (λ^{μ} ; t) is a level HX left ideal of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$.For, f(X) $\in$ U(f(λ^{μ}) ; t) and f(Y) $\in \Re_2$,

ii. (f(λ^{μ}))(f(X) f(Y))$\geq$ (f(λ^{μ}))f(X)
$\geq$ t
(f(λ^{μ}))(f(X)(f(Y))$\geq$ t.
f(X) f(Y) $\in$ U(f(λ^{μ}); t).

Hence, U(f(λ^{μ}) ; t) is a level HX right ideal of a fuzzy HX right ideal f(λ^{μ}) of a HX ring $\Re_2$.

4.9 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ^{μ} be a fuzzy HX ideal on $\Re_1$. If f : $\Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the image of a level HX ideal U(λ^{μ} ; t) of a fuzzy HX ideal λ^{μ} of a HX ring $\Re_1$ is a level HX ideal U(f(λ^{μ}); t) of a fuzzy HX ideal f (λ^{μ}) of a HX ring $\Re_2$.

Proof

It is clear.

4.10 Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^{α} be a fuzzy HX right ideal on $\Re_2$. If f: $\Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $U(\eta^{\alpha}; t)$ be a level HX right ideal of a fuzzy HX right ideal η^{α} of a HX ring $\Re_2$ then $U(f^{-1}(\eta^{\alpha}); t)$ is a level HX left ideal of a fuzzy HX left ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Proof

Let R_1 and R_2 be any two rings and f : $\Re_1 \rightarrow \Re_2$ be a homomorphism. Let η^{α} be a fuzzy HX right ideal of a HX ring $\Re_2$.Clearly, $f^{-1}(\eta^{\alpha})$ is a fuzzy HX left ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$. Let $U(\eta^{\alpha}; t)$ be a level HX right ideal of a fuzzy HX right ideal η^{α} of the HX ring $\Re_2$.Let X ,Y $\in \Re_1$ then f(X) ,f(Y) $\in \Re_2$.Choose $t \in [0, 1]$ in such a way that f(X), f(Y)$\in U(\eta^{\alpha}; t)$ and hence ,f(Y)–f(X) $\in U(\eta^{\alpha}; t)$.Then, $\eta^{\alpha}(f(X)) \geq t$, $\eta^{\alpha}(f(Y)) \geq t$ and $\eta^{\alpha}(f(Y)-f(X)) \geq t$ .For this $t \in [0, 1]$, f(X) $\in U(\eta^{\alpha}; t)$ and f(Y)$\in \Re_2$ then f(X)f(Y) $\in U(\eta^{\alpha}; t)$, as $U(\eta^{\alpha}; t)$ be a level HX right ideal of a fuzzy HX right ideal η^{α} of the HX ring $\Re_2$.Then, $\eta^{\alpha}(f(X)) \geq t$, $\eta^{\alpha}(f(X)f(Y)) \geq t$.
We have to prove that $U(f^{-1}(\eta^{\alpha}); t)$ is a level HX left ideal of a fuzzy HX left ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.
Now, Let X , Y $\in U(f^{-1}(\eta^{\alpha}); t)$.

i.
$$\begin{aligned}
(f^{-1}(\eta^{\alpha}))(X-Y) &= \eta^{\alpha}(f(X-Y)) \\
&= \eta^{\alpha}(f(Y)-f(X)) \\
&\geq t \\
(f^{-1}(\eta^{\alpha}))(X-Y)) &\geq t \\
X-Y &\in U(f^{-1}(\eta^{\alpha}); t).
\end{aligned}$$

Let f(X) $\in U(\eta^{\alpha}; t)$ and f(Y)$\in \Re_2$ then f(X)f(Y)$\in U(\eta^{\alpha}; t)$, as $U(\eta^{\alpha}; t)$ be a level HX right ideal of a fuzzy HX right ideal η^{α} of the HX ring $\Re_2$.

ii.
$$\begin{aligned}
(f^{-1}(\eta^{\alpha}))(YX) &\geq (f^{-1}(\eta^{\alpha}))(X) \\
&= \eta^{\alpha}(f(X)), \\
&\geq t \\
(f^{-1}(\eta^{\alpha}))(YX) &\geq t \\
YX &\in U(f^{-1}(\eta^{\alpha}); t).
\end{aligned}$$

Hence, $U(f^{-1}(\eta^{\alpha}); t)$ is a level HX left ideal of a fuzzy HX left ideal of a HX ring $\Re_1$.

4.11 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^{α} be a fuzzy HX left ideal on $\Re_2$. If f : $\Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $U(\eta^{\alpha}; t)$ be a level HX left ideal of a fuzzy HX left ideal η^{α} of a HX ring $\Re_2$ then $U(f^{-1}(\eta^{\alpha}); t)$ is a level HX right ideal of a fuzzy HX right ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Proof

It is clear.

4.12 Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η^{α} be a fuzzy HX ideal on $\Re_2$. If f : $\Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $U(\eta^{\alpha}; t)$ be a level HX ideal of a fuzzy HX ideal η^{α} of a HX ring $\Re_2$ then $U(f^{-1}(\eta^{\alpha}); t)$ is a level HX ideal of a fuzzy HX ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$.

Proof

It is clear.

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