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Solving Differential-Algebraic Equations by Adomian Decomposition Method

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ABSTRACT

This paper presents Adomian decomposition method (ADM) for solution of deferential algebraic equations (DAE). We illustrate the method with one example of DAEs systems and series solutions are obtained. The solutions are compared with exact solutions. The numerical results are found to be very accurate when compared with analytical solutions.
Keywords: Adomian Decomposition Method , Deferential Algebraic Equations

I. INTRODUCTION

Adomian decomposition method has been applied to a wide class of problems arising in the field of physics, medicine, engineering, biology and chemical reactions [1], [3], [7], [11]. Recently, a large amount of research works have been devoted to application of ADM for solving a wide variety of non-linear equations including algebraic, deferential, partial deferential, deferential delay and integro-dierential equations [13], [15]. In this paper, we applied the Adomian decomposition method for numerical solution of deferential algebraic equations.

Differential-Algebraic Equations can be found in scientist and engineering applications such as circuit analysis, computer-aided designs, power systems, chemical process simulation and optimal control [2]. In addition, many important mathematical models can be expressed in terms of DAEs [13]. In recent years, DAEs have been solved by implicit Runge-Kutta method [4], Pade Series [12] and a host of others. Some more general approaches for solution of DAEs can be found in [5], [9], and [10].

II. METHODS AND MATERIAL

The Adomian Decomposition Method

Consider a system of deferential equations

$$\left.\begin{aligned} y_1' &= f_1(x, y_1, \ldots, y_n) \\ y_2' &= f_2(x, y_1, \ldots, y_n) \\ &\vdots \\ y_n' &= f_n(x, y_1, \ldots, y_n) \end{aligned}\right\} \qquad (1)$$

Where each equation in the system represents the first derivative of one of the unknown functions as a mapping Depending on the dependent variable x and n unknown functions $f_1, \ldots\ldots f_n$.

In compact form, equation (1) becomes

$$Ly_i = f_1 y_1' = f_1(x, y_1, \ldots, y_n), \quad i = 1, 2, \ldots, n \qquad (2)$$

where L is the linear operator $\frac{d}{dx}$ with the inverse $L^{-1} = \int_0^x (\cdot)dx$. Applying the inverse operator on (2), we get the following canonical form which is suitable for applying Adomian decomposition method

$$y_i = y_i(0) + \int_0^x f_i = (x, y_1, \ldots, y_n)dx, \quad i = 1, 2, \ldots, n \tag{3}$$

The series solution of (3) gives

$$y_i = \sum_{j=0}^{\infty} f_{i,j} \tag{4}$$

And the integrand in (3) as the sum of the following series.

$$f_i = (x_i, y_1, \ldots, y_n) = \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{1,1}, \ldots, f_{i,j}) \tag{5}$$

where $A_(i,j)(f_{i,0}, f_{1,1}, \ldots, f_{i,n})$ are called Adomian polynomials [4]. Substituting (4) and (5) into (3), we obtain

$$\begin{aligned}\sum_{j=0}^{\infty} f_{i,j} &= y_i(0) + \int_0^x A_{i,j}(f_{i,0}, f_{1,1}, \ldots, f_{i,j}) \\ &= y_i(0) + \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{1,1}, \ldots, f_{i,n}) \end{aligned} \tag{6}$$

Using (6), we define

$$\begin{aligned} f_{i,0} &= y_i(0) \\ f_{i,i+1} &= \int_0^x A_{i,n}(f_{i,0}, f_{1,1}, \ldots, f_{i,n})dx, \quad n = 0, 1, 2, 3, \ldots \end{aligned} \tag{7}$$

A WORKED EXAMPLE

We illustrate the Adomian procedure with the aid of the following example.

Consider the following system of deferential-algebraic equations

$$U' - xV' + u - (1 + x)V = 0 \tag{8}$$

(8) with initial condition

$$u(0) = 1, \quad v(0) = 0$$

The analytical solution of (8) is given by

$$\left.\begin{aligned} U(x) &= e^{-x} + \sin x \\ V(x) &= \sin x \end{aligned}\right\} \tag{9}$$

(9) Equation (8) can be written as

$$u' = -u + x\cos x + (1 + x)\sin x \tag{10}$$

(10) Using the inverse operator $L^{-1} = \int_0^x (\cdot)dx$, we have

$$U = 1 + \int_0^x (x\cos x + (1 + x)\sin x dx - \int_0^x u\, dx \tag{11}$$

(11) From the alternate algorithm for computing the Adomian polynomials [7], the Adomian procedure is

$$U_{1,0} = 14x\sin x + \sin x - x\cos x, \quad u_{1n+1} = -\int_0^x u_{1,n}dx \tag{12}$$

After eight steps, we get the Adomian solution

$$\begin{aligned} U'(x) &= 1 - x + 1.5x^2 - 0.166666667x^3 - 0.125x^4 \\ &\quad -0.008333333x^5 + 0.009722222x^6 - 0.000198413x^6 \end{aligned} \tag{13}$$

(13) Using (9) and (13), a comparison of Adomian solution $u^*(x)$) together with exact ($u(x)$) is presented in the table below. The exact error is estimated by $|U^*(x) - u(x)|$.

III. RESULTS AND DISCUSSION

A Comparison of Adomian and Analytical Solutions.

x	$U^*(x)$	$U(x)$	Error
1.0	0.914820752	0.914820752	0.0000
0.2	0.858464438	0.858464409	2.84×10^{-8}
0.3	0.829474279	0.829474106	1.73×10^{-7}
0.4	0.826087348	0.826079786	7.56×10^{-6}
0.5	0.846243451	0.846221057	2.24×10^{-5}
0.6	0.887597286	0.887575784	2.15×10^{-5}
0.7	0.947536405	0.947150403	3.86×10^{-4}
0.8	1.023209698	1.022904982	3.05×10^{-4}
0.9	1.111634257	1.102404253	9.23×10^{-3}
1.0	1.209325640	1.20521564	4.11×10^{-3}

IV. CONCLUSION

We have discussed and applied Adomian decomposition method for solutions of deferential-algebraic equations. The method is very efficient and it yields series solutions that coverage rapidly to analytical solutions.

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