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# Finite Additive Fuzzy Group, Finite Multiplicative Fuzzy Group and Finite Fuzzy Field

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## ABSTRACT

In this paper, we define finite Additive fuzzy group, finite Multiplicative fuzzy group and finite fuzzy field with examples.
**Keywords:** Finite Fuzzy group, Finite Fuzzy Field,

## I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [1], and the notion of fuzzy subgroups was introduced by Rosenfeld [2], who showed how some basic notions of group theory could be extended in an elementary manner to fuzzy groups. The purpose of this paper is to introduce some basic concepts of finite additive fuzzy group, finite multiplicative fuzzy group and finite fuzzy field.

## II. METHODS AND MATERIAL

**2. Preliminaries**

**2.1 Definition 1**. Let $(G,*)$ be an algebraic system, where $*$ is a binary operation. $(G,*)$ is called a group if the fillowing conditions are satisfied:

$(i)$ $*$ is an associative operation.
$(ii)$ There is an identity.
$(ii)$ Every element in $G$ have inverse.

**2.2 Definition 2.** Let $X$ be a non empty set, called universal set ,then a fuzzy set on $X$ is defined as a collection of order of pairs

$A = \{(x, \mu_A(x)) | x \in X\}$

where $\mu_A$ is a function $\mu_A(x): X \to [0,1]$ ,called the membership function.

**2.3 Definition 3.** Let $G$ be a group. A fuzzy set $A$ of $G$ is said to be fuzzy subgroup if it is satisfying the following axioms:

$(i)$ $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$
$(ii) \mu_A(x^{-1}) \geq \mu_A(x)$ for all $x, y \in G$

**2.4 Definition 4.** Let $A$ fuzzy set of a set X, For $t \in [0, 1]$, the level subset of $A$ is the set
$A_t = \{x \in X \,|\mu_A(x) \geq t\}$ **.**This is called a fuzzy level subset of $A$.

**2.5 Definition 5.** Let $A$ be a fuzzy subgroup of a group $G$ . The subgroup $A_t$ of $G$ for $t \in [0, 1]$ such that $t \leq \mu_A(e)$ is called a level subgroup of .

**2.6 Definition 6.** A fuzzy subgroup $A$ of a group $G$ is called fuzzy normal if $\mu_A(xy) = \mu_A(yx)$.

**2.7 Definition 7. Let Gbe a group. A fuzzu subgroup** $\mu$ of G is called normal if $\mu(x) = \mu(y^{-1}xy)$ for all $x, y \in G$

**2.8 Definition 8. Let** $\mu$ be an upper fuzzy subgroup of G for any $x \in G$. The smallest positive integer $n$ such that $\mu(x^n) = \mu(e)$ is called an upper fuzzy order of. If there does not exist such $n$ then $x$ is said to have an infinite upper fuzzy order. We shall denote the upper fuzzy order of $x$ by $O(\mu(x))$

**e.g., let $G = \{e, a, b, ab\}$ be the klein four group and let**

$\mu = \left\{\left(e, \frac{1}{4}\right), \left(a, \frac{3}{4}\right), \left(b, \frac{3}{4}\right), \left(ab, \frac{1}{4}\right)\right\}$ **be an** upper fuzzy subgroup , then $O(\mu(ab)) = 1 \; and \; O(\mu(a)) = 2$

### 3. Finite Additive Fuzzy Group:

Let $G$ be a finite abelian group of order $p$ where $p$ is prime, also $g_0$ is a generator of $G$.

$(G, +) \equiv (Z_p, +_p)$, $Z_p = \{[0], [1], [2], \ldots\ldots [p - 1]\}$

$[m] +_p [n] = [l]$ where $(m + n) - l \equiv 0 \; mod \; p$ ., $(i.e., 0 \leq l \leq p - 1)$

Let $\mu$ be a membership function on $G$ then $\mu$ is said to be additively groupable if

$\mu[0] = 0$, $\mu[g_0] = q$ such that $\mu[mg_0] \leq mq \leq 1$ , $q \leq 1$.

For each $m$ with $1 \leq m \leq p - 1$ , where we have $G = \{0, g_0, 2g_0, 3g_0, \ldots, (p - 1)g_0\}$ with operation of addition, and $pg_0 = 0 \; in +_p$ sense

Next we define modified membership function $\mu^*: G \to [0,1]$ Such that

$$\mu^*(x) = \begin{cases} \mu(m' g_0) & if \; x = mg_0 \; where \; m \equiv m' (mod \; p) \; and \; 1 \leq m' \leq p - 1 \\ 0 & if \; x = npg_0, n \in N \end{cases}$$

Then $(G, \mu^*)$ is said to be additive fuzzy group.
And $\forall x \in G, \mu^*(x) \leq mq \; where \; 0 \leq m \leq p - 1$.

**Example:** Let $(Z_{12}, +_{12})$ be a group then

$$H_1 = \{[0]\}$$
$$H_2 = \{[0], [6]\}$$
$$H_3 = \{[0], [4], [8]\}$$
$$H_4 = \{[0], [3], [6], [9]\}$$
$$H_5 = \{[0], [2], [4], [6], [8], [10]\}$$
$$H_6 = \{[0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]\}$$

Are subgroups ,Also here

$H_1 \subset H_3 \subset H_5 \subset H_6$ and $H_1 \subset H_2 \subset H_4 \subset H_6$

Define $\mu: Z_{12} \to [0,1]$

$$\mu(x) = \begin{cases} 0 & if x = [0] \\ 0.2 & if \; x \in \{[4], [8]\} \\ 0.4 & if \; x \in \{[2], [6], [8]\} \\ 0.5 & if \; x \in \{[1], [3], [5], [7], [9], [11]\} \end{cases}$$

$$\mu^*(x) = \begin{cases} 0 & if x = [0] \\ 0.2 & if \; x \in \{[4], [8]\} \\ 0.4 & if \; x \in \{[2], [6], [8]\} \\ 0.5 & if \; x \in \{[1], [3], [5], [7], [9], [11]\} \end{cases}$$

Thus $(Z_{12}, \mu^*)$ is additive fuzzy group.

### 4. Finite Multiplicative Fuzzy Group:

Let $G = \{0, g_0, 2g_0, 3g_0, \ldots, (p - 1)g_0\}$ ,$p$ is prime with operation of multiplicative modulo $p$, denoted as $p$ then${g_0}^p = e \; in \; \cdot_p$ sense and

$${g_0}^m = \begin{cases} {g_0}^n & , where \; m = tp + n, 1 \leq n \leq p - 1 \\ e & , \quad if \; m = tp \end{cases}$$

Let $\mu: G \to [0,1]$ be a membership function such that

$\mu(e) = 1 \; and \; \mu({g_0}^n) \geq q^n$

where $\mu(g_0) = q \; and \; 1 \leq n \leq p - 1$ here $q < 1$ then $\mu$ is said to be multiplicatively groupable. Then we define modified membership function

$\mu^*: G \to [0,1]$ Such that

$$\mu^*(x) = \begin{cases} \mu({g_0}^n) & if \; x = {g_0}^m \; such \; that \; m = tp + n \; and \; 1 \leq n \leq p - 1 \\ 1 & if \; x = {g_0}^{np}, n \in N \cup \{0\} \end{cases}$$

Then $(G, \mu^*)$ is said to be multiplicative fuzzy group.
And $\forall x \in G, \mu^*(x) \geq q^n where \; 0 \leq n \leq p - 1$.
Where $x = g^m = g^{tp+n} \; and \; \mu^*(x) = \mu^*(g^{tp+n}) = \mu(g^n) \geq q^n$, $(t \in N \cup \{0\} \; and \; 0 \leq n \leq p - 1)$

**Example:** Let $(Z_{11}^{*}, \times_{11})$ be a group then

$$H_1 = \{[1]\}$$
$$H_2 = \{[1],[10]\}$$
$$H_3 = \{[1],[3],[4],[5],[9]\}$$
$$H_4 = \{[1],[2],[3],[4],[5],[6],[7],[8],[9],[10]\}$$

Are subgroups ,Also here

$H_1 \subset H_2 \subset H_4$ and $H_1 \subset H_3 \subset H_4$

Define $\mu: Z_{11}^{*} \rightarrow [0,1]$

$$\mu(x) = \begin{cases} 1 & if x = [1] \\ 0.8 & if\ x \in \{[10]\} \\ 0.6 & if\ x \in \{[2],[3],[4],[5],[6],[7],[8],[9]\} \end{cases}$$

$$\mu^{*}(x) = \begin{cases} 1 & if x = [1] \\ 0.8 & if\ x \in \{[10]\} \\ 0.6 & if\ x \in \{[2],[3],[4],[5],[6],[7],[8],[9]\} \end{cases}$$

Thus $(Z_{11}^{*}, \mu^{*})$ is Multiplicative fuzzy group.

## III. RESULTS AND DISCUSSION

**Finite Fuzzy Field**

Let $F$ be a finite field of the type $Z_p = \{[0],[1],[2],\ldots..[p-1]\}$ under $+_p$ and $\cdot_p$ in the usual sense , let $g$ be a group generator of $Z_p$ with respect to $p$-mod multiplication $\cdot_p$ . see that $g$ Can also be treated as a group generator with respect to $p$-mod addition $+_p$ then for any $x \in F$

We can have two representations of $x$ ; one with respect to $+_p$ , i.e., $x = n_1 g$ and another with respect to $\cdot_p$ i.e., $x = g^{n_2}$, Thus ., $x = n_1 g = g^{n_2}$

i.e., $xg^{-1} = n_1 e = g^{n_1 - 1}$ if $g \neq 0$.

If $\mu: G \rightarrow [0,1]$ be a membership function, then it is algebraic if $q^{n_2} \leq \mu(x) = n_1 q$ , $\forall\, x \in F$ ,

For some $n_1$ and $n_2$ such that $1 \leq n_1, n_2 \leq p-1$ , then $(F, \mu)$ is said to be a finite fuzzy field.

**Theorem:**

Let $F$ be a finite field of the type $Z_p = \{[0],[1],[2],\ldots..[p-1]\}$ under $+_p$ and $\cdot_p$ and $g$ be a group generator of $Z_p$ with respect to $p$-mod multiplication $\cdot_p$ , $\mu(g) = q$ then $q \geq \frac{1}{p-1}$ .

**Proof:** Let $F$ be a finite field of the type $Z_p = \{[0],[1],[2],\ldots..[p-1]\}$,

i.e., $F = \{0,\ g, 2g = g^2, 3g = g^3, \ldots..ng = g^n, \ldots..(p-1)g = g^{p-1}\}$

we must have $(p-1)g = g^{p-1} = e$ because we say that $g$ is a generator of $F$ with respect to $\cdot_p$

Then it means that $g$ is a generator of $F' = \{e, g, g^2, \ldots.., g^{p-2}\}$ and $g^{p-1} = e$

Therefore $\mu\{(p-1)g\} \leq (p-1)q$

$$1 \leq (p-1)q$$
$$(\because\ \mu\{(p-1)g\} = \mu(e) = 1)$$
$$q \geq \frac{1}{p-1} .$$

## IV. CONCLUSION

Finite group, finite field is very useful in real word application. We define finite fuzzy group and finite fuzzy field with possibly maximal different membership values.

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