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Solving Elasto-Static Problem in Plane Stress Case By Meshfree Method

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ABSTRACT

Computational numerical simulation has increasingly become a very important approach for solving complex practical problems in engineering and science. It provides an alternative tool of scientific investigation, instead of carrying out expensive, time-consuming experiments in laboratories. Mesh free (MF) methods are among the breed of numerical analysis technique that are being vigorously developed to avoid the drawbacks that traditional methods like Finite Element method (FEM) possess. The Element Free Galerkin (EFG) method is a meshless method in which only a set of nodes and a description of model's boundary are required to generate the discrete equations. Although it is considered meshless, the EFG utilizes a background mesh to assembly the equations system that describes the problem. In this paper the EFG method is applied to 2-D beam problem and results obtained using MATLAB program are compared with the analytical solution by using Timoshenko Beam Theory.
Keywords : EFG, MLS shape functions, weight functions, Meshfree, Matlab

I. INTRODUCTION

The development of the finite element method (FEM) in the 1950s was one of the most important advances in the field of numerical methods. The FEM is a robust and thoroughly developed method, and hence it is widely used in engineering fields due to its versatility for complex geometry and flexibility for many types of linear and non-linear problems. This mesh based numerical methods (FEM, FDM, CFD etc.) despite of great success; suffer from difficulties in some aspects, which limit their applications in many complex problems such as crack propagation, problems with phase change, large-strain deformations, etc.

The finite element methods are well established and powerful computational techniques which are used for modelling and analysis of physical phenomena in different fields of engineering and applied sciences, but it is with some shortcomings that rely on meshes or elements that are connected together by nodes in a properly predefined manner. The following limitations of FEM are becoming increasingly evident [1]:

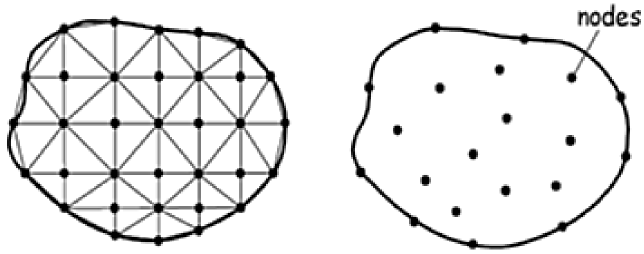
- In stress calculations, the stresses obtained using FEM packages are discontinuous and often less accurate. The need for full compatibility in the assumed displacement field in the FEM results in the loss of freedom in the shape function construction.
- When handling large deformation, considerable accuracy can be lost and the computation can even break down because of element distortions.
- It is rather difficult to simulate both crack growth with arbitrary and complex paths and phase transformations due to discontinuities that do not coincide with the original nodal lines.
- It is very difficult to simulate the breakage of material into a large number of fragments as FEM is essentially based on continuum mechanics, in which the elements formulated cannot be broken. The elements can either be totally ''eroded'' or stay as a whole piece. This usually leads to a misrepresentation of the breakage path. Serious error can occur because the nature of the problem is nonlinear, and therefore the results are highly path dependent.

- Remesh approaches have been proposed for handling these types of problems in FEM. In the remesh approach, the problem domain is remeshed at steps during the simulation process to prevent the severe distortion of meshes and to allow the nodal lines to remain coincident with the discontinuity boundaries. For this purpose, complex, robust, and adaptive mesh generation processors have to be developed. However, these processors are only workable for 2D problems. There are no reliable processors available for creating quality hexahedral meshes for 3D problems due to technical difficulty.
- Adaptive processors require ‘‘mappings’’ of field variables between meshes in successive stages in solving the problem. This mapping process often leads to additional computation as well as a degradation of accuracy. In addition, for large 3D problems, the computational cost of remeshing at each step becomes very high, even if an adaptive scheme is available.
- FDM works very well for a large number of problems, especially for solving fluid dynamics problems. It suffers from a major disadvantage in that it relies on regularly distributed nodes. Therefore, studies have been conducted for a long time to develop methods using irregular grids. Efforts in this direction are still on.

II. METHODS AND MATERIAL

2. Mesh Free Method

A recent strong interest is focused on the next generation computational methods meshfree methods, which are expected to be superior to conventional mesh based FEM in many applications[5]**.** The key idea of the meshfree methods is to provide accurate and stable numerical solutions for integral equations or PDEs with all kinds of possible boundary conditions with a set of arbitrarily distributed nodes (or particles) without using any mesh that provides the connectivity of these nodes or particles.



(a) FEM: nodes and Elements (b) MF: only nodes

Figure 1: Modelling in the FEM and MF

Meshless methods are used to establish systems of algebraic equations for the domain altogether of a problem without a predefined mesh. These methods operate with a set of distributed points inside the domain Ω (fig.1) as well as with sets of points distributed on its boundary to represent (but not discretize) the domain of the problem and its boundary. This set of distributed points does not generate a mesh, meaning that it is not required any information about the relations between these points (Liu, 2004) (Belytschko et al, 1996).

The principal attraction of mesh free methods is the possibility of simplifying adaptivity and problems with moving boundaries and discontinuities, such as phase changes or cracks. In crack growth problems, for example, nodes can be added around a crack tip to capture the stress intensity factors with the desired accuracy; this nodal refinement can be moved with a propagating crack through a background arrangement of nodes associated with the global geometry. Adaptive meshing for a large verity of problems including linear and nonlinear stress analyses can be effectively treated by these methods in a simple manner.

3. Element Free Galerkin Method (EFG):

The Element Free Galerkin (EFG) method proposed by Belytschko et al (1994) is based on the diffuse element method developed by Nayroles et al (1992). In EFG method only a set of points and the description of the model of boundaries are necessary to generate the discrete equations. In EFG we use the moving least square (MLS) method for constructing the shape functions. Moving least square method was first proposed by Lancaster and Salkauskas (1981), as an interpolation method. It was used in element free methods by Belytschko *et al.* (1994), with use of Lagrange multiplier to invoke essential boundary. [2, 3]

3.1 System of Equation:

Consider a displacement function u(x) of a field variable defined on the domain Ω, the approximated value of u(x) can be represented as,

$$u(x) \approx \hat{u}\,(\mathrm{x}) = \sum_{i=1}^{m} p_i(x)a_i(x) = P^T(x)a(x)$$

Where, *P* represents the polynomial basis function, m is the number of polynomial coefficients and *a(x)* is the unknown coefficient matrix.
For 2-D problems,

$P^T(x) = [1, x, y]$ Linear, m=3 and $a^T(x) = [a_0(x) \;\; a_1(x) \;\; a_2(x) \;, \ldots a_m(x)]$

The unknown parameters *a(x)* at any given point are determined by minimizing the difference between the local approximation at that point and the nodal parameters u_i. Let the nodes whose supports include *x* be given local node numbers 1 to *n.* In order to determine the unknown coefficients *a*, a functional *J* is constructed. It sum up the weighted quadratic error for all nodes inside the support domain as

$$J = \sum_{i=1}^{n} W(x - x_i)(\hat{u} - u_i)^2 = \sum_{i=1}^{n} W(x - x_i)(P^T(x_i)a(x) - u_i)^2$$

Where n is the number of nodes in the neighbourhood of *x* for which the weight function, $W(x - x_i) \neq 0$, and u_i refers to the nodal parameter of u at $x = x_i$.

The weights functions like cubic weight function, quartic weight, exponential weight etc, perform two actions, one as a medium of imparting smoothness or desired continuity to the approximation and other one, more important, is the establishment of the local nature of the approximation. The polynomial basis and the weight function together cast a major influence on the performance of the MLS method. Then we want to minimize this functional, so we differentiate with respect to the unknown vector *a(x)*, containing the coefficient,

$$\frac{\partial J}{\partial a} = 0$$

By inserting the expression for *J*, the equation ends up with

$$\frac{\partial J}{\partial a} = \sum_{i=1}^{n} W(x - x_i) \frac{\partial (p^T(x_i)a(x) - u_i)^2}{\partial a}$$

$$= \sum_{i=1}^{n} W(x - x_i) 2(P^T(x_i)a(x) - u_i)\,p(x_i) = 0$$

$$= \sum_{i=1}^{n} W(x - x_i)P(x_i)P^T(x_i)a(x) \;=\; \sum_{i=1}^{n} W(x - x_i)P(x_i)u_i$$

This can be written in a compact matrix form as,

A(x) a(x) = B(x) U(x)

Where the matrices are given by,

$$A(x) = \sum_{n=1}^{n} W(x - x_i)P(x_i)P^T(x_i) \in M(m \times m)$$

$$B(x) = [W(x - x_i)P(x_i) \ldots . W(x - x_n)P(x_n)] \in M(m \times n)$$

$$U(x) = \begin{bmatrix} u_1 \\ . \\ . \\ u_n \end{bmatrix} \in M(n \times 1)$$

The unknown vector *a(x)* can now be determined as,

$$A(x) = A^{-1}(x)\, B(x)\, U(x)$$

By inserting this expression in, we get a new formulation of the displacement field,

$$\hat{u} = P^T(x)a(x) = \underbrace{P^T(x)A^{-1}(x)B(x)}_{\phi(x)}U(x) = u(x) = \sum_{i=1}^{n} \phi_i(x)u_i$$

So the displacement in a point x are approximated as a sum of shape functions multiplied with respectively displacement.

The discrete equation system is obtained by imposition of boundary conditions using Lagrange's multipliers in a weak form of a problem of linear elasticity and by making use of the approximation equations for field variables [2]:

$$\begin{bmatrix} K & G \\ G^T & 0 \end{bmatrix} \begin{bmatrix} U \\ \lambda \end{bmatrix} = \begin{bmatrix} F \\ q \end{bmatrix}$$

Where,

$$K_{IJ} = \int_{\Omega} B_I^T D B_J \, d\Omega$$

$$G_{IK} = -\int_{\Gamma_u} \Phi_t N_K \, d\Gamma$$

$$f_t = \int_{\Gamma t} \Phi_t \bar{t} \, d\Gamma + \int_{\Omega} \Phi_t \bar{t} \, d\Omega$$

$$q_k = -\int_{\Gamma_u} N_k \bar{u} \, d\Gamma$$

$$B_t = \begin{bmatrix} \phi_{I,x} & 0 \\ 0 & \phi_{I,y} \\ \phi_{I,y} & \phi_{I,x} \end{bmatrix}$$

$$N_k = \begin{bmatrix} N_k & 0 \\ 0 & N_k \end{bmatrix}$$

$$D = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{(1-v)}{2} \end{bmatrix}$$

In which, K is the stiffness matrix, **G** is the boundary condition matrix, **U** is the nodal displacements vector, **λ** is the Lagrange multipliers, **F** is the force vector and **q** is a boundary condition vector, and E and v are Young's modulus and Poisson's ratio, respectively.

4. Numerical Examples

In this section, a plane stress Timoshenko beam problem is solved using an EFG program written in MATLAB. This example serves to illustrate the accuracy of the EFG method by comparing it to the exact solution for both the displacements and stresses.

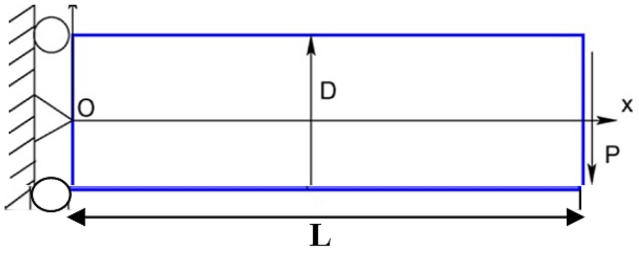


Figure 2 : Timoshenko Beam

Consider a beam of length L = 48 unit subjected to parabolic traction at the free end as shown in figure. The beam has characteristics height D=12 unit and is considered to be of unit depth and is assumed to be in a state of plane stress with P= 1000 unit, ν = 0.3 and E= 3.0 x 107.

The exact analytical solution of Timoshenko beam is given by the following equations [2]. The expressions for displacements in *x* direction, u_x, and in *y* direction, u_y, are respectively:

$$u_x = -\frac{Py}{6EI_m}\left[(6L-3x)x+(2+\nu)\left(y^2-\frac{D^2}{4}\right)\right]$$

$$u_y = -\frac{P}{6EI_m}\left[3\nu y^2(L-x)+(4+5\nu)\frac{D^2x}{4}+(3L-x)x^2\right].$$

Where *P*, is the maximum load applied, *E* is the modulus of elasticity, *x* and *y* are the coordinates in *x* axis and *y* axis for
the analyzed nodal point and I_m is the inertial moment= *D*3/12. The stresses are given by:

$$\sigma_x = -\frac{P(L-x)y}{I_m} \qquad \sigma_y = 0 \qquad \sigma_{xy} = -\frac{P}{2I_m}\left(\frac{D^2}{4}-y^2\right)$$

III. RESULTS AND DISCUSSION

5. Numerical Results

The solutions were obtained using a linear basis function with cubic spline weight function. In this paper, a set of uniform distributed scattered nodes is chosen, and a mesh of background cells is developed only for integration. The displacement and stress values along different section are plotted and comparative performance is evaluated with exact analytical solution.

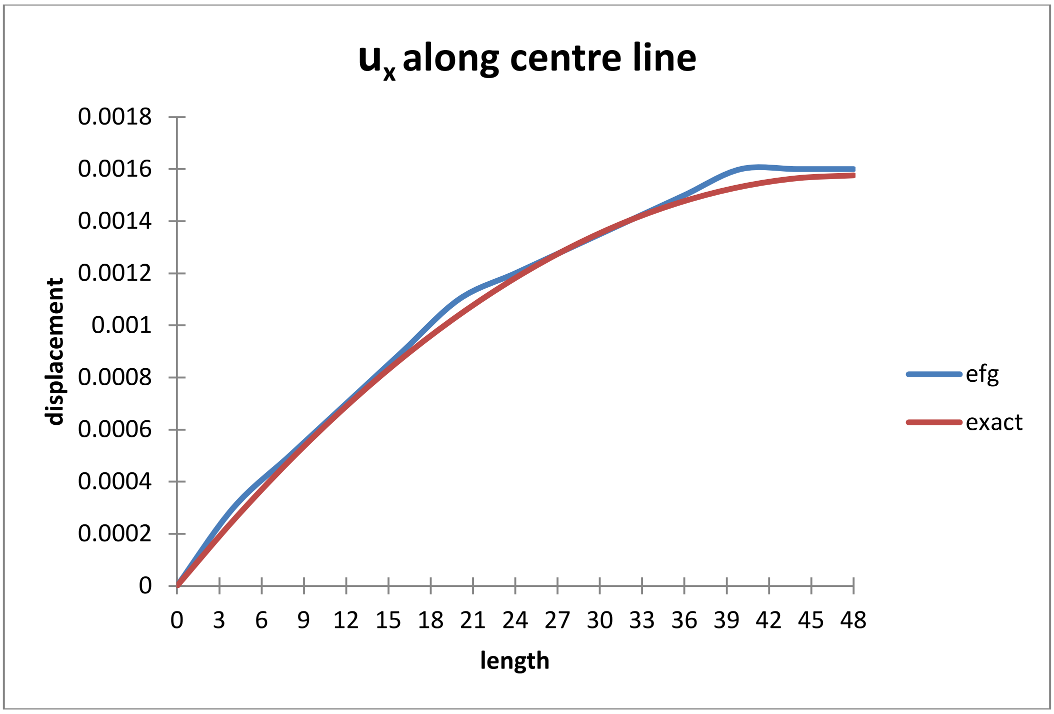


Figure 3 : Displacement for EFG and Exact

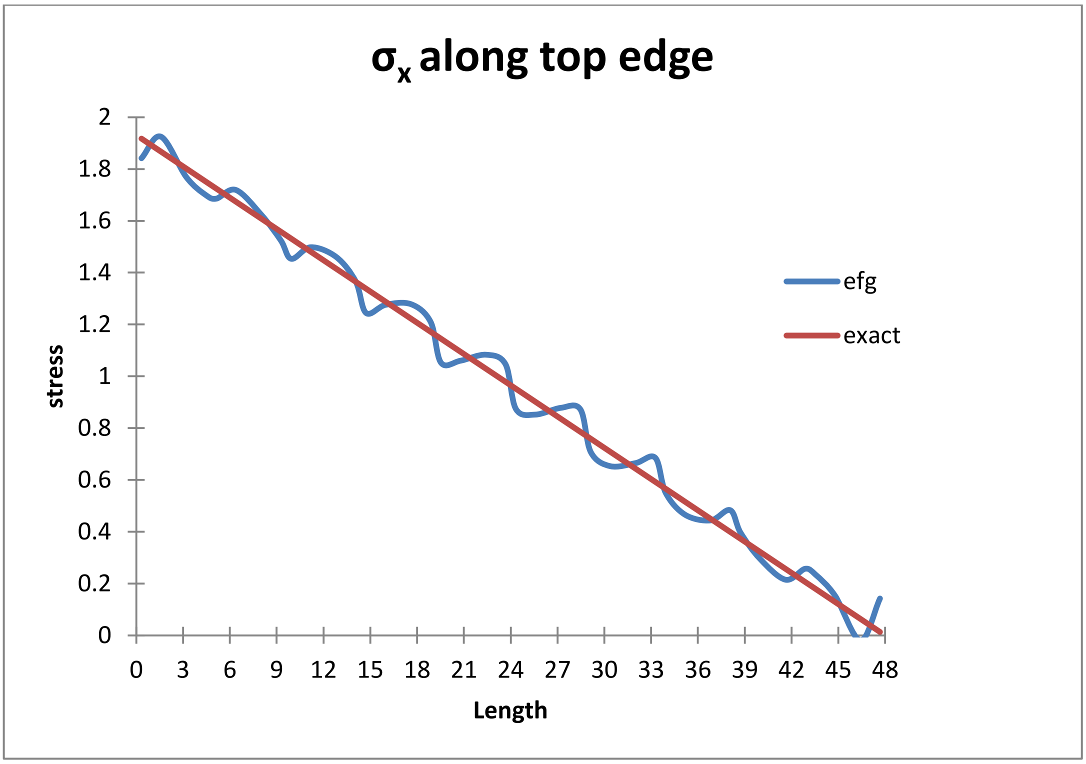


Figure 4 : σ_X for EFG and Exact

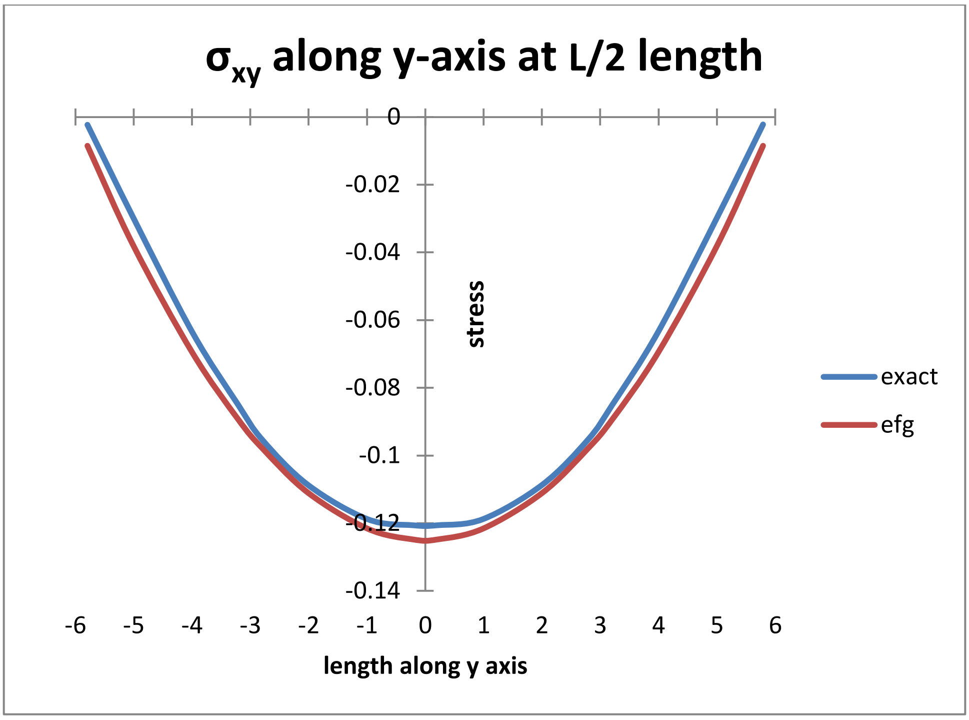


Figure 5 : σ_{xy} for EFG and Exact

IV. CONCLUSION

The details of the Element Free Galerkin (EFG) method and its numerical implementation have been presented for 2-D beam problem. It is observed that EFG method is quite promising in the performance as the results calculated from analytical solution and proposed mesh free method are quite same. We also verify that the results for normal stress and displacement fields are better than the response of the shear stress field because we utilize a linear basis in the approach.

The running time of the two-dimensional EFG program written for this paper is substantially greater than that of a comparable finite element program. However, the potential for meshless methods for certain classes of problems diminishes the importance of these disadvantages.

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