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Optimum Value of Dimensionless Size of the Support Domain for Element Free Galerkin Mesh Free Method

Prof. Sanjaykumar D. Ambaliya*, Prof. Pradip V. Savaliya

Department of Mechanical Engineering, Government Engineering College, Surat, Gujarat, India

ABSTRACT

Finite Element Method (FEM) is an established numerical solution technique for engineering problems in various fields. Mesh free (MF) methods are among the breed of numerical analysis technique that are being vigorously developed to avoid the drawbacks that traditional methods like Finite Element method (FEM) possess. The Element Free Galerkin (EFG) method is a meshless method in which only a set of nodes and a description of model's boundary are required to generate the discrete equations.The aim of this paper is to find optimum value of the dimensionless size of support domain, (d_{max}) for different weight function and nodal distribution. The EFG method is applied to one dimensional structural problem of a bar and results obtained using MATLAB program A comparison of finite element solution has been also performed to compare the accuracy of the solutions.
Keywords: Meshfree, FEM, EFG, weight functions, Dimensionless size of support domain, Matlab

I. INTRODUCTION

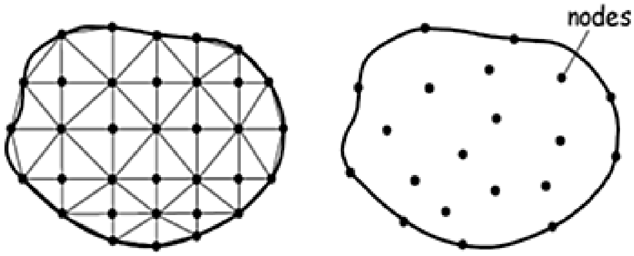
The development of the finite element method (FEM) in the 1950s was one of the most important advances in the field of numerical methods. The FEM is a robust and thoroughly developed method, and hence it is widely used in engineering fields due to its versatility for complex geometry and flexibility for many types of linear and non-linear problems. This mesh based numerical methods (FEM, FDM, CFD etc.) despite of great success; suffer from difficulties in some aspects, which limit their applications in many complex problems such as crack propagation, problems with phase change, large-strain deformations, etc. [1]

In recent years, meshless methods have been developed as alternative numerical approaches in efforts to eliminate known drawbacks of the finite element method (FEM). The main objective in developing meshless methods was to eliminate, or at least reduce, the difficulty of meshing and remeshing of complex structural elements. The nature of the various approximation functions employed by meshless methods allows the descretization or redescretization of problem domains by simply adding or deleting nodes where desired. Nodal connectivity to form an element as in FEM method is not needed, only nodal coordinates and their domain of influence (d_{max}) are necessary to discretize the problem domain. Meshless methods may also reduce other problems associated with the FEM, such as solution degradation due to locking and severe element distortion [1]. There are several meshless methods under current development, including the Element-Free Galerkin (EFG) method proposed by Belytschko, the Reproducing Kernel Particle Method (RKPM) proposed by Liu, Smooth Particle Hydrodynamics (SPH) method proposed by Gingold and Monaghan, Meshless Local Petrov-Galerkin (MLPG) method proposed by Atluri, and some other methods [5]. The well-establish EFG method use shape functions which are derived from moving least square (MLS) approximation.

II. METHODS AND MATERIAL

2. Element Free Galerkin Method (EFG):

The Element Free Galerkin (EFG) method proposed by Belytschko et al (1994) is based on the diffuse element method developed by Nayroles et al (1992). In EFG method only a set of points (node) and the description of the model of boundaries are necessary to generate the discrete equations.



(a) FEM: nodes and elements (b) MF: only nodes

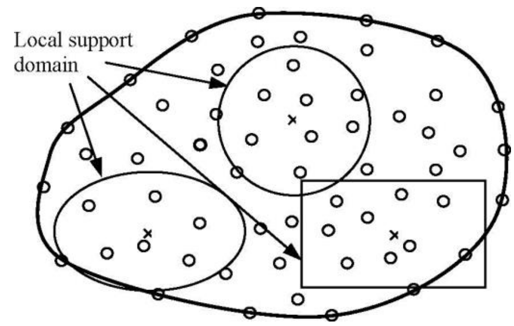
Figure 1: Modelling in the FEM and MF

The Boundary representation in meshfree methods is done only by the arbitrary distribution of the nodes, which may or may not be uniform. Fig. 1 shows the boundary representation for meshfree methods only by the nodes and for FEM using the nodes & elements. The field variables of interest are specified on these nodes.

In EFG we use the moving least square (MLS) method for constructing the shape functions. Moving least square method was first proposed by Lancaster and Salkauskas (1981), as an interpolation method. It was used in element free methods by Belytschko *et al.* (1994), with use of Lagrange multiplier to invoke essential boundary. [2, 3]

3. Support Domains

Because there is no connectivity between the nodes, you have to decide which nodes x_i should influence on the approximation for a point *x*. It is not computationally possible to use all the nodes in Ω, therefore we introduce a very important expression called support domain (Domain of influence). A local support domain of a point *x* determines the number of nodes to be used to support or approximate the function value at *x*. The support domain can have different shapes and its dimension and shape can be different for different points of interest *x*, as shown in Figure 2 they are usually circular or rectangular.



X: point of interest **0**: field node

Figure 2. Support Domain

The dimensions of domain of influence affects the accuracy of the interpolation at the point of interest, therefore the selection of suitable dimension of support domain is very important. To define the support domain for a point *x*, the dimension of the support domain d_s is determined by

$$d_s = d_{\max} d_c$$

Where, d_{max} is the dimensionless size of the support domain, d_c is the characteristic length that relates to the nodal spacing near the point *x*. If the nodes are uniformly distributed d_c is simply the distance between the two neighbouring nodes.

4. System of Equation:

Consider the descretization of the domain 0 < ***x*** < 1 given by a set of evenly spaced nodes. Each node has a corresponding ‘nodal parameter’ or u_i associated with it, which is a parameter governing the function u(x) at that point. For the displacement function u(x) of a field variable, the approximated value $\hat{u}$ (x) can be represented as [2].

$$u(x) \approx \hat{u}(\mathrm{x}) = \sum_{i=1}^{m} p_i(x) a_i(x) = P^T(x) a(x)$$

Where, *P* represents the polynomial basis function, m is the number of polynomial coefficients and *a(x)* is the unknown coefficient matrix.

For 1-D problems, P^T (x) = [1, x], Linear m = 2 and

$$a^T(x) = [a_0(x) \quad a_1(x) \quad a_2(x) \ ,... a_m(x)]$$

In matrix form the shape function *Φ is* obtained as,

$$\phi_I(x) = \sum_{i=1}^{n} P_I(x)(A^{-1}(x)B(x)) = p^T A^{-1} B_I$$

Using shape function, we get a new formulation of the displacement field,

$$\hat{u} = P^T(x)a(x) = \underbrace{P^T(x)A^{-1}(x)B(x)}_{\phi(x)} U(x) = u(x) = \sum_{i=1}^{n} \phi_i(x) u_i$$

So the displacement in a point *x* are approximated as a sum of shape functions multiplied with respectively displacement.

The discrete equation system is obtained by imposition of boundary conditions using Lagrange's multipliers in a weak form of a problem of linear elasticity and by making use of the approximation equations for field variables [2] as follows:

$$\begin{bmatrix} K & G \\ G^T & 0 \end{bmatrix} \begin{Bmatrix} u \\ \lambda \end{Bmatrix} = \begin{Bmatrix} f \\ q \end{Bmatrix}$$

$$K_{IJ} = \int_0^1 \phi_I^T{,_x} E \phi_{J,x} dx$$

$$G_{IK} = -\phi_K \big|_{\Gamma_u I}$$

$$f_I = \phi I t_x \big|_{\Gamma t} = \int_0^1 \phi_I b dx$$

$$q_k = -\bar{u}_k$$

In which, K is the stiffness matrix, **G** is the boundary condition matrix, **u** is the nodal displacements vector, λ is the Lagrange multipliers, **f** is the force vector and **q** is a boundary condition vector, and *E* is Young's modulus.

4.1 Weight Functions

The weights functions like cubic weight function, quartic weight, exponential weight etc, perform two actions, one as a medium of imparting smoothness or desired continuity to the approximation and other one, more important, is the establishment of the local nature of the approximation. The weight functions chosen for construction of shape function are as follows:

$$\text{exponential}: w(\bar{s}) = \begin{Bmatrix} e^{-(\pi/\alpha)^2} & for \bar{s} \le 1 \\ 0, & for \bar{s} > 1 \end{Bmatrix}$$

$$\text{cubicspline}: w(\bar{s}) = \begin{Bmatrix} \frac{2}{3} - 4\bar{s}^2 + 4\bar{s}^3 \; for \bar{s} \le \frac{1}{2} \\ \frac{4}{3} - 4\bar{s} + 4\bar{s}^2 - \frac{4}{3}\bar{s}^3 \; for \frac{1}{2} < \bar{s} \le 1 \\ 0, \qquad for \; \bar{s} \end{Bmatrix}$$

$$\text{Quartic spline: } w(s_I) = \begin{Bmatrix} 1 - 6s_I{}^3 + 8s_I{}^4 - 3s_I{}^4, s_I \le 1 \\ 0, \qquad s_i > 1 \end{Bmatrix}$$

Where, s = $|x-x_I|/dI$ *and, dI* is the radius of influence domain or radius of support domain of the node.

5. Numerical Examples

The Element Free Galerkin method is used for obtaining the displacement parameter at the end of the bar by applying the developed MATLAB code.

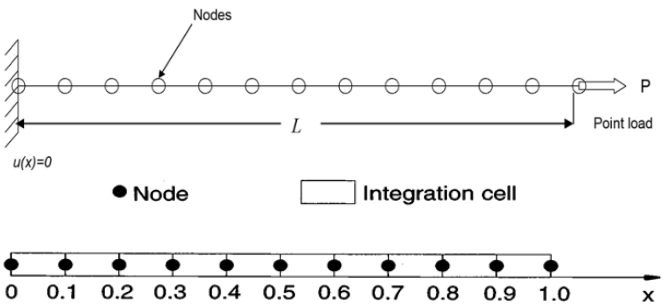


Figure 3. 1-D bars with node and integration cell

The displacement of the bar is fixed at the left end, and the right end is traction free. The bar has a constant cross sectional area of unit value, and modulus of elasticity E. The integration cells are constructed for the one-point gauss-quadrature integration of the nodal discrete equations. The number of integration cells is equal to (Number of nodes -1).

III. RESULTS AND DISCUSSION

Various results for optimum value of domain of influence parameter for end node for different weight function and nodal distribution along the bar are plotted using one point gauss quadrature integration method in MATLAB platform.

Table 1: d_{max} for irregular cubic spline

Sr. No.	1	2	3	4	5	6
d_{max}	2	2.1	2.2	2.3	2.4	2.5
efg	0.3348	0.3342	0.3338	0.3338	0.3343	0.3358
exact	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

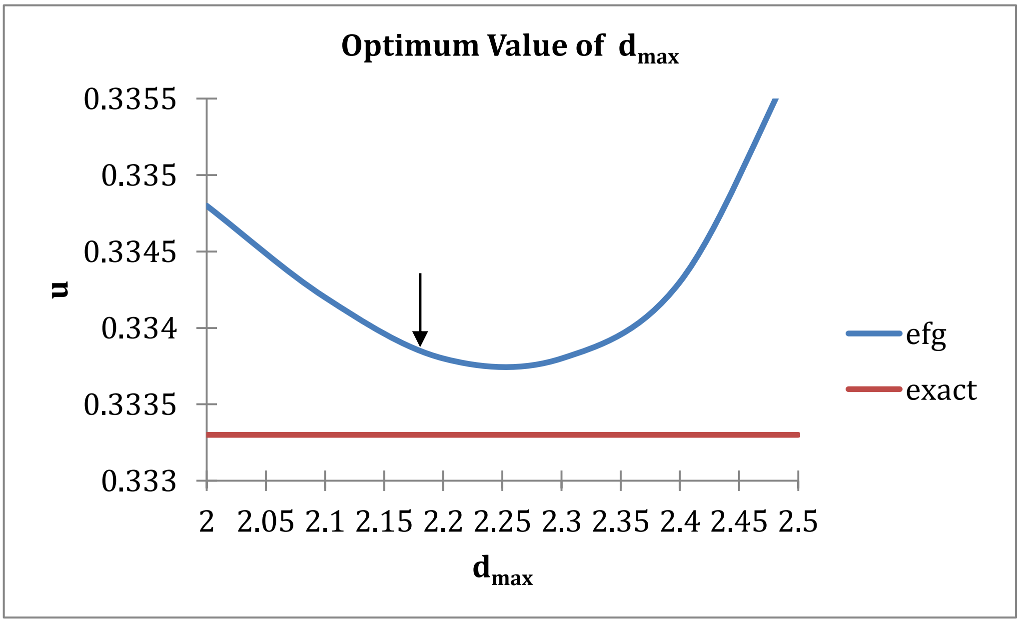


Figure 4: d_{max} for irregular cubic spline

Table 2 : d_{max} for regular cubic spline

Sr. No.	1	2	3	4	5	6	7	8
d_{max}	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7
efg	0.3324	0.3324	0.3324	0.3325	0.3327	0.333	0.3336	0.3345
exact	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

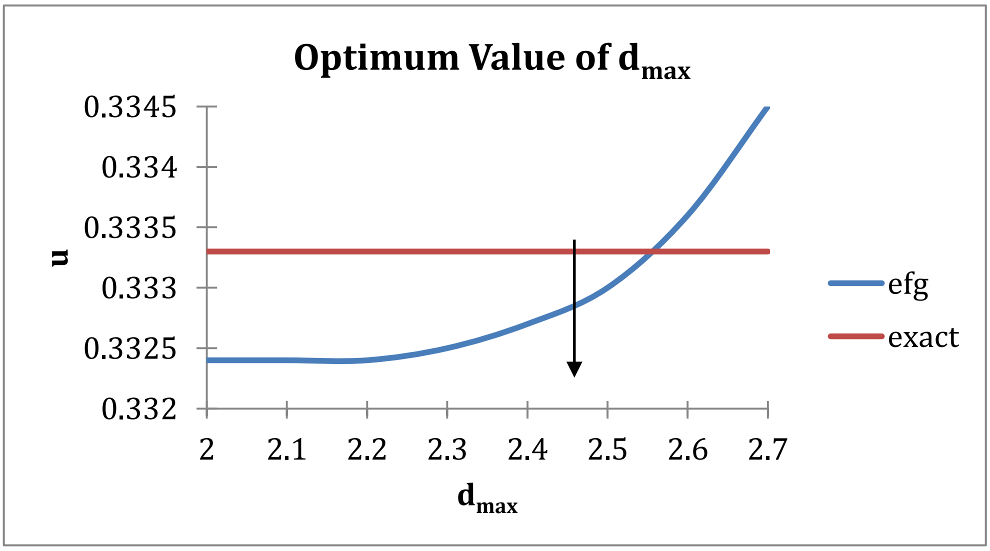


Figure 5 : d_{max} for regular cubic spline

Table 3 : d_{max} for irregular exponential

Sr. No.	1	2	3	4	5	6	7	8
d_{max}	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4
efg	0.3355	0.3352	0.3349	0.3347	0.3347	0.3348	0.3351	0.3357
exact	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

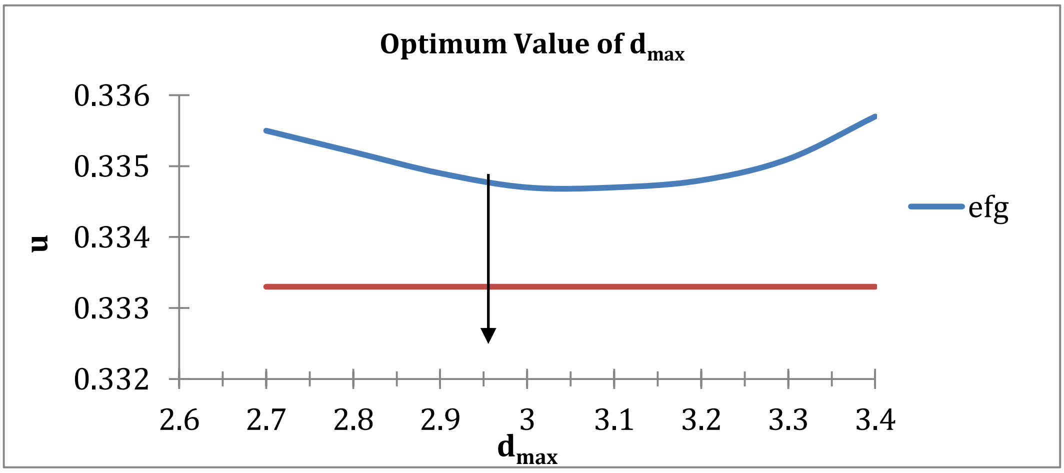


Figure 6: d_{max} for irregular exponential

Table 4: d_{max} for regular exponential

Sr. No.	1	2	3	4	5	6	7	8	9	10	11
d_{max}	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6
efg	0.3324	0.3324	0.3323	0.3323	0.3323	0.3323	0.3324	0.3325	0.3327	0.333	0.3334
exact	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

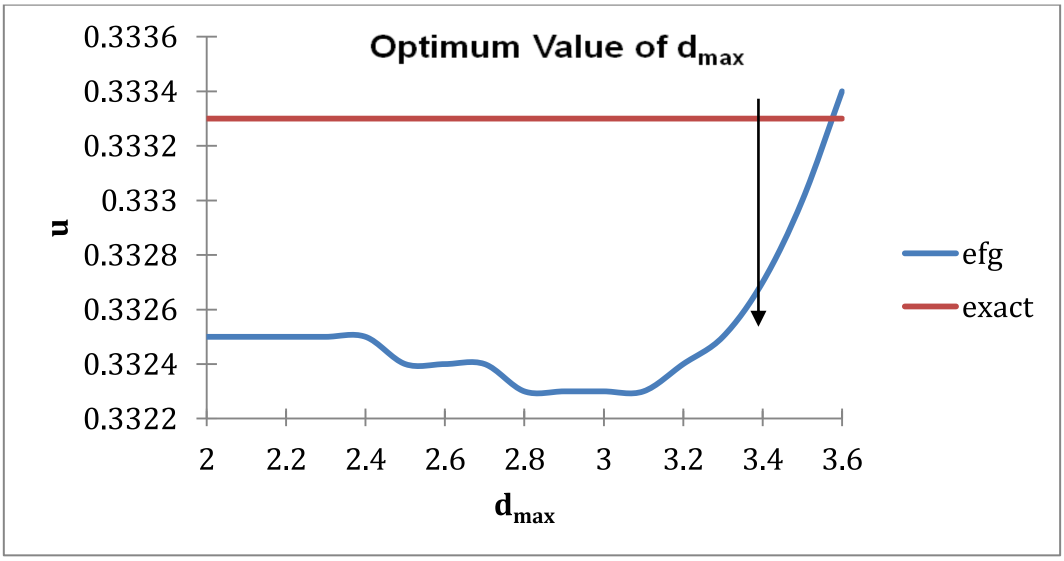


Figure 7: d_{max} for regular exponential

Table 5: d_{max} for irregular quartic spline

Sr. No.	1	2	3	4	5	6
d_{max}	1.8	1.9	2	2.1	2.2	2.3
efg	0.3363	0.3353	0.3342	0.3339	0.3345	0.3368
exact	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

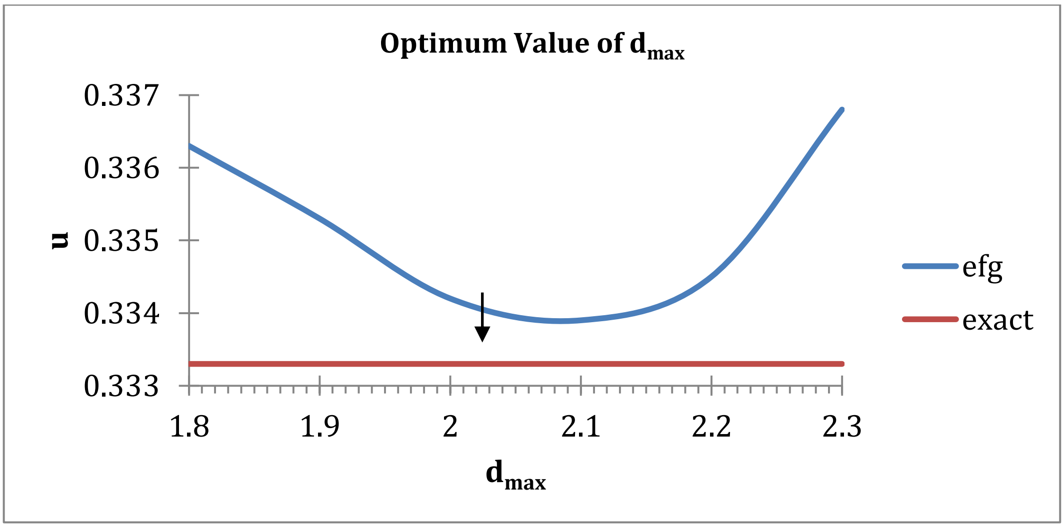


Figure 8: d_{max} for irregular quartic spline

Table 6: d_{max} for regular quartic spline

Sr. No.	1	2	3	4	5	6
d_{max}	2	2.1	2.2	2.3	2.4	2.5
efg	0.3324	0.3325	0.3328	0.3335	0.3345	0.3358
exact	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

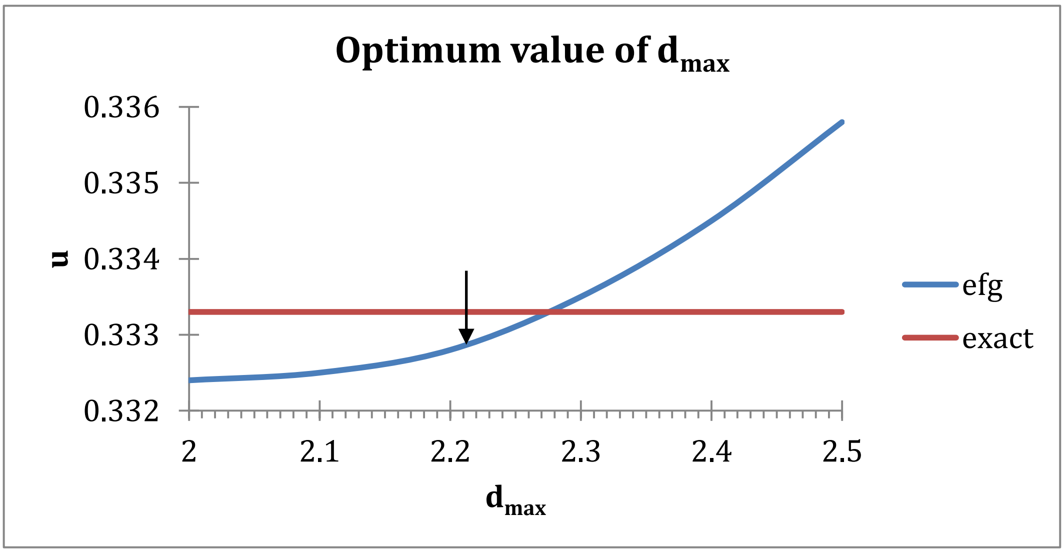


Figure 9: d_{max} for regular quartic spline

IV. CONCLUSION

The effect of support domain d_{max} for 1D problem was studied and the optimum values for various weight functions and regular – irregular nodal distribution were obtained as follows.

For irregular cubic spline 2.25, regular cubic spline 2.55, irregular exponential 3.05, regular exponential 3.5, irregular quartic spline 2.09 and for regular quartic spline it was found 2.27.

From variation of solution for displacement parameter at the end of the bar with respect to d_{max}, it is clear that there is an optimum value for the d_{max} where we can get the best approximation to the exact solution.

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