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A Study on Type – 2 Triangular Mixed Fuzzy Numbers

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ABSTRACT

The concept of a type – 2 fuzzy set which is an extension of the concept of an ordinary fuzzy set was introduced by Zadeh [5]. Type – 2 fuzzy sets possess a great expressive power and are conceptually quite appealing. In this paper, the concept of type – 2 triangular mixed fuzzy numbers (T2TMFNs) is proposed. Also arithmetic operations on T2TMFNs and ranking function of T2TMFNs are defined. Relevant numerical examples are included.
Keywords : Type – 2 fuzzy set, Type – 2 fuzzy number, Type – 2 Triangular fuzzy number.

I. INTRODUCTION

After Zadeh's work [4], uncertainty can be classified in to two types – probabilistic uncertainty and fuzzy uncertainty, through people were aware of fuzzy uncertainty before the mathematical formulation of fuzziness by Zadeh. Fuzziness can be represented in different ways. One of the most useful representations is membership function. Also depending the nature or shape of membership function a fuzzy number can be classified in different ways, such as triangular fuzzy number, trapezoidal fuzzy number etc.

The concept of a type – 2 fuzzy set, an extension of the concept of an ordinary fuzzy set, was introduced by Zadeh[5]. A type – 2 fuzzy set is characterized by a membership function, ie, the membership value for each element of this set is a fuzzy set in [0,1]. The concept of a type – 2 triangular fuzzy number was presented by Stephen Dinagar and Anbalagan[3]. This paper, deals with the another form of type – 2 triangular fuzzy number called type – 2 triangular mixed fuzzy number (T2TMFN). In section -II, some basic definitions are given. In section – III, the concept of type – 2 triangular mixed fuzzy numbers, ranking function, arithmetic operations on type – 2 triangular mixed fuzzy numbers are defined. In section – IV, numerical examples are presented. In section – V, conclusion is included.

II. METHODS AND MATERIAL

1. Preliminaries

A. Definition: (Zadeh) Type-2 fuzzy set

A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0,1].

B. Definition

The type-2 fuzzy sets are defined by functions of the formμ_A: x → χ ([0,1]) where χ ([0,1]) denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1]. An example [2] of a membership function of this type is given in the following figure

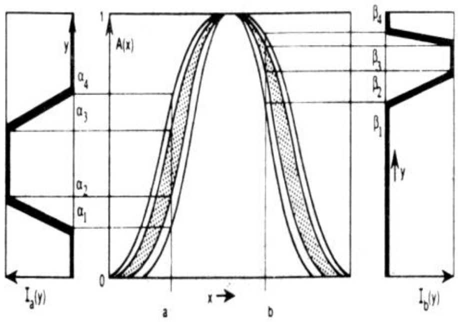

Figure 1. Illustration of the concept of a fuzzy set of type-2.

C. Definition: Type-2 fuzzy number [3]

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

(i) $\tilde{\tilde{A}}$ is normal,

(ii) $\tilde{\tilde{A}}$ is a convex set,

(iii) The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

D. Definition: Type-2 triangular fuzzy number

A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}} = \{(x,(\mu_A^1(x), \mu_A^2(x), \mu_A^3(x)); x \in R\}$ and $\mu_A^1(x) \le \mu_A^2(x) \le \mu_A^3(x)$, for all $x \in R$. Denote $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, where $\tilde{A}_1 = (A_1^L, A_1^N, A_1^U)$, $\tilde{A}_2 = (A_2^L, A_2^N, A_2^U)$ and $\tilde{A}_3 = (A_3^L, A_3^N, A_3^U)$ are same type of fuzzy numbers.

2. Type – 2 triangular mixed fuzzy numbers(T2TMFNs)

E. Notations

$\tilde{A}^2$ = closed interval approximation, $\tilde{A}^3$ = triangular fuzzy number,
$\tilde{A}^4$ = trapezoidal fuzzy number, $\tilde{A}^5$ = piecewise quadratic fuzzy number, etc.,
$\tilde{k}^5 = (k, k, k, k, k)$, where k is constant.

F. Definition

Let $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$ be a type – 2 triangular fuzzy number. Then there may be a choice that $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$ are different types of fuzzy numbers ie. $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$ or $\tilde{\tilde{A}} = (\tilde{A}_1^5, \tilde{A}_2^3, \tilde{A}_3^4)$ or $\tilde{\tilde{A}} = (\tilde{A}_1^5, \tilde{A}_2^5, \tilde{A}_3^4)$ etc. This type of type – 2 triangular fuzzy number is called type – 2 triangular mixed fuzzy numbers.

G. Ranking Function

Let F(R) be the set of all type-2 normal triangular fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function $\mathbb{R}: F(R) \to R$ which maps each fuzzy number into R.

(i) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$, then
$\mathbb{R}(\tilde{\tilde{A}}) = (4\sum_{i=1}^3 \tilde{A}_1^i + 3\sum_{i=1}^4 \tilde{A}_2^i + 6\sum_{i=1}^2 \tilde{A}_3^i)/36$.

(ii) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^5)$, then we define
$\mathbb{R}(\tilde{\tilde{A}}) = (20\sum_{i=1}^3 \tilde{A}_1^i + 15\sum_{i=1}^4 \tilde{A}_2^i + 12\sum_{i=1}^5 \tilde{A}_3^i)/180$.

(iii) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^2, \tilde{A}_3^5)$, then we define
$\mathbb{R}(\tilde{\tilde{A}}) = (10\sum_{i=1}^3 \tilde{A}_1^i + 15\sum_{i=1}^2 \tilde{A}_2^i + 6\sum_{i=1}^5 \tilde{A}_3^i)/90$.

(iv) Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1^4, \tilde{A}_2^4, \tilde{A}_3^6)$, then we define
$\mathbb{R}(\tilde{\tilde{A}}) = (6\sum_{i=1}^4 \tilde{A}_1^i + 6\sum_{i=1}^4 \tilde{A}_2^i + 4\sum_{i=1}^6 \tilde{A}_3^i)/72$, etc.

H. Arithmetic Operations

Let $\tilde{\tilde{A}} = (\tilde{A}_1^3, \tilde{A}_2^4, \tilde{A}_3^2)$ and $\tilde{\tilde{B}} = (\tilde{B}_1^4, \tilde{B}_2^3, \tilde{B}_3^2)$ be two type – 2 triangular mixed fuzzy numbers then

1). Addition and subtraction:

$\tilde{\tilde{A}} \pm \tilde{\tilde{B}} = (\tilde{A}_1^3 \pm \tilde{k}^3, \tilde{A}_2^4 \pm \tilde{k}^4, \tilde{A}_3^2 \pm \tilde{k}^2)$, where $k = (4\sum_{i=1}^4 \tilde{B}_1^i + 3\sum_{i=1}^3 \tilde{B}_2^i + 6\sum_{i=1}^2 \tilde{B}_3^i)/36$.

2). Scalar multiplication:

If $\alpha \ge 0$ and $\alpha \in R$, then
$\alpha\tilde{\tilde{A}} = (\alpha\overrightarrow{\tilde{A}_1^3}, \alpha\overrightarrow{\tilde{A}_2^4}, \alpha\overrightarrow{\tilde{A}_3^2})$, where $\tilde{A}_1^3 = (a, b, c)$ then $\overrightarrow{\tilde{A}_1^3} = (a, b, c)$.
If $\alpha < 0$ and $\alpha \in R$, then
$\alpha\tilde{\tilde{A}} = (\alpha\overleftarrow{\tilde{A}_1^3}, \alpha\overleftarrow{\tilde{A}_2^4}, \alpha\overleftarrow{\tilde{A}_3^2})$, where $\tilde{A}_1^3 = (a, b, c)$ then $\overleftarrow{\tilde{A}_1^3} = (c, b, a)$.

3). Multiplication:

$$\tilde{\tilde{A}} \times \tilde{\tilde{B}} = \left(\frac{k'\overrightarrow{\tilde{A}_1^3}}{36}, \frac{k'\overrightarrow{\tilde{A}_2^4}}{36}, \frac{k'\overrightarrow{\tilde{A}_3^2}}{36}\right), \text{where } k' = (3\sum_{i=1}^4 \tilde{B}_1^i + 4\sum_{i=1}^3 \tilde{B}_2^i + 6\sum_{i=1}^2 \tilde{B}_3^i), k' \ge 0.$$

$$\tilde{\tilde{A}} \times \tilde{\tilde{B}} = \left(\frac{k'\overleftarrow{\tilde{A}_3^2}}{36}, \frac{k'\overleftarrow{\tilde{A}_2^4}}{36}, \frac{k'\overleftarrow{\tilde{A}_1^3}}{36}\right), k' < 0.$$

4). Division:

$$\tilde{\tilde{A}} \div \tilde{\tilde{B}} = \left(\frac{36\overrightarrow{\tilde{A}_1^3}}{k'}, \frac{36\overrightarrow{\tilde{A}_2^4}}{k'}, \frac{36\overrightarrow{\tilde{A}_3^1}}{k'}\right), k' \geq 0.$$

$$\tilde{\tilde{A}} \div \tilde{\tilde{B}} = \left(\frac{36\overleftarrow{\tilde{A}_3^2}}{k'}, \frac{36\overleftarrow{\tilde{A}_2^4}}{k'}, \frac{36\overleftarrow{\tilde{A}_1^3}}{k'}\right), k' < 0$$

III. RESULTS AND DISCUSSION

Numerical example

Example for operations on type – 2 triangular mixed fuzzy numbers

Let $\tilde{\tilde{A}} = ((-1,3,4),[2,4],(2,3,4,7))$ and $\tilde{\tilde{B}} = ([0,2],(0,1,3,4),(2,3,4))$ be two type – 2 triangular mixed fuzzy numbers then

1) $\mathbb{R}(\tilde{\tilde{A}}) = ((-1,3,4),[2,4],(2,3,4,7)) = (4(6) + 6(6) + 3(16))/36$ = (24+36+48)/36 = 3.
2) $\mathbb{R}(\tilde{\tilde{B}}) = \left([0,2],(0,1,3,4),(2,3,4)\right) = (6(2) + 3(8) + 4(9))/36 = 2$
3) $\tilde{\tilde{A}} + \tilde{\tilde{B}} = \left((-1,3,4),[2,4],(2,3,4,7)\right) + ([0,2],(0,1,3,4),(2,3,4))$
   = ((1,5,6),[4,6],(4,5,6,9)).
4) $\tilde{\tilde{A}} - \tilde{\tilde{B}} = \left((-1,3,4),[2,4],(2,3,4,7)\right) - ([0,2],(0,1,3,4),(2,3,4))$
   = ((-3, 1, 2), [0, 2], (0, 1, 2, 5))
5) $\tilde{\tilde{A}} \times \tilde{\tilde{B}} = \left((-1,3,4),[2,4],(2,3,4,7)\right) \times ([0,2],(0,1,3,4),(2,3,4))$
   = ((-2,6,8),[4,8],(4,6,8,14)).
6) $\tilde{\tilde{A}} \div \tilde{\tilde{B}} = \left((-1,3,4),[2,4],(2,3,4,7)\right) \div ([0,2],(0,1,3,4),(2,3,4))$
   = ((-0.5, 1.5, 2), [1, 2], (1, 1.5, 2, 3.5)).

IV. CONCLUSION

In this paper, the definition of type-2 triangular mixed fuzzy number, ranking function, arithmetic operations on type – 2 triangular mixed fuzzy numbers are defined. Type –2 trapezoidal mixed fuzzy numbers, type – 2 piecewise quadratic mixed fuzzy numbers, etc., can be studied in future.

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