© 2016 IJSRSET | Volume 2 | Issue 5 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section: Engineering and Technology

Some Special Types of Type-2 Triangular Fuzzy Number

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ABSTRACT

Type-2 fuzzy sets are fuzzy sets whose membership values are fuzzy sets on the interval [0, 1]. Zadeh, as an extension of fuzzy sets, proposed this concept. Type-2 fuzzy sets possess a great expressive power and are conceptually quite appealing. In this paper, some special types of type-2 triangular fuzzy number are proposed. Arithmetic operations and numerical examples are also included.
Keywords: Triangular fuzzy number, Trapezoidal fuzzy number, Pentagonal fuzzy number, Type-2 fuzzy set, Type-2 fuzzy number, Type-2 triangular fuzzy number.

I. INTRODUCTION

Uncertainty is an attribute of information. For systems being controlled using the type-1 fuzzy logic systems, such uncertainty leads to fuzzy rules whose antecedents or consequents are uncertain, which in turn translates into uncertain antecedent or consequent membership functions. Type-1 fuzzy sets are not able to directly model such uncertainties because their membership functions are totally crisp. On the other hand, type-2 fuzzy sets are able to model such uncertainties because their membership functions are themselves fuzzy. Membership functions of type-1 fuzzy sets are two-dimensional, whereas membership functions of type-2 fuzzy sets are three-dimensional. It is the new third-dimension of type-2 fuzzy sets that provides additional degrees of freedom that make it possible to directly model uncertainties. Type-2 fuzzy sets are difficult to understand and use because: (1) the three-dimensional nature of type-2 fuzzy sets makes them very difficult to draw; (2) there is no simple collection of well-defined terms that let us effectively communicate about type-2 fuzzy sets, and then be mathematically precise about them; and (3) using type-2 fuzzy sets is computationally more complicated than using type-1 fuzzy sets.

The concept of a type-2 fuzzy set, which is an extension of the concept of an ordinary fuzzy set, was introduced by Zadeh [8]. A fuzzy relation of higher type has been regarded as one way to increase the fuzziness of a relation and, according to Hisdal [1], “Increased fuzziness in a description means increased ability to handle in exact information in a logically correct manner”. According to Jhon [2], “Type-2 fuzzy sets allow for linguistic grades of membership, thus assisting in knowledge representation, and they also offer improvement on inferring with type-2 fuzzy sets”. Type-2 fuzzy sets have already been used in a number of applications. Stephen Dinagar and Anbalagan [4] presented new ranking function and arithmetic operations on generalized type-2 trapezoidal fuzzy numbers. Stephen Dinagar and Latha [5] introduced type-2 triangular fuzzy matrices.

This paper is organized as follows. In section-II, some basic definitions are given. In section-III, the definition of some special types of type-2 triangular fuzzy number, proposed ranking function and arithmetic operations on some special types of type-2 triangular fuzzy number are presented. In section-IV, relevant numerical examples are presented. In section-V, conclusion is also included.

II. METHODS AND MATERIAL

1. Preliminaries

A. Definition: Fuzzy Set

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set A in a universe of discourse X is defined as the following set of pairs:

$$A = \{(x, \mu_A(x)); x \in X\}.$$

Here μ_A: $X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x ϵ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

B. Definition: Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one x ϵ X such that $\mu_A(x) = 1$.

C. Definition: Triangular fuzzy number

For a triangular fuzzy number A(x), it can be represented by A(a,b,c;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \frac{c-x}{c-b}, & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

D. Definition: Trapezoidal fuzzy number

For a trapezoidal fuzzy number A(x), it can be represented by A(a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{for } c \leq x \leq d \\ 0 & \text{for } x > d \end{cases}$$

E. Definition: Pentagonal fuzzy number

For a pentagonal fuzzy number A(x), it can be represented by A(a,b,c,d,e;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ \frac{x-b}{c-b} & \text{for } b \leq x \leq c \\ 1 & \text{for } x = c \\ \frac{d-x}{d-c} & \text{for } c \leq x \leq d \\ \frac{e-x}{e-d} & \text{for } d \leq x \leq e \\ 0 & \text{for } x > e \end{cases}$$

F. Definition: (Zadeh) Type-2 fuzzy set

A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0,1].

G. Pictorial Representation:

The type-2 fuzzy sets are defined by functions of the form μ_A: $x \rightarrow \chi([0,1])$ where $\chi([0,1])$ denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1]. An example [3] of a membership function of this type is given in the following figure.

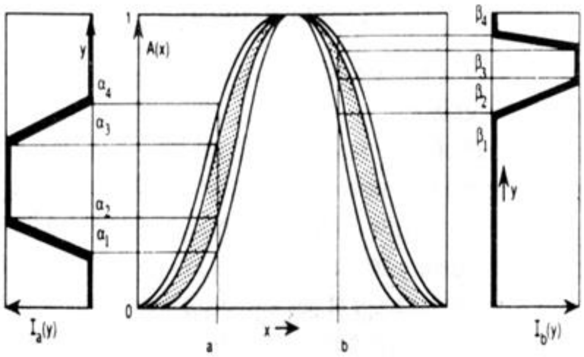

Figure 2. Illustration of the concept of a fuzzy set of type-2.

H. Definition: Type-2 fuzzy number [4]

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

(i) $\tilde{\tilde{A}}$ is normal,

(ii) $\tilde{\tilde{A}}$ is a convex set,

(iii) The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

I. Definition: Type-2 triangular fuzzy number

A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}} = \{(x,(\mu_A^1(x),\mu_A^2(x),\mu_A^3(x)); x \in R\}$ and $\mu_A^1(x) \leq \mu_A^2(x)$

$\leq \mu_A^3(x)$, for all $x \epsilon R$. Denote $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, where $\tilde{A}_1 = (A_1^L, A_1^N, A_1^R)$, $\tilde{A}_2 = (A_2^L, A_2^N, A_2^R)$ and $\tilde{A}_3 = (A_3^L, A_3^N, A_3^R)$ are same type of fuzzy numbers.

J. Definition: Type-2 trap-triangular fuzzy number

A type-2 trap-triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}} = \{(x, (\mu_A^1(x), \mu_A^2(x), \mu_A^3(x)); x \epsilon R\}$ and $\mu_A^1(x) \leq \mu_A^2(x) \leq \mu_A^3(x)$, for all $x \epsilon R$. Denote $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, where $\tilde{A}_1 = (A_1^L, A_1^M, A_1^N, A_1^R)$, $\tilde{A}_2 = (A_2^L, A_2^M, A_2^N, A_2^R)$ and $\tilde{A}_3 = (A_3^L, A_3^M, A_3^N, A_3^R)$ are same type of trapezoidal fuzzy numbers.

2. Special Types of Type-2 Triangular Fuzzy Number

K. Arithmetic operations on type-2 trap-triangular fuzzy numbers

Let $\tilde{\tilde{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) = ((a_1^L, a_1^M, a_1^N, a_1^R), (a_2^L, a_2^M, a_2^N, a_2^R), (a_3^L, a_3^M, a_3^N, a_3^R))$ and $\tilde{\tilde{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = ((b_1^L, b_1^M, b_1^N, b_1^R), (b_2^L, b_2^M, b_2^N, b_2^R), (b_3^L, b_3^M, b_3^N, b_3^R))$ be two type-2 trap-triangular fuzzy numbers. Then we define,

1) Addition:

$\tilde{\tilde{a}} + \tilde{\tilde{b}} = ((a_1^L + b_1^L, a_1^M + b_1^M, a_1^N + b_1^N, a_1^R + b_1^R), (a_2^L + b_2^L, a_2^M + b_2^M, a_2^N + b_2^N, a_2^R + b_2^R), (a_3^L + b_3^L, a_3^M + b_3^M, a_3^N + b_3^N, a_3^R + b_3^R))$

2) Subtraction:

$\tilde{\tilde{a}} - \tilde{\tilde{b}} = ((a_1^L - b_3^R, a_1^M - b_3^N, a_1^N - b_3^M, a_1^R - b_3^L), (a_2^L - b_2^R, a_2^M - b_2^N, a_2^N - b_2^M, a_2^R - b_2^L), (a_3^L - b_1^R, a_3^M - b_1^N, a_3^N - b_1^M, a_3^R - b_1^L))$

3) Scalar multiplication:

If $k \geq 0$ and $k \epsilon R$ then

$k\tilde{\tilde{a}} = ((ka_1^L, ka_1^M, ka_1^N, ka_1^R), (ka_2^L, ka_2^M, ka_2^N, ka_2^R), (ka_3^L, ka_3^M, ka_3^N, ka_3^R))$

and if $k < 0$ and $k \epsilon R$ then

$k\tilde{\tilde{a}} = ((ka_3^R, ka_3^N, ka_3^M, ka_3^L), (ka_2^R, ka_2^N, ka_2^M, ka_2^L), (ka_1^R, ka_1^N, ka_1^M, ka_1^L))$

4) Multiplication:

Define $\sigma b = b_1^L + b_1^M + b_1^N + b_1^R + b_2^L + b_2^M + b_2^N + b_2^R + b_3^L + b_3^M + b_3^N + b_3^R$.

If $\sigma b \geq 0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_1^L\sigma b}{12}, \frac{a_1^M\sigma b}{12}, \frac{a_1^N\sigma b}{12}, \frac{a_1^R\sigma b}{12}\right), \left(\frac{a_2^L\sigma b}{12}, \frac{a_2^M\sigma b}{12}, \frac{a_2^N\sigma b}{12}, \frac{a_2^R\sigma b}{12}\right), \left(\frac{a_3^L\sigma b}{12}, \frac{a_3^M\sigma b}{12}, \frac{a_3^N\sigma b}{12}, \frac{a_3^R\sigma b}{12}\right)\right)$$

If $\sigma b < 0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_3^R\sigma b}{12}, \frac{a_3^N\sigma b}{12}, \frac{a_3^M\sigma b}{12}, \frac{a_3^L\sigma b}{12}\right), \left(\frac{a_2^R\sigma b}{12}, \frac{a_2^N\sigma b}{12}, \frac{a_2^M\sigma b}{12}, \frac{a_2^L\sigma b}{12}\right), \left(\frac{a_1^R\sigma b}{12}, \frac{a_1^N\sigma b}{12}, \frac{a_1^M\sigma b}{12}, \frac{a_1^L\sigma b}{12}\right)\right)$$

5) Division:

Whenever $\sigma b \neq 0$ we define division as follows:

If $\sigma b > 0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{12a_1^L}{\sigma b}, \frac{12a_1^M}{\sigma b}, \frac{12a_1^N}{\sigma b}, \frac{12a_1^R}{\sigma b}\right), \left(\frac{12a_2^L}{\sigma b}, \frac{12a_2^M}{\sigma b}, \frac{12a_2^N}{\sigma b}, \frac{12a_2^R}{\sigma b}\right), \left(\frac{12a_3^L}{\sigma b}, \frac{12a_3^M}{\sigma b}, \frac{12a_3^N}{\sigma b}, \frac{12a_3^R}{\sigma b}\right)\right)$$

If $\sigma b < 0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{12a_3^R}{\sigma b}, \frac{12a_3^N}{\sigma b}, \frac{12a_3^M}{\sigma b}, \frac{12a_3^L}{\sigma b}\right), \left(\frac{12a_2^R}{\sigma b}, \frac{12a_2^N}{\sigma b}, \frac{12a_2^M}{\sigma b}, \frac{12a_2^L}{\sigma b}\right), \left(\frac{12a_1^R}{\sigma b}, \frac{12a_1^N}{\sigma b}, \frac{12a_1^M}{\sigma b}, \frac{12a_1^L}{\sigma b}\right)\right)$$

L. The proposed ranking function

Let F(R) be the set of all type-2 normal trap-triangular fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function $\check{R}: F(R) \rightarrow R$ which maps each fuzzy number into R.

Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, $= ((A_1^L, A_1^M, A_1^N, A_1^R), (A_2^L, A_2^M, A_2^N, A_2^R), (A_3^L, A_3^M, A_3^N, A_3^R))$ then we define $\check{R}(\tilde{\tilde{A}}) = (A_1^L + A_1^M + A_1^N + A_1^R + A_2^L + A_2^M + A_2^N + A_2^R + A_3^L + A_3^M + A_3^N + A_3^R) / 12$.

Also we define orders on F(R) by

$$\check{R}(\tilde{\tilde{A}}) \geq \check{R}(\tilde{\tilde{B}}) \text{ if and only if } \tilde{\tilde{A}} \gtrsim_{\check{R}} \tilde{\tilde{B}},$$
$$\check{R}(\tilde{\tilde{A}}) \leq \check{R}(\tilde{\tilde{B}}) \text{ if and only if } \tilde{\tilde{A}} \lesssim_{\check{R}} \tilde{\tilde{B}}$$
$$\text{and } \check{R}(\tilde{\tilde{A}}) = \check{R}(\tilde{\tilde{B}}) \text{ if and only if } \tilde{\tilde{A}} \approx_{\check{R}} \tilde{\tilde{B}}.$$

M. Definition: Type-2 pent-triangular fuzzy number

A type-2 pent-triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}} = \{(x, (\mu_A^1(x), \mu_A^2(x), \mu_A^3(x)); x \epsilon R\}$ and $\mu_A^1(x) \leq \mu_A^2(x) \leq \mu_A^3(x)$, for all $x \epsilon R$. Denote $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, where $\tilde{A}_1 = (A_1^L, A_1^M, A_1^N, A_1^O, A_1^R)$, $\tilde{A}_2 = (A_2^L, A_2^M, A_2^N, A_2^O, A_2^R)$ and $\tilde{A}_3 = (A_3^L, A_3^M, A_3^N, A_3^O, A_3^R)$ are same type of pentagonal fuzzy numbers.

N. Arithmetic operations on type-2 pent-triangular fuzzy numbers

Let $\tilde{\tilde{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) = ((a_1^L, a_1^M, a_1^N, a_1^O, a_1^R), (a_2^L, a_2^M, a_2^N, a_2^O, a_2^R), (a_3^L, a_3^M, a_3^N, a_3^O, a_3^R))$ and $\tilde{\tilde{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = ((b_1^L, b_1^M, b_1^N, b_1^O, b_1^R), (b_2^L, b_2^M, b_2^N, b_2^O, b_2^R), (b_3^L, b_3^M, b_3^N, b_3^O, b_3^R))$ be two type-2 pent-triangular fuzzy numbers. Then we define,

1) Addition:

$\tilde{\tilde{a}} + \tilde{\tilde{b}} = ((a_1^L + b_1^L, a_1^M + b_1^M, a_1^N + b_1^N, a_1^O + b_1^O, a_1^R + b_1^R), (a_2^L + b_2^L, a_2^M + b_2^M, a_2^N + b_2^N, a_2^O + b_2^O, a_2^R + b_2^R), (a_3^L + b_3^L, a_3^M + b_3^M, a_3^N + b_3^N, a_3^O + b_3^O, a_3^R+b_3^R))$

2) Subtraction:

$\tilde{\tilde{a}} - \tilde{\tilde{b}} = ((a_1^L - b_3^R, a_1^M - b_3^O, a_1^N - b_3^N, a_1^O - b_3^M, a_1^R - b_3^L), (a_2^L - b_2^R, a_2^M - b_2^O, a_2^N - b_2^N, a_2^O - b_2^M, a_2^R - b_2^L), (a_3^L - b_1^R, a_3^M - b_1^O, a_3^N - b_1^N, a_3^O - b_1^M, a_3^R - b_1^L))$

3) Scalar multiplication:

If $k \geq 0$ and $k \in R$ then
$k\tilde{\tilde{a}} = ((ka_1^L, ka_1^M, ka_1^N, ka_1^O, ka_1^R), (ka_2^L, ka_2^M, ka_2^N, ka_2^O, ka_2^R), (ka_3^L, ka_3^M, ka_3^N, ka_3^O, ka_3^R))$
and if $k < 0$ and $k \in R$ then
$k\tilde{\tilde{a}} = ((ka_3^R, ka_3^O, ka_3^N, ka_3^M, ka_3^L), (ka_2^R, ka_2^O, ka_2^N, ka_2^M, ka_2^L), (ka_1^R, ka_1^O, ka_1^N, ka_1^M, ka_1^L))$

4) Multiplication:

Define $\sigma b = b_1^L+b_1^M+b_1^N+b_1^O+b_1^R+b_2^L+b_2^M+b_2^N+b_2^O+b_2^R+b_3^L+b_3^M+b_3^N+b_3^O+b_3^R$.
If $\sigma b \geq 0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_1^L\sigma b}{15}, \frac{a_1^M\sigma b}{15}, \frac{a_1^N\sigma b}{15}, \frac{a_1^O\sigma b}{15}, \frac{a_1^R\sigma b}{15}\right), \left(\frac{a_2^L\sigma b}{15}, \frac{a_2^M\sigma b}{15}, \frac{a_2^N\sigma b}{15}, \frac{a_2^O\sigma b}{15}, \frac{a_2^R\sigma b}{15}\right), \left(\frac{a_3^L\sigma b}{15}, \frac{a_3^M\sigma b}{15}, \frac{a_3^N\sigma b}{15}, \frac{a_3^O\sigma b}{15}, \frac{a_3^R\sigma b}{15}\right)\right)$$

If $\sigma b<0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_3^R\sigma b}{15}, \frac{a_3^O\sigma b}{15}, \frac{a_3^N\sigma b}{15}, \frac{a_3^M\sigma b}{15}, \frac{a_3^L\sigma b}{15}\right), \left(\frac{a_2^R\sigma b}{15}, \frac{a_2^O\sigma b}{15}, \frac{a_2^N\sigma b}{15}, \frac{a_2^M\sigma b}{15}, \frac{a_2^L\sigma b}{15}\right), \left(\frac{a_1^R\sigma b}{15}, \frac{a_1^O\sigma b}{15}, \frac{a_1^N\sigma b}{15}, \frac{a_1^M\sigma b}{15}, \frac{a_1^L\sigma b}{15}\right)\right)$$

5) Division:

Whenever $\sigma b \neq 0$ we define division as follows:
If $\sigma b>0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{15a_1^L}{\sigma b}, \frac{15a_1^M}{\sigma b}, \frac{15a_1^N}{\sigma b}, \frac{15a_1^O}{\sigma b}, \frac{15a_1^R}{\sigma b}\right), \left(\frac{15a_2^L}{\sigma b}, \frac{15a_2^M}{\sigma b}, \frac{15a_2^N}{\sigma b}, \frac{15a_2^O}{\sigma b}, \frac{15a_2^R}{\sigma b}\right), \left(\frac{15a_3^L}{\sigma b}, \frac{15a_3^M}{\sigma b}, \frac{15a_3^N}{\sigma b}, \frac{15a_3^O}{\sigma b}, \frac{15a_3^R}{\sigma b}\right)\right)$$

If $\sigma b<0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{15a_3^R}{\sigma b}, \frac{15a_3^O}{\sigma b}, \frac{15a_3^N}{\sigma b}, \frac{15a_3^M}{\sigma b}, \frac{15a_3^L}{\sigma b}\right), \left(\frac{15a_2^R}{\sigma b}, \frac{15a_2^O}{\sigma b}, \frac{15a_2^N}{\sigma b}, \frac{15a_2^M}{\sigma b}, \frac{15a_2^L}{\sigma b}\right), \left(\frac{15a_1^R}{\sigma b}, \frac{15a_1^O}{\sigma b}, \frac{15a_1^N}{\sigma b}, \frac{15a_1^M}{\sigma b}, \frac{15a_1^L}{\sigma b}\right)\right)$$

O. The proposed ranking function

Let F(R) be the set of all type-2 normal pent-triangular fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function $\check{R}$: F(R) →R which maps each fuzzy number into R.

Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, $= ((A_1^L, A_1^M, A_1^N, A_1^O, A_1^R), (A_2^L, A_2^M, A_2^N, A_2^O, A_2^R), (A_3^L, A_3^M, A_3^N, A_3^O, A_3^R))$ then we define $\check{R}(\tilde{\tilde{A}}) = (A_1^L + A_1^M + A_1^N + A_1^O + A_1^R + A_2^L + A_2^M + A_2^N+A_2^O+A_2^R+A_3^L+A_3^M+A_3^N+A_3^O+A_3^R) / 15$.
Also we define orders on F(R) by

$\check{R}(\tilde{\tilde{A}}) \geq \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \gtrsim_{\check{R}} \tilde{\tilde{B}}$,

$\check{R}(\tilde{\tilde{A}}) \leq \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \lesssim_{\check{R}} \tilde{\tilde{B}}$

and $\check{R}(\tilde{\tilde{A}}) = \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \approx_{\check{R}} \tilde{\tilde{B}}$.

III. RESULTS AND DISCUSSION

Numerical Example

P. Example for operations on type-2 trap-triangular fuzzy numbers

Let $\tilde{\tilde{a}}$ = ((2, 3, 5, 6), (3, 4, 6, 7), (7, 8, 10, 11))
and $\tilde{\tilde{b}}$ = ((−2, −1, 1, 2), (−1, 0, 2, 3), (3, 4, 6, 7)).
Then $\check{R}(\tilde{\tilde{a}})$ = 72 / 12 = 6 and $\check{R}(\tilde{\tilde{b}})$ = 24 / 12 = 2.

Now

1) $\tilde{\tilde{a}} + \tilde{\tilde{b}}$ = ((0, 2, 6, 8), (2, 4, 8, 10), (10, 12, 16, 18))

2) $\tilde{\tilde{a}} - \tilde{\tilde{b}}$ = ((−5, −3, 1, 3), (0, 2, 6, 8), (5, 7, 11, 13))

3) $5\tilde{\tilde{a}}$ = ((10, 15, 25, 30), (15, 20, 30, 35), (35, 40, 50, 55))

4) $-3\tilde{\tilde{a}}$ = ((−33, −30, −24, −21), (−21, −18, −12, −9), (−18, −15, −9, −6))

5) $\tilde{\tilde{a}} \times \tilde{\tilde{b}}$ = ((4, 6, 10, 12), (6, 8, 12, 14), (14, 16, 20, 22))

6) $\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}}$ = ((1, 1.5, 2.5, 3), (1.5, 2, 3, 3.5), (3.5, 4, 5, 5.5))

Q. Example for operations on type-2 pent-triangular fuzzy numbers

Let $\tilde{\tilde{a}}$ = ((1, 2, 4, 6, 7), (3, 4, 6, 8, 9), (5, 6, 8, 10, 11)) and $\tilde{\tilde{b}}$ = ((−3, −2, 0, 2, 3), (−2, −1, 1, 3, 4), (2, 3, 5, 7, 8)).

Then $\check{R}(\tilde{\tilde{a}})$ = 90 / 15 = 6 and $\check{R}(\tilde{\tilde{b}})$ = 30 / 15 = 2.

Now

1) $\tilde{\tilde{a}} + \tilde{\tilde{b}}$ = ((−2, 0, 4, 8, 10), (1, 3, 7, 11, 13), (7, 9, 13, 17, 19))

2) $\tilde{\tilde{a}} - \tilde{\tilde{b}}$ = ((−7, −5, − 1, 3, 5), (−1, 1, 5, 9, 11), (2, 4, 8, 12, 14))

3) $5\tilde{\tilde{a}}$ = ((5, 10, 20, 30, 35), (15, 20, 30, 40, 45), (25, 30, 40, 50, 55))

4) $-3\tilde{\tilde{a}}$ = ((−33, −30, −24, −18, −15), (−27, −24, −18, −12, −9), (−21, −18, −12, −6, −3))

5) $\tilde{\tilde{a}} \times \tilde{\tilde{b}}$ = ((2, 4, 8, 12, 14), (6, 8, 12, 16, 18), (10, 12, 16, 20, 22))

6) $\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}}$ = ((0.5, 1, 2, 3, 3.5), (1.5, 2, 3, 4, 4.5), (2.5, 3, 4, 5, 5.5))

IV. CONCLUSION

In this article, some special types of type-2 triangular fuzzy number are defined. Also arithmetic operations on some special types of type-2 triangular fuzzy number are discussed. These arithmetic operations on some special types of type-2 triangular fuzzy number may be utilized in further works. Also in future, some special types of type-2 trapezoidal fuzzy number and some special types of type-2 pentagonal fuzzy number may be defined.

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