© 2016 IJSRSET | Volume 2 | Issue 5 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section: Engineering and Technology

Encircled Energy Factor in the PSF of an Amplitude Apodised Optical System

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ABSTRACT

In the present paper, the Encircled Energy Factor (EEF) of an optical system has been studied with an amplitude apodisation filter. It has been found that for a given percentage of light flux within the diffraction pattern, the value of the encircled radius increases gradually with apodisation parameter.
Keywords: Fourier Optics, Apodisation, Encircled Energy Factor(EEF (δ)).

I. INTRODUCTION

The encircled energy factor measures the fraction of the total energy in the PSF, which lies within a circle of specified radius with its center at the origin of the co-ordinate system defining the focused Gaussian plane. It is well-known that the image of a point source of light obtained with a diffraction-limited system is not a point. There is a spread of light flux over a considerable region of space in the focused plane and the actual nature of this spread is controlled by various factors viz., the size and the shape of aperture, aberrations and the type of the non-uniformity of transmission. The spread of this light flux at and near the focus of an optical system can manifest in two important ways in decreasing the value of the central intensity in the Airy pattern and increasing the geometrical size of the ideal point image. Important of this was first realized by LOMMEL[1] who developed a mathematical theory of light distribution, in three-dimensions at and near the focus of an optical system with a circular aperture. LOMMEL was also successful in verifying some of his theoretical results experimentally.

Subsequent to the works of LOMMEL, a few more studies were reported in this direction. Thus to mention a few of these studies, intensity distributions near the edges of the geometrical shadow were studied by STRUVE [2]. The effects of amount of defocus from the distribution of intensity at a point away from the Gaussian focal plane were considered by SCHWARTZCHILD [3]. The conclusions drawn from these studies, led RAYLEIGH [4] to point the important of the encircled energy factor as an image quality assessment parameter. Strongly enough, no interest was shown at all by any investigator on this topic for over a period of nearly four decades in spite of the important work by Rayleigh on encircled energy factor. We come across only one related work by DUBYE[5] during this long period of time, who had studied the pattern of the diffracted field at and away from the focus and established a few more general features of the far-field pattern of the diffraction-limited systems.

II. METHODS AND MATERIAL

Mathematical Expression for EEF:

The encircled energy factor (EEF) is defined as the ratio of the flux inside a circle of radius ‘δ’ centered on the diffraction head to the total light flux in the diffraction pattern. Thus the EEF denoted by E(δ), can be written as

$$E(\delta)=\frac{\int_0^{2\pi}\int_0^{\delta}\left|G_\beta(0,\ z)\right|^2 zdz\,d\phi}{\int_0^{2\pi}\int_0^{\infty}\left|G_\beta(0,\ z)\right|^2 zdz\,d\phi}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Where φ is the azimuthally angle since the integration over φ introduces just a constant factor 2π in both the numerator and denominator, above expression reduces to,

$$E(\delta)=\frac{\int_0^{\delta}\left|G_\beta(0,\ z)\right|^2 zdz}{\int_0^{\infty}\left|G_\beta(0,z)\right|^2 zdz}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where $G_\beta(0,z)$ is the light amplitude in the image plane at point z units away from the diffraction head. The denominator in the expression (2) represents the total flux; the amplitude of the light diffracted in the far field region with rotationally symmetric pupil function can be expressed by the known expression as

$$G_\beta(0,z)=2\int_0^1 f(r)J_0(zr)rdr\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3)$$

Substituting for the denominator of the equation (2) in terms of the expression τ for $G_\beta(0,\ z)$, we obtain for co-sinusoidal filters,

$$E(\delta)=\frac{\int_0^{\delta}\left[\int_0^1 f_\beta(r)J_0(zr)rdr\right]^2 zdz}{\int_0^1\left|f_\beta(r)\right|^2 rdr}\ \ldots\ldots\ (4)$$

Where $f_\beta(r)=\dfrac{1+\beta\,C\text{os}\,\pi r^2}{1+\beta}$

Substituting $f_\beta(r)$ for in the above expression (4) we finally obtain

$$E(\delta)=\frac{\int_0^{\delta}\left[\int_0^1\left(\frac{1+\beta\,Cos\pi r^2}{1+\beta}\right)J_0(zr)rdr\right]^2 zdz}{\int_0^1\left(\frac{1+\beta\,Cos\pi r^2}{1+\beta}\right)^2 rdr}\ \ldots\ (5)$$

III. RESULTS AND DISCUSSION

The EEF for increasing values of δ starting from δ=0.0 to 15 has been computed using the expression (5) the computed results have been presented graphically in figure the figure shows the variation of encircled energy factor EEF with δ for various values of apodisation parameter β when observations are made in the perfectly focused plane corresponding to y=0. In the figure we have presented these results with the normalized EEF for various values of β and for y=0. It is observed from the figure that for given percentage of light flux within the diffraction pattern 50% of the value of δ increases gradually with β. This is true for all values of the enclosed energy flux below 70% of the total flux in the image the behavior of the variation of EEF with δ shows a very slight, almost insignificant, departure from the above pattern. The most important conclusion from this study is that the high concentration of light flux in a small circle will definitely play a dominant role in the resolution aspect of optical systems with these filters.

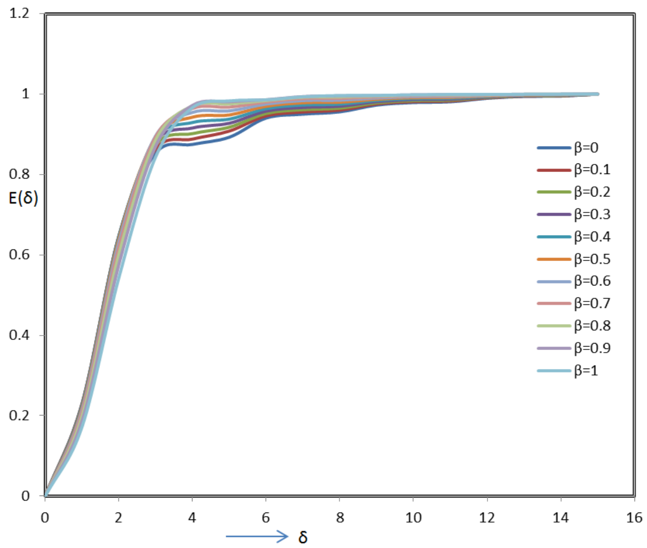

Figure 1. Curves showing EEF (δ) for various values of β

Table 1. Values of EEF (δ) for various values of β and δ

δ	β=0	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5	β=0.6	β=0.7	β=0.8	β=0.9	β=1
0	0	0	0	0	0	0	0	0	0	0	0
1	0.2306	0.2298	0.2286	0.2267	0.2239	0.22	0.2147	0.2077	0.1987	0.1871	0.1727
2	0.6445	0.6465	0.6475	0.6471	0.6448	0.64	0.6318	0.6192	0.6012	0.5763	0.5433
3	0.8535	0.8634	0.8729	0.8816	0.8889	0.8941	0.8963	0.8942	0.8862	0.8704	0.8445
4	0.8749	0.8882	0.9018	0.9157	0.9295	0.9427	0.9546	0.9645	0.9709	0.9722	0.9665
5	0.8929	0.9082	0.9178	0.9277	0.9379	0.9482	0.9583	0.9676	0.9755	0.9811	0.9832
6	0.9405	0.946	0.9518	0.9577	0.9637	0.9697	0.9753	0.9802	0.984	0.9861	0.9857
7	0.9501	0.9555	0.9611	0.9669	0.9728	0.9785	0.9839	0.9886	0.9922	0.994	0.9934
8	0.9559	0.9603	0.965	0.9699	0.9749	0.9799	0.9848	0.9893	0.9931	0.9957	0.9965
9	0.9728	0.9755	0.9783	0.9812	0.9842	0.9872	0.9901	0.9927	0.9949	0.9964	0.9968
10	0.979	0.9813	0.9837	0.9862	0.9887	0.9912	0.9936	0.9957	0.9973	0.9982	0.9981
11	0.9809	0.9829	0.985	0.9872	0.9895	0.9918	0.994	0.996	0.9976	0.9988	0.9991
12	0.9896	0.9907	0.9918	0.9929	0.9941	0.9953	0.9964	0.9975	0.9984	0.999	0.9992
13	0.9943	0.9949	0.9956	0.9964	0.9971	0.9978	0.9985	0.999	0.9995	0.9997	0.9995
14	0.995	0.9956	0.9962	0.9968	0.9974	0.9981	0.9987	0.9992	0.9997	0.9999	0.9999
15	1	1	1	1	1	1	1	1	1	1	1

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