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A Different Approach on Pythagorean Triangle, Which Satisfies $(\textit{one leg of right triangle } 2pq) - 6\frac{(Area)}{(Perimeter)} = K$

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ABSTRACT

Patterns of Pythagorean Triangle for each of which $(\textit{one leg of right triangle } 2pq) - 6\frac{(Area)}{(Perimeter)}$ may be expressed as a positive integer(K). A few interesting relations are also given.

Keywords: Pythagorean Triangle, Nasty Numbers.

I. INTRODUCTION

The method of obtaining three non-zero integers x, y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians [1 to 7]. In [8 to 13] special Pythagorean Problems are studied. In this communication, we present yet another interesting Pythagorean triangle where in each of which

$\left(\begin{matrix}\textit{one leg of right}\\ \textit{triangle } 2pq\end{matrix}\right) - 6\frac{(Area)}{(Perimeter)} = K$. A few interesting relation among the solutions are given.

Notation

1. $T_{m,n} = n\left[1+\frac{(n-1)(m-2)}{2}\right]$ = Polygonal number of rank n, with sides m.

II. METHODS AND MATERIAL

The most cited solution of the Pythagorean equation $x^2 + y^2 = z^2$ is

$x = 2pq$
$y = p^2 - q^2$ where $p > q > 0$ (1)
$z = p^2 + q^2$

Denoting the Area and the Perimeter of the above Pythagorean triangle by A and P respectively, the assumption that the ratio A/P can be expressed as the equation $\left(\begin{matrix}\textit{one leg of right}\\ \textit{triangle } 2pq\end{matrix}\right) - 6\left(\frac{A}{P}\right) = K$ where K is a non–zero integer, which leads to the equation

$$3q^2 - pq = K \quad (2)$$

In which follows we are present pattern of integral solution of (2) and thus in view of (1), the integral solution of (2) are obtained.

Pattern I

Let $K = \alpha^3$ which leads (2) as

$$3q^2 - pq = \alpha^3 \quad (3)$$

Choosing $q = \alpha^2, 3q - p = \alpha$ in (3) and solving we get

$$p = 3\alpha^2 - \alpha \text{ and } q = \alpha^2 \quad (4)$$

In which follows, we obtain the values of x, y, z

$$x = 2pq \Rightarrow x(\alpha) = 6\alpha^4 - 2\alpha^3 \quad (5)$$

$$y = p^2 - q^2 \Rightarrow y(\alpha) = 8\alpha^4 - 6\alpha^3 + \alpha^2 \quad (6)$$

$$z = p^2 + q^2 \Rightarrow z(\alpha) = 10\alpha^4 - 6\alpha^3 + \alpha^2 \quad (7)$$

Few examples are given

α	p	q	x	y	z
4	44	16	1408	1680	2192
5	70	25	3500	4275	5525
7	140	49	13720	17199	22001
9	234	81	37908	48195	61317
11	352	121	85184	109263	138545

Properties

1. $x + 2\alpha^3$is six times a Quartic integer.
2. $x(\alpha) + 2\alpha^3$is a nasty number.
3. $2(z - y)$is a perfect square.
4. $3(z - y)$is a nasty number.
5. $z - y \equiv 0 \ (mod\ 2)$
6. $x + y + z \equiv 0 \ (mod\ 2)$
7. $(x + y + z) - 4\alpha(4\alpha - 1)\, T_{5,\alpha} = 0$
8. $x - (2\alpha)^2 T_{5,\alpha} = 0$
9. $2(y + z)$is a perfect square
10. $y - 2\alpha^2 T_{10,\alpha} - T_{4,\alpha} = 0$
11. $z - 4\alpha^2 T_{7,\alpha} - T_{4,\alpha} = 0$
12. $(x + y - z) - 2\alpha^2 T_{6,\alpha} = 0$

Pattern II

Assuming $K = \alpha^4 (\alpha > 0)$ which leads (2) as

$$3q^2 - pq = \alpha^4 \quad (8)$$

Case i:

Choosing $q = \alpha^2, 3q - p = \alpha^2$ in (8) and solving we get

$$p = 2\alpha^2 \text{ and } q = \alpha^2 \quad (9)$$

In which follows, we obtain the values of x, y, z

$$x = 2pq \Rightarrow x(\alpha) = 4\alpha^4 \quad (10)$$

$$y = p^2 - q^2 \Rightarrow y(\alpha) = 3\alpha^4 \quad (11)$$

$$z = p^2 + q^2 \Rightarrow z(\alpha) = 5\alpha^4 \quad (12)$$

Few examples are given

α	p	q	x	y	z
2	8	4	64	48	80
8	128	64	16384	12288	20480
10	200	100	40000	30000	50000
12	288	144	82944	62208	103680
14	392	196	153664	115248	192080

Properties

1. $x + y + z$ is twelve times a Quartic integer.
2. $6(x - y)$is a nasty number.
3. $6\,(z - y)$is a nasty number.
4. $z + x - y$is a nasty number.
5. $2(x + y - z)$is a perfect square.
6. $y + z - x$is a perfect square.

Case ii:

Choosing $q = \alpha^3, 3q - p = \alpha$ in (8) and solving we get

$$p = 3\alpha^3 - \alpha \text{ and } q = \alpha^3 \quad (13)$$

In which follows, we obtain the values of x, y, z

$$x = 2pq \Rightarrow x(\alpha) = 6\alpha^6 - 2\alpha^4 \quad (14)$$

$$y = p^2 - q^2 \Rightarrow y(\alpha) = 8\alpha^6 - 6\alpha^4 + \alpha^2 \quad (15)$$

$$z = p^2 + q^2 \Rightarrow z(\alpha) = 10\alpha^6 - 6\alpha^4 + \alpha^2 \quad (16)$$

Few examples are given

α	p	q	x	y	z
4	188	64	24064	31248	39440
5	370	125	92500	121275	152525
7	1022	343	701092	926835	1162133
10	2990	1000	2980000	7949100	9940100
3	78	27	4212	5355	6813

Properties

1. $2(z - y)$is a perfect square.
2. $(z - y) - 2(T_{4,\alpha})^3 = 0.$
3. $(x + y + z) - 4\alpha(4\alpha - 1)\, T_{5,\alpha} = 0.$
4. $y - k - 2k\, T_{10,\alpha} = 0$where $k = \alpha^2$.
5. $y = 3kT_{8,k} - 2(k - 1)\, T_{3,k}$where $k = \alpha^2$.
6. $x + y + z \equiv 0 \ (mod\ 2)$
7. $z - k - 4k\, T_{7,k} = 0$where $k = \alpha^2$.

Pattern III

Assuming $K = \alpha^5 (\alpha > 0)$ which leads (2) as

$$3q^2 - pq = \alpha^5 \quad (17)$$

Case i:

Choosing $q = \alpha^3, 3q - p = \alpha^2$ in (17) and solving we get

$$p = 3\alpha^3 - \alpha^2 \text{ and } q = \alpha^3 \quad (18)$$

In which follows, we obtain the values of x, y, z

$x = 2pq \Rightarrow x(\alpha) = 6\alpha^6 - 2\alpha^5$ (19)

$y = p^2 - q^2 \Rightarrow y(\alpha) = 8\alpha^6 - 6\alpha^5 + \alpha^4$ (20)

$z = p^2 + q^2 \Rightarrow z(\alpha) = 10\alpha^6 - 6\alpha^5 + \alpha^4$ (21)

Few examples given

α	p	q	x	y	z
2	20	8	320	336	464
4	176	64	22528	26880	35072
5	350	125	87500	106875	138125
8	1472	512	1507328	1904640	2428928
10	2900	1000	5800000	7410000	9410000

Properties

1. $2(z - y)$is perfect square.
2. $3(z - y)$is nasty number.
3. $x(\alpha) - 2\alpha^4\,[T_{4,\alpha} + T_{6,\alpha}] = 0$.
4. $y(\alpha) - \alpha^4\,[2T_{8,\alpha} + 1] = 0$.
5. $z(\alpha) - \alpha^4\,[4T_{7,\alpha} + 1] = 0$.
6. 2(y + z) is a perfect square.
7. $y + z - 8\left(T_{5,\alpha}\right)^2 = 0$.
8. $(z - x) - 4\alpha^2 T_{3,\alpha^2} + \alpha^3 T_{8,\alpha} = 0$.

Case ii:

Choosing $q = \alpha^4, 3q - p = \alpha$ in (17) and solving we get

$$p = 3\alpha^4 - \alpha \text{ and } q = \alpha^4 \quad (22)$$

In which follows, we obtain the values of x, y, z

$x = 2pq \Rightarrow x(\alpha) = 6\alpha^8 - 2\alpha^5$ (23)

$y = p^2 - q^2 \Rightarrow y(\alpha) = 9\alpha^8 - 6\alpha^5 + \alpha^2 - \alpha^4$ (24)

$z = p^2 + q^2 \Rightarrow z(\alpha) = \alpha^8 + 9\alpha^6 - 6\alpha^5 + \alpha^2$ (25)

Properties

1. $(y - z) = 2k\,[(2k)^2\,(k - 1) - T_{3,k}\,)$ where $k = \alpha^2$.
2. $x \equiv 0\ (mod\ 2)$.
3. $(y - x) - \alpha^2\,(3\alpha^6 + 1) + 8\alpha T_{3,\alpha} = 0$.
4. $z(\alpha) - \alpha^5(\alpha^2(\alpha + 9) - 6)$is a perfect square.
5. $y(\alpha) - \alpha^4(3\alpha(3\alpha^3 - 2) - 1)$ is a perfect square

Few examples are given

α	p	q	x	y	z
1	2	1	4	3	5
2	46	16	1472	1860	2372
3	241	81	39042	51520	64642
4	764	256	391168	518160	619232
5	1870	625	2337500	3106275	3887525

Pattern IV

Let $K = \alpha\beta, \quad \beta > \alpha > 0$ which leads (2) as

$$3q^2 - pq = \alpha\beta \quad (26)$$

Choosing $q = \beta, 3q - p = \alpha$ in (26) and solving we get

$$p = 3\beta - \alpha \text{ and } q = \beta \quad (27)$$

In which follows, we obtain the values of x, y, z

$x = 2pq \Rightarrow x(\alpha,\beta) = 6\beta^2 - 2\alpha\beta$ (28)

$y = p^2 - q^2 \Rightarrow y(\alpha,\beta) = 8\beta^2 - 6\alpha\beta + \alpha^2$ (29)

$$z = p^2 + q^2 \Rightarrow z(\alpha, \beta) = 10\beta^2 - 6\alpha\beta + \alpha^2 \quad (30)$$

Few examples are given

α	β	p	q	x	y	z
1	2	5	2	20	21	29
2	3	7	3	42	40	58
4	5	11	5	110	96	146
6	7	17	7	238	240	338
7	8	18	8	288	260	388

Recurrence relation

1. $x(\alpha + 1, \beta + 1) - 2x(\alpha, \beta) + x(\alpha - 1, \beta - 1) = 8$
2. $y(\alpha + 1, \beta + 1) - 2y(\alpha, \beta) + y(\alpha - 1, \beta - 1) = 8$
3. $z(\alpha + 1, \beta + 1) - 2z(\alpha, \beta) + z(\alpha - 1, \beta - 1) = 10$

Properties

1. $x(1, \beta) \equiv 0 \ (mod\ 2)$
2. $y(1, \beta) - 2T_{10,\beta} - 1 = 0.$
3. $z(1, \beta) - 4T_{7,\beta} - 1 = 0$
4. $x(1, \beta) - 12T_{3,\beta} + 8\beta = 0$
5. $y(1, \beta) - 1 \equiv 0(mod2)$
6. $z(1, \beta) - 1 \equiv 0(mod2)$
7. $z(1, \beta) - x(1, \beta)$is a perfect square.
8. 8. $z(1, \beta) - y(1, \beta)$ is two times a perfect square.
9. $x(1, \beta) + y(1, \beta) - z(1, \beta) + 2\beta$is a perfect square.
10. $x(1, \beta + 1) - 12T_{3,\beta} - 4(\beta + 1) = 0$
11. $y(1, \beta + 1) - 16T_{3,\beta} - (2\beta + 3) = 0$
12. $z(1, \beta + 1) - 20T_{3,\beta} - (4\beta + 5) = 0$
13. $3[y(1, \beta + 1) - x(1, \beta + 1)] + 3$is a nasty number.
14. $z(1, \beta + 1) - x(1, \beta + 1)$is a perfect square.
15. $z(1, \beta + 1) - y(1, \beta + 1)$is a perfect square.

III. CONCLUSION

One may search for other patterns of Pythagorean triangle under consideration.

IV. REFERENCES

[1] L.E. Dickson, History of the Theory of Numbers, Chelsea Publishing Company, New York, 1952.

[2] Daniel Shanks, "Solved and Unsolved Problems in Number Theory", Spartan Books, New York, 1971.

[3] Paulo Ribenboim, "Fermat's Last Theorem for Amateurs". Sprinter-Verlag, New York, 1999.

[4] Albert H. Beiler, "Recreations in the Theory of Numbers", Dover Publications, New York, 1963.

[5] S.B. Malik, "Basic Number Theory", Vikas Publishing House Pvt. Limited, New Delhi,

[6] L.J. Mordell, Diophatine Equations, Academic Press, New York, 1969.

[7] Niven, Ivan, Zuckermann, S. Herbert, Montgomery and L.Hugh, An introduction to the Theory of Numbers, Jhon Wiley and Sons. Inc, New York, 2004.

[8] M.A. Gopalan and S. Devibala, Pythagorean Triangle: A Treassure House, Proceeding of the KMA National Seminar on Algebra, Number Theory and Applications to Codeing and Cryptanalysis, Little Flower College, Guruvayur, September 16-18, 2004.

[9] M.A. Gopalan and R. Anbuselvi, A Special Pythagorean Triangle, ActaCienciaIndica XXXI M, No. 1, p. 053, 2005.

[10] M.A. Gopalan and S. Devibala, On a Pythagorean Problem, ActaCienciaIndia XXXIIM, No. 4, p. 1451, 2006.

[11] B.L. Bhatia and SupriyaMohanty, Nasty Numbers and their Characterizations, Mathematical Education, p. 34-37, July – Sep. 1985.