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A Different Approach on A Pythagorean Triangle which Satisfies

$$p(Hypotonuse) - 4p\frac{(Area)}{(Perimeter)} = \beta^2$$

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ABSTRACT

We obtain non-trivial values for the sides of the Pythagorean triangle such that $p(Hypotonuse) - 4p\frac{(Area)}{(Perimeter)} = \beta^2$. A few interesting relations between the sides of the Pythagorean triangle are presented.

Keywords: Integral Solutions, Pythagorean Triangles

I. INTRODUCTION

One well known set of solutions of the Pythagorean equation $x^2 + y^2 = z^2$ are $x = 2uv, y = u^2 - v^2$ and $z = u^2 + v^2$. Many mathematicians has been used this set of solutions to obtain the non-zero integral values for x, y and z [1-3]. As a new approach, in this paper we introduce another set of solutions $x = 2U + 1, y = 2U^2 + 2U$ and $z = 2U^2 + 2U + 1$ for the equation $x^2 + y^2 = z^2$. By using this solution we obtain three non-zero integers x,y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ [4-6]. In this communication, we present yet another interesting Pythagorean triangle where in each of which the ratio $p(Hypotonuse) - 4p\frac{(Area)}{(Perimeter)}$ may be expressed as a perfect square.

II. METHODS AND MATERIAL

Taking $A > 0$ to be the generators of the Pythagorean triangle (x, y, z), the assumption that $p(Hypotonuse) - 4p\frac{(Area)}{(Perimeter)} = \beta^2$ leads to the Pellian equation $Y^2 = DX^2 + p$ where $D = 2p$, not a perfect square and $U = X$.

For the clear understanding we consider the following two cases:

i) $p = 9$ (odd number) so that $D = 18$
ii) $p = 12$ (even number) so that $D = 24$

Case (i):

When $p = 9$ the equation

$$Y^2 = DX^2 + p \qquad (1)$$

Becomes

$$Y^2 = 18X^2 + 9 \qquad (2)$$

Let $(x_0, y_0) = (12,51)$ be the initial solution of (2).

Consider the Pellian

$$Y^2 = 18X^2 + 1 \qquad (3)$$

Let $(\widetilde{x_0}, \widetilde{y_0}) = (4,17)$ be a solution of (3)

Using Brahmagupta lemma the general solution $(\widetilde{x_n}, \widetilde{y_n})$ of equation (3) is given by

$$\widetilde{y_n} + \sqrt{18}\widetilde{x_n}) = (17 + 4\sqrt{18})^{n+1} \quad (4)$$

Where $n = 0,1,2,3\ldots$

Since irrational roots occur in pairs

$$\widetilde{y_n} - \sqrt{18}\widetilde{x_n}) = (17 - 4\sqrt{18})^{n+1} \quad (5)$$

Where $n = 0,1,2,3$ ...

From equation (4) and (5), we obtain

$$\widetilde{y_n} = \frac{1}{2}[(17 + 4\sqrt{18})^{n+1} + (17 - 4\sqrt{18})^{n+1}] \quad (6)$$

and

$$\widetilde{x_n} = \frac{1}{2\sqrt{18}}[(17 + 4\sqrt{18})^{n+1} - (17 - 4\sqrt{18})^{n+1} \quad (7)$$

Using the equations (6) and (7), the solutions of equation (2) is given by

$$U_{n+1} = X_{n+1} = \frac{1}{2\sqrt{18}}[(12\sqrt{18} + 51)(17 + 4\sqrt{18})^{n+1} - (12\sqrt{18} - 51)(17 - 4\sqrt{18})^{n+1}]$$

$n = -1,0,1,2$ ...

$$Y_{n+1} = \frac{1}{2\sqrt{18}}[(51\sqrt{18} + 216)(17 + 4\sqrt{18})^{n+1} - (51\sqrt{18} - 216)(17 - 4\sqrt{18})^{n+1}]$$

$n = -1,0,1,2$ ...

Numerical Examples

n	U_{n+1}	Y_{n+1}
-1	12	51
0	408	1731
1	7332	31155
2	249264	1057539

Observations:

1. Recurrence relations for X and Y are
   $X_{n+3} - 4X_{n+2} - 509X_{n+1} = 0$ and
   $Y_{n+3} - 4Y_{n+2} - 509Y_{n+1} = 0$
2. For all values of n, Y is even and Y is odd
3. For all values of n, X_{n+1} is divisible by 4 and Y_{n+1} is divisible by 3

Case (Ii):

When $p = 12$ the equation (1) leads to

$$Y^2 = 24X^2 + 12 \quad (8)$$

Let $(x_0, y_0) = (1,6)$ be the initial solution of (8).

To obtain the general solution of (8) consider the Pellian equation

$$Y^2 = 24X^2 + 1 \quad (9)$$

Let $(\widetilde{x_0}, \widetilde{y_0}) = (1,5)$ be a solution of (9)

Using Brahmagupta lemma the general solution $(\widetilde{x_n}, \widetilde{y_n})$ of equation (9) is given by

$$\widetilde{y_n} + \sqrt{24}\widetilde{x_n} = (5 + \sqrt{24})^{n+1} \quad (10)$$

Where $n = 0,1,2,3\ldots$

Since irrational roots occur in pairs

$$\widetilde{y_n} - \sqrt{24}\widetilde{x_n} = (5 - \sqrt{24})^{n+1} \quad (11)$$

Where $n = 0,1,2,3$ ...

From equation (10) and (11), we obtain

$$\widetilde{y_n} = \frac{1}{2}[(5 + \sqrt{24})^{n+1} + (5 - \sqrt{24})^{n+1}] \quad (12)$$

and

$$\widetilde{x_n} = \frac{1}{2\sqrt{18}}[(5 + \sqrt{24})^{n+1} + (5 - \sqrt{24})^{n+1}] \quad (13)$$

Using the equations (6) and (7), the solutions of equation (8) is given by

$$U_{n+1} = X_{n+1} = \frac{1}{2\sqrt{24}}[(\sqrt{24} + 6)(5 + \sqrt{24})^{n+1} - (\sqrt{24} - 6)(5 - \sqrt{24})^{n+1}] \quad n = -1,0,1,2 \ldots$$

$$Y_{n+1} = \frac{1}{\sqrt{24}}[(3\sqrt{24} + 12)(5 + \sqrt{24})^{n+1} + (3\sqrt{24} - 12)(5 - \sqrt{24})^{n+1}] \quad n = -1,0,1,2 \ldots$$

Numerical Examples

n	U_{n+1}	Y_{n+1}
-1	1	6
0	11	54
1	109	534
2	1079	5286

Observations:

1. Recurrence relations for X and Y are
   $X_{n+3} - 10X_{n+2} + X_{n+1} = 0$ and
   $Y_{n+3} - 10Y_{n+2} + Y_{n+1} = 0$
2. For all values of n, X is odd and Y is even.
3. For all values of n, Y_{n+1} is divisible by 6.
4. $X_{n+3} + X_{n+1} \equiv 0(mod 10)$
5. $Y_{n+3} + Y_{n+1} \equiv 0(mod 10)$
6. $X_{n+3} + X_{n+2} + X_{n+1} \equiv 0(mod 11)$
7. $Y_{n+3} + Y_{n+2} + Y_{n+1} \equiv 0(mod 11)$

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