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# Geometric Thickness for the General Graphs

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## ABSTRACT

In this method the user gives a graph as input which is then processed. After processing the input graph is determined whether it is a planar graph or not. If the input graph is planar then a planar embedding of the given input graph is shown as the output having only one color. If the input graph is non planar then n layers of the given input graph is displayed where each layer is of different colors. Then each layer is also called as the Geometric-Thickness. The geometric thickness θ(G) of a graph G is the smallest integer t such that there exist a straight-line drawing of G and a partition of its straight-line edges into t subsets, where each subset induces a planar drawing. Over a decade ago, Hutchinson, Shermer, and Vince proved that any n-vertex graph with geometric thickness two can have at most 6n − 18 edges, and for every n ≥ 8 they constructed a geometric thickness two graph with 6n − 20 edge, but we taken the 6n-18 edges. And also we do the NP-hardness of coloring graphs of geometric thickness.

**Keywords:** Graph,Embedding,Graph Thickness

## I. INTRODUCTION

### Degrees and Thickness:

The concept of graph thickness has often been connected to electronic circuits, which might be printed in layers so that each layer is crossing-free, that is, a Planar graph. For practical reasons, the degrees in such graphs cannot be arbitrarily large. Bose and Prabhu consider the special case in which all degrees are 4 or less. Among their results, they show that if p # 1 (mod 4), then the "degree-4thickness"of KP is [(p+ 3)/4]. For p = 5 and 9, this formula does hold of course, but the question remains unsettled for p = 4k + 1whenk >3. Halton & asksa reverse question of sorts: among all graphs with maximum degree d, what is the maximum thickness T(d)? He observed that Z'(d) > L(d+ 5)/4] since that is the thickness of Kd , d. On the other hand, Petersen's theorem stating that every2k-regular graph can be decomposed into k 2-regulargraphs implies that Z'(d) < L(d+ 1)/2J. Clearly, T(l) = T(2) = 1 and 2"(3) = T(4) = 2. Halton conjectures that 2'(6) = 2: every graph with maximumdegree6 or less is biplanar . I would be quite surprised if this were so. More generally, he conjectures that every graph with maximum degree 4r – 2 has thickness  or less.

## II. METHODS AND MATERIAL

### Coloring Problem

Suppose that we have two maps ,one of countries on the earth and another of colonies on the moon, say, and that each country can have one colony on the moon. A proper coloring of the two maps has these properties.

(1) Each country has the same color as its colony.
(2) Adjacent countries have different colors.
(3) Adjacent colonies have different colors.

The basic question is this: what is the most colors needed for coloring any such pair of maps? Since each map has a graphs its dual ,by identifying each country vertex with its colony vertex, we see that the union of the two graphs is biplanar .So, in effect, we are working for the maximum chromatic number among all graphs of thickness2. The standard argument based on degrees (implied by Euler's formula)shows that it is at moat12. On the other hand, Sulanke(see [13]) observed that Kll –

C5 has thickness2 and chromatic number9. Therefore, we have the following result.

**Biembeddings**

Included in the early work on thickness is an extension of biplanarity to other surfaces. Given a surface S, the S-thickness of a graph G is the minimum number of S-embeddable graphs for which the union is G. If S is the projective plane, torus,or double-torus, then the S-thickness of KP is L(p+ 5)/6J, L(P+ 4)/6J, snd L(P+ 3)/6J, respectively.(In each of the next two surfaces, the Klein bottle and the triple-torus, only five-sixths of the thicknesses of KP are known.) The natural extension of biplanarity to other surfaces S is this: a graph is called S-bi embeddable if it is the union of two S-embeddable graphs. Most of the work on this topic, which was formally begun by Anderson and Cook ,involves complete graphs and orientable surfaces. Let B(h) denote the maximum order of a complete graph that is biembeddable on the orientable surface Sh, the sphere with h handles. (In the literature, most of the results are stated for B(h) + 1; that is, for the minimum P for which KP is not %-biembeddable.) Euler’s formula implies that B(h) < 1(1/2)(13 + 73+ 96h)J. From the thickness results cited earlier, we can deduce the following results.

**Theorem 1:**

For h S 3, the maximum order B(h) of a complete graph that is Sh-biembeddable satisfies B(O) = 8,

B(1) = 13, B(2) = 14, and B(3) = 15 or 16.

We note that if B(3) = 16,then KM must be the union of two triangulations of S3. We also note that if we let ~(k) denote the corresponding number for the non orientable surface with k cross caps, then for the projective plane we have~(l) = 12, and depending on whetherK13 is the union of two K leinbottle graphsor not, we have3(2) = 12or 13. Anderson and Cook [23]also consider combinations of surfaces S and S’, considering graphs that are the unionof an S-embeddable graphandanS’-embeddable graph. We denote by B(i, j) the maximum order of a complete graph that is the union of an Sicandan Sj-embeddable graph. In other words, B(i, j) denotes the maximum order of an Si-embeddable graph G whose complement~ is Sj-embeddable. The Euler bound here is B(i, j) S [(1/2)(13+ ~73 +48(i +j))]. A variety of sporadic results, some that cover infinitely many pairs of surfaces, have been found by Anderson and White, Anderson and Cabaniss and Jackson. A comprehensive survey of bi-embeddings was given by Cabaniss. Here we state some of what is known when one of the surfaces is the plane or the torus .

**Theorem 2:**

(a)The minimum order of a complete graph that is the union of a planar graph and an Sh- embeddable graph is 8 if h = O;11or 12if h = 1; 12or 13if h = 2; 13if h = 3; and 13or 14 if h = 4. (b) The maximum order of a complete graph that is the union of a toroidal graph and an Sh-embeddable graph is 13 if h = 1 or 2, and 16if h = 6.

We find the case in which one of the graphs is planar particularly interesting. A variation of this problem is this question: what is the minimum genus of the complement of a planar graph of order p? Euler’s formula implies that it is at least [P(P– 13)/121+ 2. For small l values of p, we have the following corollary.

**Corollary:**

The minimum genus of the complement of a planar graph of order p is o, if p <8, 1, if p = 9, 10, or 11, 1 or 2, if p = 12, 2 or 3, if p = 13, 3 or 4, if p = 14.

In closing, we note that if K12is the union of a planar graph and a toroidal graph, then these must both be triangulations of their respective surfaces, and the same is true if K13is the union of a planar graph and a double-toroidal graph.

**Geometric Thickness:**

The thickness of a graph drawing is the minimum k ∈ N such that the edges of the drawing can be partitioned into k non-crossing sub drawings; that is, each edge is assigned one of k colours such that edges with same colour do not cross. Every planar graph can be drawn with its vertices at pre-specified locations . Thus a graph with thickness k has a drawing with thickness k [44]. However, in such a drawing the edges might be highly curved. This motivates the notion of geometric thickness. A drawing (ϕV,ϕE) of a graph G is geometric if the image of each edge ϕE(vw) is a straightline-segment (by definition, with endpointsϕV(v) and ϕV(w)). Thus a geometric drawing of a graph is determined by the positions of its vertices. We thus refer to ϕV as a geometric drawing. The geometric thickness of a graph G, denoted by θ(G), is the minimum k ∈ N such that there is a geometric drawing of G with thickness k. Kainen first defined geometric thickness under the name of real linear thickness, and it has also been called rectilinear thickness. By the Fary–Wagner theorem, a graph has geometric thickness 1 if and only if it is planar. Graphs of geometric thickness 2, the so-called doubly linear graphs, were studied by Hutchinson etc... The outer thickness (respectively, arboricity, star-arboricity) of a graph drawing is the minimum k ∈ N such that the edges of the drawing can be partitioned into k outer non- crossing subdrawings (non-crossing forests, non-crossing star-forests).Again a graph with outer-thickness (arboricity, star-arboricity) k has a drawing with outer thickness (arboricity, star-arboricity) k.

## III. RESULTS AND DISCUSSION

### EXAMPLE FOR GEOMETRIC THICKNESS :

Two Graphs with 6n−19 Edges Let K0 9 be the graph obtained by deleting an edge from K9. In this section we first construct a geometric thickness two representation Γ of K0 9 that has 6n−19edges. We then show how to add vertices in Γ such that for any n ≥ 9 one canconstruct a geometric thickness two graph with 6 n−19 edges.

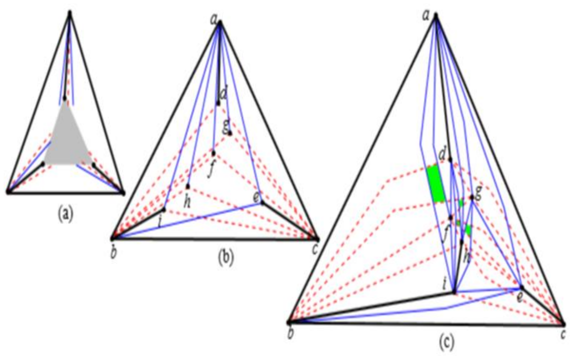
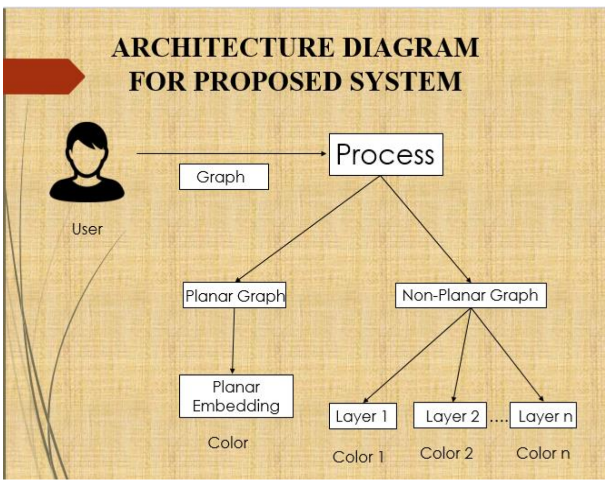



**Figure 1.** (a) Illustration for the shared edges (bold). (b) Initial point set. (c) A geometric thickness two representation Γ of K0 9, where the planar layers are shown in red (dashed) and blue (thin). Black edges can belong to either red or blue layer. Free quadrangles are shown in green (shaded). Some edges are drawn with bends for clarity.

Hutchinson et al. [14, Theorem 6] proved that if any geometric thickness two graph with 6n−18 edges exists, then the convex hull of its geometric thickness two representation must be a triangle. This representation is equivalent to the union of two plane triangulations that share at least six common edges, i.e., the three outer edges and the other three edges are adjacent to the three outer vertices, as shown in black in Figure 1(a). Since each triangulation contains 3n−6 edges, the upper bound of 2(3n−6)−6 = 6n−18 follows. These prop- erties of an edge maximal geometric thickness two representation motivated us to examine pairs of triangulations that create many edge crossings while drawn simultaneously. In particular, we first created a set of points interior to the con- vex hull such that addition of straight line segments from each interior point to the three points on the convex hull creates two plane drawings that, while drawn simultaneously, contain a crossing in all but the six common edges. Fig- ure 1(b) illustrates such a scenario. We then tried to extend each of these two planar drawings to a triangulation by adding new edges such that every new edge crosses at least one initial edge. We found multiple distinct point sets for which all but one newly added edge cross at least one initial edge, resulting in multiple distinct geometric thickness two representations with 2(3n−6)−7 = 6n−19 edges. For example, see Figure 1(c), where the underlying graph is K0 9. Let Γ be a geometric thickness two representation. A triangle in Γ is empty if it contains exactly three vertices on its boundary, but does not contain any vertex in its proper interior, e.g., the triangle ∆ghi in Figure 1(d). A quadrangle in Γ is free if it is created by the intersection of two empty triangles but does not contain any vertices of Γ, as shown in Figure 1(d) in green.

### Architecture Diagram for Proposed System



The system uses to find planar representation of different layers by using breadth first search.The proposed system uses breadth first search instead of greedy search which reduces the time required for coloring the given non-planar graph.This uses Polynomial time concept.Using breadth first search we are splitting the layers and the cross edges are treated as another layer.Finally each layer is colored with different colors.

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