

# Performance Analysis of MIMO System over Fading Channels

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## ABSTRACT

In today's era multiple input and multiple output (MIMO) system is one of the most emerging experimentation field in wireless communication. This research paper shows the capacity of MIMO system over Rayleigh, Rician, Nakagami and weibull distributions by using Water filling Model. The probability density function is also shown in this paper for MIMO system under different fading channels which also shows the intersymbol interference between the signals due to number of antennas at transmitter and receiver side. For simulation MATLAB software is used.

**Keywords :** Multiple input multiple output (MIMO), MIMO capacity, Water filling Model

## I. INTRODUCTION

In wireless communication, multiple input multiple output (MIMO) systems specify any wireless communication system where multiple antennas are used at both sides of communication channel. So the systems that use more than one transmit and receive antenna are called as multiple input multiple output (MIMO) systems. This wireless technology enormously enhance the capacity and the range of a wireless communication system. It also reduces the bit error rate. The channel between the transmitter and receiving antenna of MIMO system is defined with a channel matrix. Channel gains among transmitter and receiver antenna combinations are the elements of MIMO channel matrix. Uncertainty in the system is generally increased by multipath environment. Antenna array utilized at the receiver, number of elements in the array, statistical properties and the spacing present between the antenna elements are the factors on which system is dependent. These multiple paths only used to create interference in other systems. MIMO can use these additional paths as an additional advantage. It can be applied to give further robustness to the radio link by battering the SNR (signal to noise ratio), or by enhancing the link data capacity. Fading decrease the performance of arrangement due to error rate but MIMO behave freely over different fading channels. It has latency for the enhancement of channel capacity. This technology has great advantage because of its use in digital television (DTV), wireless local area networks

(WLANs), metropolitan area networks (MANs), and mobile communication. MIMO is an essential element of wireless communication standards comprising : IEEE802.11ac (Wi-Fi), IEEE802.11n(Wireless Fidelity), HSPA +(3G) World Wide Interoperability for Microwave Access (4G), and LTE (4G). Home plug AV2 specification and power-line communication for 3-wire installations as a part of ITU-G hn standard are the recent area where it has been used. In adequate MIMO system there are some logical obstructions so it is necessary to enhance the channel capacity between transmitter and receiver of multiple input multiple output system and to develop the techniques to decrease fading effects.

## II. MATHEMATICAL MODEL OF MIMO SYSTEM

The MIMO system consists of group of antennas at transmitter and receiver. Let  $A_T$  are transmitter antennas and  $A_R$  are receiver antennas as shown in Figure. 1. The signal model is expressed as:

$$G = Hq + n \quad (1)$$

Here,

$G$  is ( $A_R \times 1$ ) received signal vector

$q$  is ( $A_T \times 1$ ) transmitted signal vector

$n$  is ( $A_T \times 1$ ) complex additive white Gaussian noise (AWGN) vector with variance  $\sigma$

and  $H$  is the  $(A_R \times A_T)$  channel matrix  
 The outcome of the intermediate in transmitter and receiver links shown by channel matrix.  
 Channel matrix  $H$  can be represented as:

$$H = \begin{bmatrix} h_{11} & \dots & h_{1AT} \\ \vdots & \ddots & \vdots \\ h_{AR1} & \dots & h_{ARAT} \end{bmatrix} \quad (2)$$

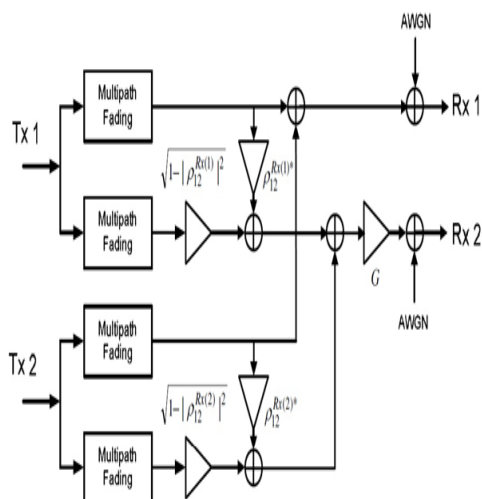


Figure. 1. General Diagram of MIMO system

Channel matrix can endeavor  $K$  corresponding coordinate auxiliary medium with distinct mean gains, where

$$K = \text{rank}(HH^H) \leq \min(A_T A_R) \quad (3)$$

To determine the consequences of channel matrix  $H$  on the capacity, Singular value decomposition (SVD) simplification may be used. So, channel matrix  $H$  can be represented as:

$$H = EDF^H \quad (4)$$

The elements of the unit matrix  $E$  ( $A_T \times A_R$ ) consists of the eigen vectors of  $HH^H$  and the elements of the unitary matrix  $F$  ( $A_T \times A_T$ ) consists of the eigen vectors of  $HH^H$ . The diagonal matrix  $D$  ( $A_R \times A_T$ ) contain elements that are positive and have real value which is known as singular values, equivalent to the Eigen values  $\lambda$  of  $HH^H$  square root.

Suppose that the channel is familiar at both transmitter and receiver (full or perfect channel sensing information CSI) so the maximal normalized capacity as for bandwidth in terms of b/s/Hz spectrum efficiency of coordinate auxiliary medium equals to :

$k$

$$C = \sum_{i=1}^k \log_2(1 + \lambda_i p_i / \sigma_N^2) \quad (5)$$

Here  $p_i$  is the power designate to all sub channel  $i$  and can be resolved to enhance the capacity by using water filling theorem so that all sub channel is filled with appropriate level  $L$  :

$$1/\lambda_i + P_1 + \dots + 1/\lambda_k + P_k = L \quad (6)$$

Or

$$P_i = L - 1/\lambda_i \quad (7)$$

So that it meet the following specification that the total transmitted power is equal to the addition of all power of sub channels or :

$$\sum_{i=1}^k P_i = P_{TX} \quad (8)$$

$P_i$  is contend to zero if  $1/\lambda_i > L$ .

A concise review of the arbitrary distributions used in this paper are as follows:

#### (A) RAYLEIGH DISTRIBUTION

In Rayleigh fading large number of reflections are experienced in the environment. It uses a mathematical approach to examine the propagations and can be used in number of conditions. In this signal fade according to Rayleigh distribution. In Rayleigh fading model a received multipath signal consists of many reflected waves having independent and identical distribution in phase and quadrature amplitude. It is used to miniature the distributed signals that arrive at receiver by different number of pathways. Weibull distribution's special case is Rayleigh distribution. So Rayleigh distribution function is given by

$$f(x) = \frac{r}{b^2} e(-\frac{x^2}{2b^2}) \quad (9)$$

This type of distribution is helped to develop the channel matrix and to calculate the capacity of the system.

$$H_{\text{rayleigh}} = \begin{bmatrix} h_{11} & \dots & h_{1AT} \\ \vdots & \ddots & \vdots \\ h_{AR1} & \dots & h_{ARAT} \end{bmatrix} \quad (10)$$

#### (B) RICIAN DISTRIBUTION

It is similar to Reyleigh fading but the difference is that Rician fading has strong dominant LOS component. Presence of line of sight among transmitter and receiver

make rician fading to occur. Amplitude gain of the signal is characterized with Rician distribution so magnitude of signal which is passing through the channel vary randomly or fade accordingly to distribution. In other words, if there is line of sight then the component which are in direct path goes into deeper fade as compared to multipath component. This type of signal is approximated by Rician distribution.

For Rician distribution the density function is given by:

$$f(x) = \frac{x}{b^2} e^{\frac{(-x^2+s^2)}{2b^2}} I_0\left(\frac{xs}{b^2}\right) \quad (11)$$

Where

The zero order  $I_0$  is first type modified Bessel function,  $s$  ( $s \geq 0$ ) is non-centrality parameter and  $b$  ( $b > 0$ ) is scale parameter.

This type of distribution is used to create the channel matrix as well as to calculate the relevant capacity for the system:

$$H_{\text{rician}} = \begin{matrix} h_{11} & \dots & h_{1AT} \\ \vdots & \ddots & \vdots \\ h_{AR1} & \dots & h_{ARAT} \end{matrix} \quad (12)$$

### (C) NAKAGAMI-M DISTRIBUTION

It is a probability distribution relevant to gamma distribution. In wireless communication area, to represent statistical fading of the multipath environments, nakagami-m distribution is other major distribution that is used and that is established through empirical evaluation. The probability density function for Nakagami-m is :

$$f(x) = \frac{2m^m x^{2m-1} e^{-mx^2}}{\Gamma(m)\Omega^m} - \frac{2x^{2m-1}}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \frac{e^{-mx^2}}{\Omega} \quad (13)$$

Here

$\Omega$  is the second moment

$m$  ( $m \geq 0.5$ ) is shape parameter or Nakagami fading parameter  $\Gamma(\cdot)$  is the standard Gamma function. It also covers wide range of fading conditions. There is one-sided Gaussian distribution when  $m=1/2$  or  $0.5$ . There is Rayleigh distribution when  $m=1$ . The Nakagami model implement a fading scheme that is extra strict than Rayleigh fading when  $m < 1$ .

The Nakagami-m distribution is availed to develop the channel matrix and to find out the relevant system capacity.

$$H_{\text{nakagami}} = \begin{matrix} h_{11} & \dots & h_{1AT} \\ \vdots & \ddots & \vdots \\ h_{AR1} & \dots & h_{ARAT} \end{matrix} \quad (14)$$

### (D) WEIBULL DISTRIBUTION

The weibull distribution is mainly used in both indoor and outdoor environments. The probability density function for weibull distribution is given by

$$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k} \quad (15)$$

The weibull distribution is availed to develop the channel matrix and to find out the relevant capacity for the system.

$$H_{\text{weibull}} = \begin{matrix} h_{11} & \dots & h_{1AT} \\ \vdots & \ddots & \vdots \\ h_{AR1} & \dots & h_{ARAT} \end{matrix} \quad (16)$$

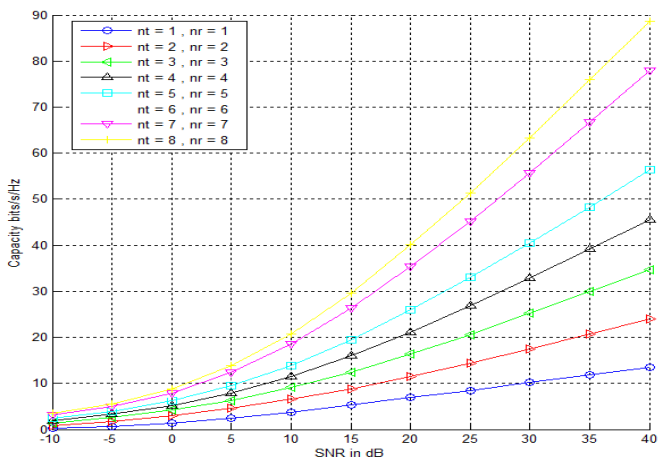
## III. SIMULATION RESULTS

MATLAB m-file is used in this paper to demonstrate the MIMO system along with to simulate the impact of different kind of distributions like Rayleigh, Rician, Nakagami-m and Weibull on the performance of MIMO system.

The simulation is done for eight transmitter and receiver antennas under different fading channels.

### (A) RAYLEIGH DISTRIBUTION

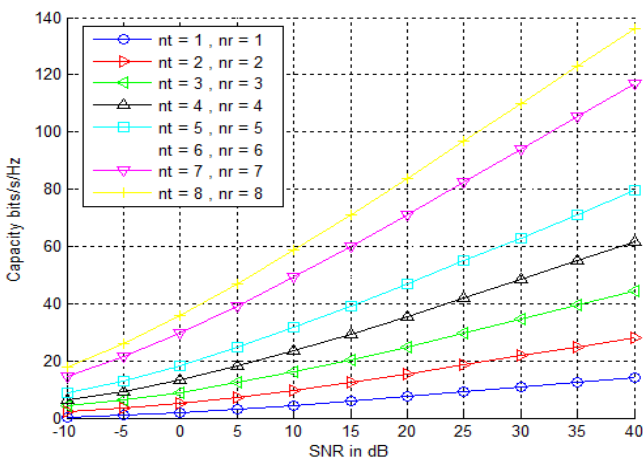
The capacity of MIMO arrangement in terms of bits/sec/Hz is determined under Rayleigh distribution for eight transmitter and receiver antennas over the range of -10dB to 40dB SNR. The results in the Figure. 2 shows the deviation of capacity with number of antennas used. The capacity of MIMO system is enhancing with the number of antennas used in transmitter and receiver side.



**Figure 2.** The channel capacity for Rayleigh Distribution

**(B) RICIAN DISTRIBUTION**

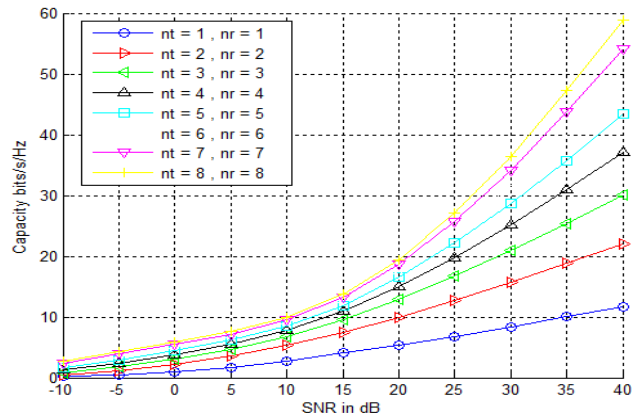
In concern of bits/sec/Hz the capacity of MIMO system is determined under Rician distribution for eight transmitter and receiver antennas over the range of -10dB to 40dB SNR. The results in the Figure.3 shows the deviation of capacity with number of antennas used. For eight antennas unique marker colors and marker symbols are used.



**Figure 3.** The channel capacity for Rician distribution

**(C) WEIBULL DISTRIBUTION**

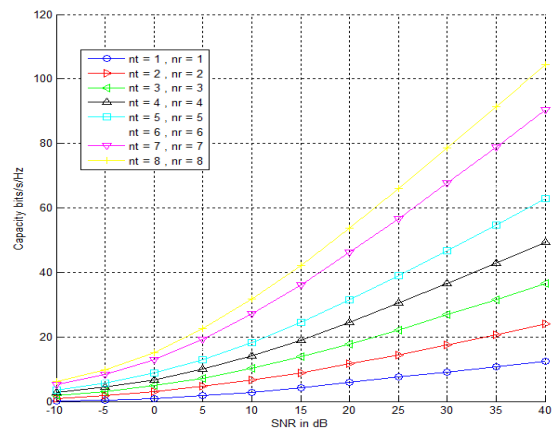
The capacity of MIMO system in terms of bits/sec/Hz is calculated under Weibull distribution for eight transmitter and receiver antennas over the range of -10dB to 40dB SNR. The results in the Figure.4 shows the deviation of capacity with number of antennas used.



**Figure 4.** The channel capacity for Weibull distribution

**(D) M-NAKAGAMI DISTRIBUTION**

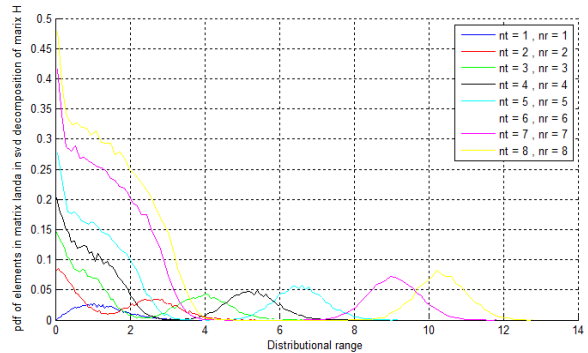
The MIMO system capacity in terms of bits/sec/Hz is determined under m- nakagami distribution for eight transmitter and receiver antennas over the range of -10dB to 40dB SNR. The results in the Figure.5 shows the deviation of capacity with number of antennas used.



**Figure 5.**The channel capacity for m-nakagami distribution

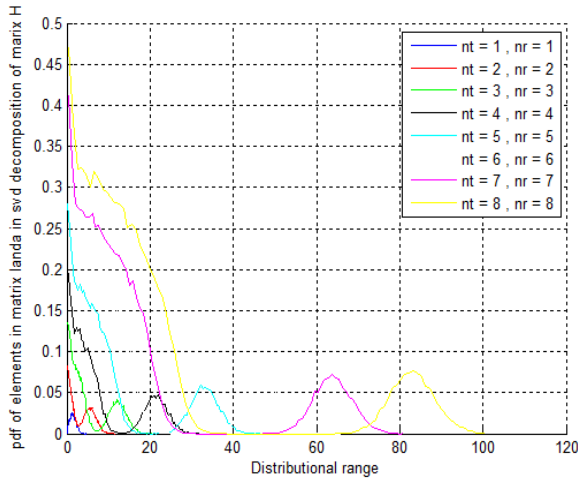
**PROBABILITY DENSITY FUNCTION FOR DIFFERENT DISTRIBUTIONS**

**(A) RAYLEIGH DISTRIBUTION**



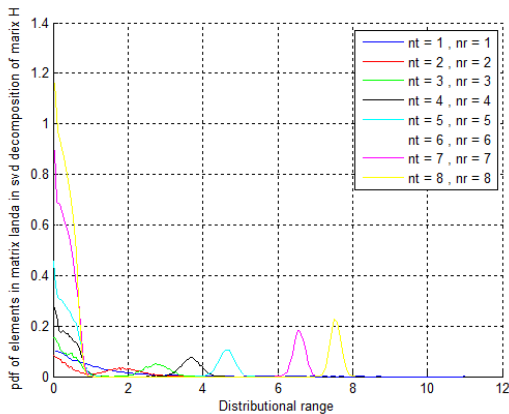
**Figure 6** Probability density function for Rayleigh distribution

**(B) RICIAN DISTRIBUTION**



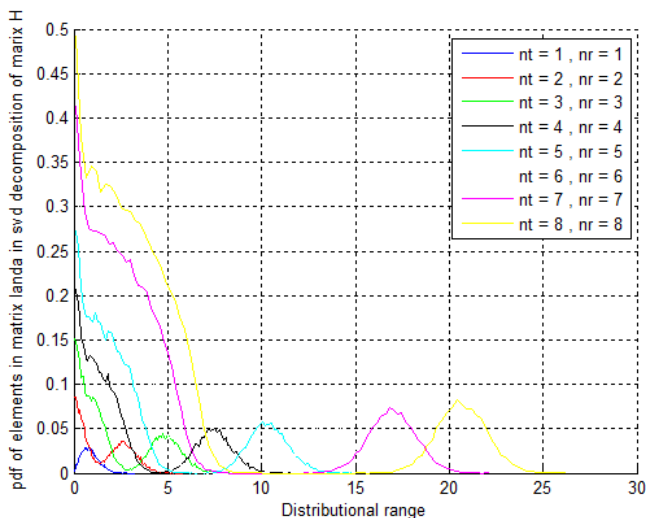
**Figure 7.** Probability density function for Rician distribution

**(C) WEIBULL DISTRIBUTION**



**Figure 8.** Probability density function for Weibull distribution

**(D) M-NAKAGAMI DISTRIBUTION**



**Figure 9.** Probability density function for M-nakagami distribution

From these Figures 6, 7, 8, 9 we can see the probability density function for MIMO system under different fading channels with same transmitter and receiver antennas. It is also concluded from Figures that if we increase number of antennas at transmitter and receiver in MIMO system, intersymbol interference will increase.

**IV. CONCLUSION**

In this paper waterfilling model is used to analyze the performance of 8x8 MIMO system. We studied 8x8 MIMO system over Rayleigh ,Rician, Weibull and m-Nakagami fading channel which give us enhanced capacity. The capacity can also be more enhanced by increasing number of antennas at transmitter and receiver side of MIMO system. The MIMO system has greater capacity over Rician fading channel and lesser over weibull fading channel within same SNR. At low SNR capacity enhances linearly but with high signal-to-noise-ratio it enhances logarithmically. It is also shown that if number of antennas will increased at transmitter and receiver side, intersymbol interference will increase.

**V. REFERENCES**

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