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Use of Hybrid PSOGSA Search Algorithm for Optimum Design of RC Beam

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ABSTRACT

A more realistic and optimum design of reinforced concrete (RC) beam using a hybrid of particle swarm optimization and gravitational search algorithm (PSOGSA) is presented in this paper. Optimal size and reinforcement of the beam element have been found by employing the technique in computer aided environment, whereby the whole process of analysis, design and optimization has been coded in C++. The analysis and design procedure follows specifications of Indian codes. Use of gravitational search along with standard particle swarm optimization technique has been found to possess a better capability to escape from local optimums with faster convergence than the standard PSO and GSA. In this approach, different variables of beam element have been considered as continuous functions and rounded off appropriately to imbibe the practical relevance of the present study. Few beam design examples have been considered to emphasis the validity of this optimum design procedure.

Keywords: Optimum Design, Particle Swarm Optimization, Gravitational Search Algorithm, Indian Design Standards.

I. INTRODUCTION

Material cost is an issue of grave importance in design and construction industry. As a solution, numerous optimization techniques have been proposed by the researchers to cut down the use of material by designing lighter structures. In reinforced concrete structures, use of three different cost items consisting of concrete, steel and formwork influence the total cost of the structure which makes the optimization process more complicated [1]. Many researchers deal these complexities by optimizing reinforced concrete structures using heuristic evolutionary algorithms. These include genetic algorithm (GA), simulated annealing (SA), harmony search (HS), ant colony optimization (ACO) and particle swarm optimization (PSO) and their hybrids obtained from combining of two or more algorithms. The hybridization of algorithms has been performed to integrate the strengths of different algorithms and to overcome the weaknesses of them. The hybrid big bang-big crunch (BB-BC) was originally developed by Erol and Eksin [2] and was proved to outperformed hybrid than classic genetic algorithms (GA) for many benchmark optimization functions. The applicability of artificial neural networks (ANN) and genetic algorithm (GA) for optimum design of singly and doubly reinforced beams has been presented in [3]. Camp [4-5] and Kaveh and Talatahari [6-7] both proposed hybrid forms of the BB-BC and demonstrated its applicability and computational efficiency in solving structural engineering optimization problems. Sahab et al. [8] presented a two-stage hybrid optimization algorithm based on a modified genetic algorithm. A hybrid particle swarm optimizer and ant colony approach PSACO which was initially introduced by Shelokar et al. [9] for the solution of the continuous unconstrained problems and recently utilized for truss structures. Kaveh and Talatahari[10] hybridized Particle swarm optimizer, ant colony strategy and harmony search scheme for optimization of truss structures.

The optimization of reinforced concrete beams have been performed using the capability of genetic algorithm in most of the previous works [11-14]. The present paper hybridizes PSO and gravitational search algorithm (GSA) for optimum design of RC beam member, and it is based on the principles of two different methods in search of optimal design solution. The new algorithm named as hybrid PSOGSA combines the social thinking feature in PSO with the local search capability of GSA. The other features of PSO and GSA are explained in detail by the researchers in their studies [15-18]. The advantages of PSO consist of easy implementation and have smaller number of parameters to be adjusted. However, it is reported that the original standard PSO had difficulties in controlling the balance between exploration (global investigation of the search place) and exploitation (the fine search around a local optimum) [19]. In order to improve this character of PSO, it is hybridized with other approaches.

II. METHODS AND MATERIAL

A. **Formulation of Optimization Problem**

In the present optimization problem, design variables are determined in such a way that the cost (objective function) becomes minimum. Some design restrictions or constraints limit the values of these design variables. The total cost of the material includes the cost of reinforcement (flexural and shear), cost of concrete and cost of formwork. Since the proposed algorithm is applicable for unconstrained and continuous optimization problem, the formulation of penalized objective function – including imposed penalties due to violation of constraints – is done to convert the constrained problem into an unconstrained one.
The cost of reinforced concrete beam is given as:

$$C = C_{st}V_{st} + C_C V_C \qquad (1)$$

C is the total cost of beam element; C_{st} cost of steel per unit volume of steel; V_{st} total volume of steel in the beam; C_C cost of concrete per unit volume of concrete; V_C total volume of concrete in the beam. The cost of formwork does not vary significantly for any particular place so can be dropped while evaluating the objective function.
Dividing equation (1) by C_C

$$\frac{C}{C_C} = \frac{C_{st}}{C_C} V_{st} + V_C$$

substituting $\frac{C}{C_C} = Z$, $\frac{C_{st}}{C_c} = \alpha$ (cost ratio) and $V_C = V_G - V_{st}$, equation (2) is obtained
V_G is the gross volume of beam. Since C_C is a constant parameter for a given place, the objective function Z represents total cost of beam required to minimize.

$$\text{Minimize } Z = (\alpha - 1)V_{st} + V_G \qquad (2)$$

Volume of steel V_{st} depends upon area of steel and its provided length. Similarly gross volume of concrete depends upon cross sectional area and length of beam.

In the present study, all input design parameters have been considered fixed. These include span of beam, grade of reinforcement and concrete, intensity of gravity loads, end moments, effective cover of concrete and cost ratio (ratio of per unit cost of reinforcement to per unit cost of concrete). The design variables of the beam considered in the present model are width (b_B) and effective depth (d_B) of the beam for cross section design and area of longitudinal reinforcement and shear reinforcement (A_{st}) have been calculated as dependent design parameters. Designs constraints considered in the present study not only considers Indian code provisions for RC beam design (IS 456: 2000), but also few practical aspects as well [20].

Condition for moment of resistance of the section at end supports and at the point of maximum sagging moment

For a given beam, the cross-sectional dimensions (depth and width) and area of steel to be provided at the ends and centre shall be such that moment of resistance of beam is greater than the actual hogging moment at its ends and actual sagging moment that the beam is subjected to at its centre respectively.

$$0.87\, f_y A_{stend}\left(d_B - \frac{f_y A_{stend}}{f_{ck}\, b_B}\right) > M_h$$

$$0.87\, f_y A_{stmid}\left(d_B - \frac{f_y A_{stmid}}{f_{ck}\, b_B}\right) > M_S$$

Condition for depth of beam from limit state of serviceability: deflection consideration

As per code [20], for spans upto 10 m, the vertical deflection of a continuous beam shall be considered within limits if the ratio of its span (l) to its effective depth is less than 26. For spans above 10 m, factor 26 is to be multiplied by $\frac{10}{l}$, it can be expressed as

$\frac{l}{d_B} \leq 26$, when span $\leq$ 10 m (l and d_B are in m)

$\frac{l}{d_B} \leq 26\left(\frac{10}{l}\right)$, when span > 10 m ($l$ and d_B are in m)

Condition for minimum width of beam

From practical point of view, the beam shall be wide enough to accommodate at least two bars of tensile steel of given diameter or it should not be less than 300 mm.

$$b_B \geq b_{B_{min}}$$

Condition for limiting depth of neutral axis

To ensure that tensile steel does not reach its yield stress before concrete fails in compression so as to avoid brittle failure, the maximum depth of neutral axis is restrained, it can be expressed as

$$\frac{0.87\, f_y\, A_{stend}}{0.36\, f_{ck}\, b_B\, d_B} < \frac{x_m}{d_B} \quad \text{and}$$

$$\frac{0.87\, f_y\, A_{stmid}}{0.36\, f_{ck}\, b_B\, d_B} < \frac{x_m}{d_B}$$

$\frac{x_m}{d_B}$ value varies with the grade of steel and is given as:

$\frac{x_m}{d_B}$ = 0.53, when f_y = 250 N/mm^2

$\frac{x_m}{d_B}$ = 0.48, when f_y = 415 N/mm^2

$\frac{x_m}{d_B}$ = 0.46, when f_y = 500 N/mm^2

Condition for minimum and maximum tensile steel at the beam ends and in the middle of beam

The minimum and maximum area of tensile steel to be provided shall be taken as:

$$A_{stend(\min)} \geq \frac{0.85\, b_B\, d_B}{f_y} \;;\; A_{stend(\max)} \leq 0.04\, b_B\, D_B$$

$$A_{stmid(\min)} \geq \frac{0.85\, b_B\, d_B}{f_y} \quad A_{stmid(\max)} \leq 0.04\, b_B\, D_B$$

B. **Overview of Hybrid PSOGSA**

Determination of global optimal solution among all possible inputs is the aim of implementing any optimization algorithm and to improve the performance, hybridization of two or more algorithms is performed. Several heuristic algorithms have been combined to form hybrid methods for optimization problems. The basic idea of combining Standard PSO with GSA was suggested by Mirjalili and Hashim (2010) [19]. They combined social thinking ability of PSO and search capability of GSA.

In order to explain this algorithm, a system with N masses (agents) is considered in which the position of the i^{th} mass is defined as:

$$X_i = (x_i^1, .. x_i^2 \ldots, x_i^n \ldots \ldots .. x_i^d \;,\; i = 1,2,3 \ldots . N \quad (3)$$

x_i^d is the position of i^{th} mass in the d^{th} dimension, and n is the dimension of the search space. In this case, the positions of masses are the candidate solutions for the problem, which at the next iterations of the algorithm will be improved. According to Rashedi (2009), each agent's mass is calculated after the evaluation of the current population's fitness and considered as a candidate solution. After initialization of agents, their masses, gravitational force, gravitational constant, and resultant forces (4-10) among them are calculated. After calculating the accelerations and with updating the best solution so far, the velocities of all agents can be calculated using (11). Finally, the positions of agents are defined as (12). The process of updating velocities and positions will be stopped by meeting an end criterion.

$$m_i\,(\mathrm{t}) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \qquad (4)$$

$$M_i(\mathrm{t}) = \frac{m_i(\mathrm{t})}{\sum_{j=1}^{N} m_i(\mathrm{t})}$$

For any minimization problem

$$best(t) = min_{j\in(1\ldots.N)}\, fit_j(t) \qquad (5)$$

$$worst(t) = max_{j\in(1\ldots.N)}\, fit_j(t) \qquad (6)$$

In this relation, $M_i(\mathrm{t})$ and $fit_i(t)$ represent the mass and the fitness value of the agent i at t. According to the gravity law, the overall forces from a set of heavier masses are used to calculate the agent's acceleration (11) by using following equations:

$$F_{ij}^{d}(t) = \frac{G(t)\, M_j(t)\, M_i(t)}{R_{ij}(t)+\epsilon}\left(x_j^{d}(t) - x_i^{d}(t) \right) \quad (7)$$

$R_{ij}(t)$ - Euclidian distance between two agents i and j and ε - a small constant. Gravitational constant $G(t)$ is initialized at the beginning of the search and will be reduced with time to control search accuracy as follows:

$$G(t) = G_O(t) + \left(\frac{t}{t_{max}}\right)\beta \quad (8)$$

t - Current iterations, t_{max} is the maximum number of iteration. The parameters maximum number of iterations t_{max} , population size N, initial gravitational constant G_O and constant β control the performance of GSA.

$$F_i^{d}(t) = \sum_{j\in N, j\neq i} rand_j F_{ij}^{d}(t) \quad (9)$$

$$a_i^{d}(t) = \frac{F_i^{d}(t)}{M_i(t)} \quad (10)$$

This hybrid is a stochastic algorithm with a feature to select randomly, the important parameters that have an influence on the search procedure. The advantage of implementing PSOGSA is that it avoids getting trapped in local optima, and also improves upon premature convergence probability. It thereby reaches at better optimal solution in a reasonable time. The functionality of both the algorithms is combined and run parallel. The modified velocity equation becomes as stated in Eq. (11).

$$v_i^{d}(t+1) = wv_i^{d}(t) + c_1' \,.r.\, a_i^{d}(t) + c_2' \,.r.\left(p_g^{d}(t) - x_i^{d}(t)\right) \quad (11)$$

$v_i^{d}(t)$ represents velocity of agent i at iteration t, c_1' and c_2' are the positive numbers illustrating the weights of the acceleration terms that guide each particle towards the individual best and swarm best positions respectively. w is the weighing function, r is a random number between 0 and 1, $a_i^{d}(t)$ is the acceleration of agent i at iteration t , and p_g is the best solution so far. $d'^{d}_{i}(t)$- includes democratic influence of other particles on i^{th} particle in d^{th} dimension.

Each iteration updates the position of particles as (12)

$$x_i^{d}(t+1) = x_i^{d}(t) + v_i^{d}(t+1) \quad (12)$$

in which the time interval is equal to 1.0 and thus the velocity vector can be added to the position vector. It is clear that the information produced by all members of the swarm moving with an acceleration guided by GSA, is utilized by the PSO with the purpose of determining new position of each particle, and thus the phrase modified PSOGSA.

III. RESULTS AND DISCUSSION

In order to evaluate the performance of PSOGSA technique, some examples of RC beams those are the members of any RC frame structure are studied. The given set of loads (gravity loads and end moments) for the beam is shown in Figure1. The configuration and steel reinforcement are the design variables required to reach at objective criteria. Grades of concrete and steel were taken as input variables and cost ratio (α) is a parameter which varies at different places. The maximum depth to width ratio is designer's input parameter to avoid thin sections and kept between1.5 to 3 in this case.

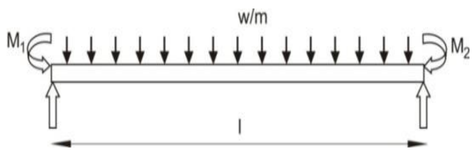

Figure1: Generalized sectional view of beam

The beams design problems have been solved initially by conventional limit state method as per IS456:2000 and then a set of solution is obtained by applying hybrid particle swarm optimization technique and gravitational search algorithm (PSOGSA). The constant parameters of the algorithm those will be found finetuned with them are as follows:

$G_O = 100$; $C_1 = 0.5$; $C_2 = 2$; β = 20(for PSOGSA)

The population size and maximum number of iterations are also initial input parameters for any population based algorithm and taken as 20 and 500 respectively in this case. The maximum number of iterations is stopping criteria in search of optimum results. It is necessary to define the upper and lower bounds of design variables for the random selection of population. These bounds are given in Table I.

TABLE I

UPPER AND LOWER BOUNDS OF DESIGN VARIABLES

Design variable	Lower bound	Upper bound	Nature of variables
Width (b)	300 mm	450 mm	Integer
Depth (D)	450 mm	900 mm	Integer
Rebar Diameter (φ)	12 mm, 16 mm, 20 mm, 25 mm		Discrete
Number of Rebars (NB)	2	12	Integer

NUMERICAL EXAMPLES

A. **Problem Definition**

Design a beam element of a frame having a given span and carries a set of loads i.e. end moments and gravity load (including self weight as well as the imposed load) on it as shown in Figure1. Five different designs are carried out for some fixed parameters such as grades of concrete and steel as M30 and Fe415 respectively. The cost ratio is kept equal to 100. Effective cover to reinforcement is 50mm for all design examples. All other input parameters are mentioned in TABLE II

TABLE II

INPUT PARAMETERS OF BEAM DESIGN EXAMPLES

Example No.	Span (m)	Gravity load (kN/m)	End moments (kN-m)	
	L	W	M_1	M_2
1	5	30	50	100
2	6	35	70	175
3	7	40	80	200
4	8	45	150	300
5	9	50	200	350

B. **General Solution Procedure**

As the algorithm is meant for continuous variable solutions and to make it suitable for realistic structure design problems, the cross sectional dimensions of beam found as continuous variables, rounded off to the nearest integer which is a multiple of ten to get a discrete optimum width and depth of the beam are calculated. This is the first stage of optimizing cross sectional dimensions of the beam.

The remaining part of the beam design problem is optimization of reinforcement. Area of reinforcement has large influence on the objective criteria. In the literature there are two ways to decide optimum reinforcement. In the first method, a database of all possible sections with number of bars used is constructed and arranged in order of moment of resistance of sections. Some researchers [1, 21-22] have prepared the database for reinforcement detailing with specific diameter rebars. The database has been constructed by incorporating all the geometric constraints but moment capacity constraint has been checked explicitly. On the other hand, in the studies [13, 23], different diameter rebars in different layers while constructing the reinforcement templets have been used but using different diameters are not common in practice. In the second approach, reinforcement topology is decided without constructing database. The commercially available bar sizes are treated as discrete variables and provided in such a way it gives area of steel almost equal to the area obtained from optimum dimensions of beam and number of rebars should be such that these can be accommodated in a designed optimum section by fulfilling the criterion of spacing between bars as per IS456:2000. So in the present study we adopt second approach for placing flexural reinforcement.

The total amount of reinforcement constitute flexural as well as shear reinforcement. Provision of shear reinforcement as per codal requirement has been done to achieve at total area of reinforcement. For the given set of input parameters, the cross-sectional dimensions of beam, area of longitudinal reinforcement at the end (Ast_1 & Ast_2) and in the middle of beam (Ast_3 & Ast_4) are obtained. The flow chart for beam design optimization has been shown in Figure 2.

All optimization runs are carried out on a standard PC with a Intel® Core™ i3 CPU M350 @2.27 GHz frequency and 3 GB RAM memory. The algorithm has been coded in Turbo C++ in installed in Window 7. (32 bit operating system). The computing time for optimization procedure for a beam design is 2 sec which is much less than those available in the literature.

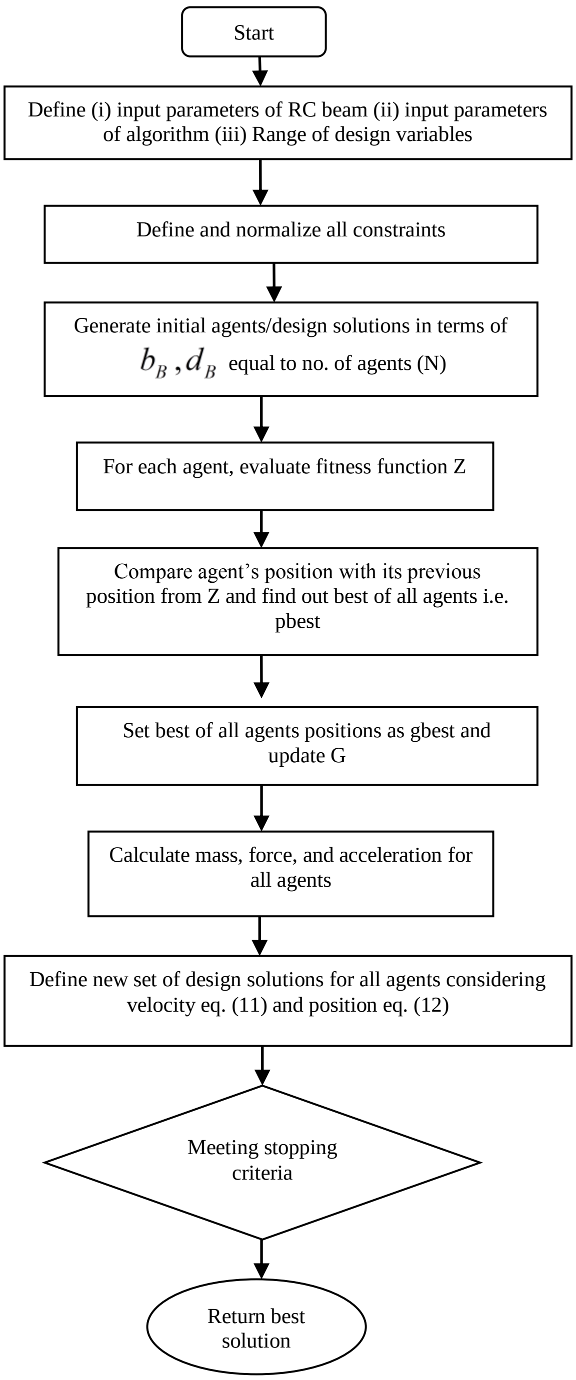

Figure2: Flow chart for RC beam optimal design using PSOGSA

Table III presents the solution set in terms of independent design variables and derived quantities to get at ready in hand solution. Table IV shows the convergence behaviour and computing time required for optimum design process. The number of iterations required to get optimal solution and time taken for optimization process indicate that the convergence is very fast using PSOGSA.

TABLE III

OPTIMAL DESIGN VALUES FOR BEAM DESIGN

STAGE 1: OPTIMAL CROSS SECTIONAL DIMENSIONS OF BEAM

	Cross section		**Top Reinforcement at each end**		**Bottom Reinforcement in middle**	
Ex	**Depth (mm)**	**Width (mm)**	**Ast_1 (mm^2)**	**Ast_2 (mm^2)**	**Ast_3 (mm^2)**	**Ast_4 (mm^2)**
1	500	300	302	604	348	1043
2	650	300	408	816	468	1403
3	750	300	398	797	563	1691
4	900	300	500	1000	679	2038
5	900	300	592	1185	894	2682

Contd…TABLE III

STAGE 2: OPTIMAL REINFORCEMENT DETAILING OF BEAM

Ex	**Top reinforcement at each end**				**Bottom reinforcement in middle**			
	NB_1	**Φ_1 (mm)**	**NB_2**	**Φ_2 (mm)**	**NB_3**	**Φ_3 (mm)**	**NB_4**	**Φ_4 (mm)**
1	2	12	2	20	2	16	10	12
2	4	12	8	12	5	12	3	25
3	2	16	4	16	5	12	6	20
4	5	12	5	16	4	16	7	20
5	3	16	6	16	3	20	9	20

TABLE IV

CONVERGENCE OF OPTIMUM DESIGN OF RC BEAM

Ex	**No. of Iterations**	**Computing time (sec)**	**Objective function Z (C/C_c)**
1	156	2	1.866971
2	141	2	2.924651
3	142	2	3.83498
4	130	2	5.338971
5	124	2	6.879218

The optimal design of RC beams is incomplete without proper reinforcement detailing. The optimal reinforcement detailing has been done to obtain quick and accurate solutions of beam design. At stage 2, the area of steel is calculated on the basis of number of bars and diameter of bars. It is noticed that the proper detailing of reinforcement has significant effect on objective function.

IV.CONCLUSION

In this study, the reinforced concrete beams subjected to end moments and gravity loads are optimized by using PSOGSA according to the rules of IS456:2000. In the numerical examples, optimum dimensions and reinforcements are investigated in the beams. The developed program is capable to find the optimum design cost efficiently in terms of objective function. A parameter called 'cost ratio' has been considered for prevalent prices of steel and concrete at a given place so as to impart practical relevance to the study instead of taking it only a piece of pure academic work The computing time for optimization process is only 2 sec. This makes the approach stronger for the practical applications in future.

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NOTATIONS

A_{stend}	Area of steel at the beam end (mm^2)
$A_{stend(min)}$	Minimum area of steel at the beam end (mm^2)
$A_{stend(max)}$	Maximum area of steel at the beam end (mm^2)
A_{stmid}	Area of steel at the middle of the beam (mm^2)
$A_{stmid(min)}$	Minimum area of steel at beam mid (mm^2)
$A_{stmid(max)}$	Maximum area of steel at the beam mid (mm^2)
b_B	Width of beam (mm)
b_{Bmin}	Minimum width of the beam (mm)
D_B	Overall depth of beam (mm)
d_B	Effective depth of beam (mm)
d_b	Max peripheral dist. among longitudinal bars of column (mm)
f_{ck}	Characteristic compressive strength of concrete (N/mm^2)
f_y	Characteristic strength of steel(N/mm^2)
l	Length of the beam (m)