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A New Approach of Solving Fuzzy Game Problem of Order 3 X 3 Using Dodecagonal Fuzzy Numbers

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ABSTRACT

Fuzzy Game problem has recently applied in Optimization technique fields. Many Practical problems require decision making in a competitive situation called a Game. In such situation it is useful to model the problems as games with fuzzy payoffs. A new approach of solving fuzzy game problem of order 3x3 using Dodecagonal fuzzy numbers and its solution is discussed using maximin-minimax principle.

Keywords : Dodecagonal Fuzzy Numbers, Fuzzy Game Problem, Fuzzy Number, Maximin-Minimax Principle, Membership Function,

I. INTRODUCTION

The Mathematical treatment of the Game theory was made available in 1944.The problem of game theory defined as the course of action (alternatives) for each competitor. The Theory of games is applied in many practical problems which we encounter in real situation. Fuzzy number may be normal (or) abnormal, triangular (or) trapezoidal (or) octagonal. A membership function and ranking of fuzzy number is defined for Dodecagonal fuzzy numbers. By using this ranking the fuzzy game problem is converted to a crisp value problem. In this paper, the fuzzy game problem is illustrated with some operations of Dodecagonal fuzzy numbers.

II. PRELIMINARIES

2.1 Definition:

Let X be non-empty set. A Fuzzy set A in X is characterized by its membership function
$A \rightarrow [0,1]$ and A(x) is interpreted as the degree of membership of element x in Fuzzy A for each $x \in X$.

The value zero is used to represent complete non-membership; the value one is used to represent complete membership and value in between are used to represent intermediate degree of membership. The mapping A is also called the membership function of fuzzy set A.

2.2 Definition:

A crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

2.3 Definition:

A Fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

i) There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$
ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.
iii) $\tilde{A}$ Must be normal and convex.

2.4 Definition:

A Fuzzy number A_{DD} is denoted by
$A_{DD} = (a_1,a_2,a_3,a_4,a_5,a_6, a_7,a_8, a_9, a_{10}, a_{11}, a_{12})$ where $a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9, a_{10}, a_{11}$ and a_{12} the real numbers and its membership function are given below

$$\mu_{\tilde{A}}(\mathbf{x}) = \begin{cases} 0 & for\ x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ 1 & for\ a_4 \le x \le a_5 \\ k+(1-k)\left(\frac{x-a_5}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k+(1-k)\left(\frac{a_8-x}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 1 & for\ a_8 \le x \le a_9 \\ k+(1-k)\left(\frac{a_{10}-x}{a_{10}-9}\right) & for\ a_9 \le x \le a_{10} \\ k & for\ a_{10} \le x \le a_{11} \\ k\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & for\ a_{11} \le x \le a_{12} \\ 0 & for\ x \ge a_{12} \end{cases}$$

Where $0 < k < 1$

2.5 Definition (Ranking of fuzzy number)

Let $\tilde{A}$ be a normal Dodecagonal fuzzy number. The value $M_0^{DoDe}(\tilde{A})$, called the measure of $\tilde{A}$ is calculated as follows:

$M_0^{Dode}(\tilde{A}) = \frac{1}{6}[(a_1+a_2+a_3+a_{10}+a_{11}+a_{12})k + (a_4+a_5+a_6+a_7+a_8+a_9)(1-k)]$ where $0 \le k \le 1$

2.6 Definition- Pure strategy

If a player known exactly what the other player is going to do is called pure strategy.

2.7 Definition- Saddle point.

If the max-min value equals the min-max value, then the game is said to have a saddle point and the corresponding strategies which give the saddle point are called optimal strategies. The amount of payoff at an equilibrium point is called the crisp game value of the game matrix.

2.8 Solution of all 3x3 matrix game

Consider the general 3x3 game matrix A = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

To solve this game we proceed as follows:

(i) Test for a saddle point.
(ii) If there is no saddle point, solve by finding equalizing strategies.

The Optimal mixed strategies for player A = (p_1, p_2) and

For player B = (q_1, q_2)

where $p_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$; $p_2 = 1 - p_1$

$q_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$; $q_2 = 1 - q_1$ and

Value of the game $V = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$

2.9 Example

Consider the following fuzzy game problem

Player A

$$\text{Player B} \begin{pmatrix} (3,4,5,6,7,8,9,10,11,12,13,14) & (-3,-2-1,0,1,2,3,4,5,6,7,8) & (-2,-1,0,1,2,3,4,5,6,7,8,9) \\ (1,2,3,4,5,6,7,8,9,10,11,12) & (0,1,2,3,4,5,6,7,8,9,10,11) & (2,3,4,5,6,7,8,9,10,11,12,13) \\ (-13,-12-11,-10,-9,-8,-7,-6,-5,-4,-3,-2) & (-1,0,1,2,3,4,5,6,7,8,9,10) & (-4,-3,-2,-1,0,1,2,3,4,5,6,7) \end{pmatrix}$$

Solution

By definition of Dodecagonal fuzzy number $\tilde{A}$ is calculated as

$M_0^{Dode}(\tilde{A}) = \frac{1}{6}[(a_1+a_2+a_3+a_{10}+a_{11}+a_{12})k + (a_4+a_5+a_6+a_7+a_8+a_9)(1-k)]$

Where $0 \le k \le 1$

STEP 1 :

Convert the given fuzzy problem into a crisp value problem.

This problem is done by taking the value of k is 0.6

We Obtain the value of the $M_0^{Dode}(a_{ij})$

$a_{11} = (3,4,5,6,7,8,9,10,11,12,13,14)$

$M_0^{Dode}(a_{11}) = \frac{1}{6}[(3+4+5+12+13+14)(0.6) + (6+7+8+9+10)(1-0.6)]$

$= \frac{1}{6}[(51)(0.6) + (40)(0.4)]$

$= \frac{1}{5}[30.6 + 16]$

$= \frac{1}{6}[46.6]$

$M_0^{Dec}(a_{11}) = 7.77$

a_{12} = (-3,-2,-1,0,1,2,3,4,5,6,7,8)

$M_0^{Dode}(a_{12}) = \frac{1}{6}$ [(-3-2-1+6+7+8)(0.6) + (0+1+2+3+4+5) (1-0.6)]

$= \frac{1}{6}$ [(15) (0.6) + (15) (0.4)]

$= \frac{1}{5}$ [9+6]

$= \frac{1}{6}$ [15]

$M_0^{Dec}(a_{12}) = 2.5$

a_{13} = (-2,-1,0,1,2,3,4,5,6,7,8,9)

$M_0^{Dode}(a_{13}) = \frac{1}{6}$ [(-2-1+0+7+8+9)(0.6) + (1+2+3+4+5+6) (1-0.6)]

$= \frac{1}{6}$ [(21) (0.6) + (21) (0.4)]

$= \frac{1}{5}$ [12.6+8.4]

$= \frac{1}{6}$ [21]

$M_0^{Dec}(a_{13}) = 3.5$

a_{21} = (1,2,3,4,5,6,7,8,9,10,11,12)

$M_0^{Dode}(a_{21}) = \frac{1}{6}$ [(1+2+3+10+11+12)(0.6) + (4+5+6+7+8+9) (1-0.6)]

$= \frac{1}{6}$ [(39) (0.6) + (39) (0.4)]

$= \frac{1}{5}$ [23.4+15.6]

$= \frac{1}{6}$ [39]

$M_0^{Dec}(a_{21}) = 6.5$

a_{22} = (0,1,2,3,4,5,6,7,8,9,10,11)

$M_0^{Dode}(a_{22}) = \frac{1}{6}$ [(0+1+2+9+10+11)(0.6) + (3+4+5+6+7+8) (1-0.6)]

$= \frac{1}{6}$ [(33) (0.6) + (33) (0.4)]

$= \frac{1}{5}$ [19.8+13.2]

$= \frac{1}{6}$ [33]

$M_0^{Dec}(a_{22}) = 5.5$

a_{23} = (2,3,4,5,6,7,8,9,10,11,12,13)

$M_0^{Dode}(a_{23}) = \frac{1}{6}$ [(2+3+4+11+12+13)(0.6) + (5+6+7+8+9+10) (1-0.6)]

$= \frac{1}{6}$ [(45) (0.6) + (45) (0.4)]

$= \frac{1}{5}$ [27+18]

$= \frac{1}{6}$ [45]

$M_0^{Dec}(a_{23}) = 7.5$

a_{31} = (-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2)

$M_0^{Dode}(a_{31}) = \frac{1}{6}$ [(-13-12-11-4-3-2)(0.6) + (-10-9-8-7-6-5) (1-0.6)]

$= \frac{1}{6}$ [(-45) (0.6) + (-45) (0.4)]

$= \frac{1}{5}$ [-27-18]

$= \frac{1}{6}$ [-45]

$M_0^{Dec}(a_{31}) = -7.5$

a_{32} = (-1,0,1,2,3,4,5,6,7,8,9,10)

$M_0^{Dode}(a_{32}) = \frac{1}{6}$ [(-1+0+1+8+9+10)(0.6) + (2+3+4+5+6+7) (1-0.6)]

$= \frac{1}{6}$ [(27) (0.6) + (27) (0.4)]

$= \frac{1}{5}$ [16.2+10.8]

$= \frac{1}{6}$ [27]

$M_0^{Dec}(a_{32}) = 4.5$

a_{33} = (-4,-3,-2,-1,0,1,2,3,4,5,6,7)

$M_0^{Dode}(a_{33}) = \frac{1}{6}$ [(-4-3-2+5+6+7)(0.6) + (-1+0+1+2+3+4) (1-0.6)]

$= \frac{1}{6}$ [(9) (0.6) + (9) (0.4)]

$= \frac{1}{5}$ [5.4+3.6]

$= \frac{1}{6}$ [9]

$M_0^{Dec}(a_{33}) = 1.5$

STEP 2

The Payoff matrix is

$$\begin{array}{cc} & \text{Player A} \\ \text{Player B} & \begin{pmatrix} 7.77 & 2.5 & 3.5 \\ 6.5 & 5.5 & 7.5 \\ -7.5 & 4.5 & 1.5 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & \text{R – Min} \\ \begin{pmatrix} 7.77 & 2.5 & 3.5 \\ 6.5 & 5.5 & 7.5 \\ -7.5 & 4.5 & 1.5 \end{pmatrix} & \begin{array}{c} 2.5 \\ \mathbf{5.5} \\ -7.5 \end{array} \\ \text{Col-max} \quad 7.77 \quad \mathbf{5.5} \quad 7.5 & \end{array}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 5.5

Minimum of 3rd row = -7.5

Maximum of 2nd column = 7.77

Maximum of 2nd column = 5.5

Maximum of 3rd column = 7.5

Max (min) = 5.5, Min (max) = 5.5

It has saddle point. The Crisp solution to the problem is saddle point = (A_2,B_2)

Value of the game = 5.5

III. CONCLUSION

In this paper, the Dodecagonal fuzzy number plays a main role in the fuzzy game problem to solve by pure stragies and its value of the game is captured. The parameter k can be adjusted for getting the desired result. If the parameter k has different values for the same fuzzy game problem, we get different fuzzy game value.

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