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On Solving Fuzzy Game Problem by Graphically Using Quadrangle Fuzzy Number

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ABSTRACT

In this paper, a new concept of fuzzy game problem using Quadrangle fuzzy numbers with some operation is introduced. By using ranking to the payoffs, we convert the fuzzy valued game problem to crisp valued game problem, which can be solved using the Graphical method. The solution of such fuzzy games with mixed strategies by minimax-maximin principle is discussed.

Keywords : Fuzzy numbers, Fuzzy ranking, Fuzzy game problem, Graphical method, Membership function , mixed strategy, Quadrangle fuzzy number, Saddle point.

I. INTRODUCTION

Game theory is a mathematical theory that deals with the general features of competitive situations like some operations of quadrangle fuzzy numbers in a formal abstract way. It places particular emphasis on the decision making processes of the adversaries.. The focus in this paper is on the minimax principle which states that each competitor will act so as to minimize one player's maximum loss (or maximize one player's minimum gain). Furthermore, even if the opponent is able to use only his knowledge of the tendencies of the first player to deduce probabilities that are different from those for the optimal mixed strategy, then the opponent still can take advantage for this knowledge to reduce the expected payoff to the first player. We have analyzed the solution of such fuzzy games with mixed strategies by minimax – maximin principle. By using graphical method.

II. PRELIMINARIES

2.1 Fuzzy Numbers

A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its

Membership function $\mu_A : \mathbb{R} \to [0,1]$ has the following characteristics

(i) A is normal. It means that there exists an $x \in \mathbb{R}$ such that $\mu_A(x) = 1$

(ii) A is convex. It means that for every $x_1, x_2 \in \mathbb{R}$,

$\mu_A(\lambda x_1 + (1-\lambda) x_2) \geq \min\{ (\mu_A(x_1)), (\mu_A(x_2)) \}, \lambda \in [0,1]$

(iii) μ_A is upper semi-continuous.

(iv) sup $(\tilde{A})$ is bounded in $\mathbb{R}$.

2.2 Quadrangle fuzzy number

For a Quadrangle fuzzy number A(x), it can be represented by A (a,b,c,d;1) with membership function $\mu_{A(x)}$ given by,

$$\mu_{A(x)} = \begin{cases} \frac{(x-a)}{b-a}, & a \leq x \leq b \\ 1, & b < x < c \\ \frac{(d-x)}{d-c}, & c \leq x \leq d \\ 0, & otherwise \end{cases}$$

2.3 Ranking of Quadrangle Fuzzy Number

Several approach for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for computing the fuzzy numbers is by the use of a ranking function. That is, for every

$\tilde{A} = (a_1{:}a_2{:}a_3{:}a_4) \in \mathbb{F}(\mathbb{R})$, the ranking function $\mathfrak{R}: \mathbb{F}(\mathbb{R}) \to \mathbb{R}$ by graded mean is defined as

$\Re(\tilde{A})=\frac{1}{4}(a+b+c+d)$

2.4 Mathematical formulation of a fuzzy game problem.

Let Player A have m strategies A1, A2 ...Am and Player B have n strategies B1,B2, ...Bn . Here, it is assumed that each player has his choices from amongst the pure strategies. Also it is assumed that player A is always the gainer and player B is always the loser. That is, all payoff are assumed in terms of player A. Let aij be the payoff which player A gains from player B if player A chooses strategy Ai and player B chooses strategy Bj. Then the payoff matrix to player A.

		Player B		
		B_1	.....	B_n
	A_1	a_{11}	......	a_{1n}
	...	....	......	......
Player A	A_m	a_{m1}	......	a_{mn}

2.5 Pure Strategies

If player known exactly got the other player is going to do is called pure strategies.

2.6 Mixed Strategies

When maximin ≠ minimax pure strategies fails, therefore each player with certain probabilistic fix action type of strategy is called mixed strategy.

2.7 Optimum Strategies

The maximin = minimax the corresponding pure strategy is called optimum strategy.

2.8 Pay off Matrix

Zero sum games with two players are called rectangular games. In this case the loss (gain) of one player is exactly equal to the gain (loss) of the other player. The gain resulting from a two persons zero sum game can be represented in the matrix form is called pay off matrix.

2.9 Saddle Point

A saddle point of a pay off matrix is that position in the pay off matrix. Where maximum of row minima, co-inside with the minimum of the column maxima. The pay off at the saddle point is called the value of the game denoted by gamma. The saddle point need not be unique. We denote the maximin value of the game by gamma ($\underline{\gamma}$) and the minimax value of the game by $\bar{\Upsilon}$.

The game is said to be fair. If $\underline{\gamma} = 0 = \bar{\Upsilon}$

The game is said to be strictly determinable. If $\underline{\gamma} = \gamma = \bar{\Upsilon}$

2.10 Two Persons Zero Sum Game

If the algebraic sum of gains and losses of all the player is zero in a gain, then such game is called zero sum game. Otherwise the game is called non-zero sum game.

2.11 Games without saddle point (mixed strategy)

Solution of 2x2 games without saddle point for any 2x2 two persons zero sum game without any saddle point having the pay off matrix for player A is

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \end{array}$$

An optimum mixed strategy $S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & p_2 \end{pmatrix}$ and $S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$

Where, $p_1 = \frac{a22-a21}{\lambda}$ and $p_2 = 1- p_1$

$q_1 = \frac{a22-a12}{\lambda}$and $q_2 = 1-q_1$

$\lambda = (a_{11}+a_{22}) - (a_{21}+a_{12})$ and $\gamma = \frac{a11\ a22-a21\ a12}{(a11+a22)-(a21+a12)}$

III. Numerical Examples

Consider the 4x2matrix for fuzzy game problem to solve Graphical method using quadrangle fuzzy number.

Player B

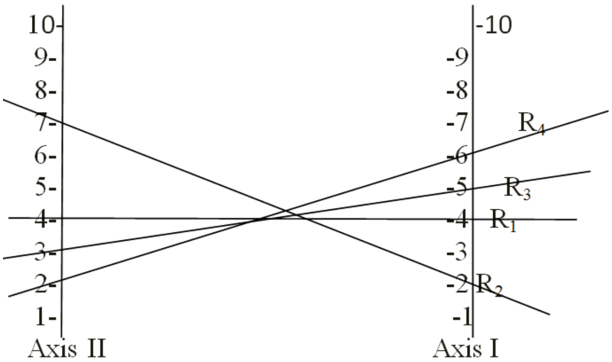
$$\text{Player A}\quad \begin{pmatrix} (1,4,5,6) & (1,3,5,7) \\ (-1,2,3,4) & (4,6,8,10) \\ (3,4,5,8) & (1,2,4,5) \\ (4,5,7,8) & (-1,2,3,4) \end{pmatrix}$$

Convert the given fuzzy problem into a crisp value by using Ranking of Quadrangle fuzzy number.

Player B

$$\text{Player A}\quad \begin{pmatrix} 4 & 4 \\ 2 & 7 \\ 5 & 3 \\ 6 & 2 \end{pmatrix}$$

Consider the two Axis. Axis 1 and Axis 2 vertically at unit distance apart.



Therefore the 2x2 matrix is R_2 and R_4

Using graphical method to convert into a 2x2 pay off matrix is given by,

Player B

$$\text{Player A}\quad \begin{pmatrix} 2 & 7 \\ 6 & 2 \end{pmatrix}$$

			Row minima	minimax
	2	7	2	
	6	2	2	2
Column maxima	6	7		
Maximin				
	6			

Therefore, there is no saddle point it is mixed strategy

$\lambda = (a_{11}+a_{22}) - (a_{21}+a_{12})$

$= (2+2) - (6+7)$

$= 4-13 = -9$

$p_1 = \frac{a22-a21}{\lambda} = \frac{2-6}{-9} = \frac{-4}{-9} = \frac{4}{9}$

$p_2 = 1 - p_1$

$= \frac{5}{9}$

$q_1 = \frac{a22-a12}{\lambda} = \frac{2-7}{-9} = \frac{5}{9}$

$q_2 = 1 - q_1$

$= \frac{4}{9}$

$$\gamma = \frac{a11\,a22 - a21\,a12}{(a11+a22)-(a21+a12)} = \frac{38}{9}$$

$$S_A = \begin{pmatrix} A_1 & A_2 \\ \frac{4}{9} & \frac{5}{9} \end{pmatrix} \qquad S_B = \begin{pmatrix} B_1 & B_2 \\ \frac{5}{9} & \frac{4}{9} \end{pmatrix}$$

IV. CONCLUSION

In this paper, a method of solving fuzzy game problem using ranking of fuzzy numbers has been considered. This method can be used to solve any 2 x m matrix with its values as Quadrangle fuzzy numbers. These values are converted into crisp values and the reduced crisp value game is then solved by any of the graphical method.

V. REFERENCES

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