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Redundancy Condition of Vertices in Coloring of Graphs

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ABSTRACT

In this article we verified the redundant condition X(G) = x(G-v) in coloring of vertices for complete graph, cyclic, simple graph, friendship graph, fan graph, wheel graph, tree and bipartite graphs. These characterizations are verified through examples.

Keywords : Bipartite Graph, Complete Graph, Cyclic Graph, Fan Graph, Friendship Graph, Simple Graph, Snake Graph, Tree, Wheel Graph.

I. INTRODUCTION

A graph is a collection of points and lines connecting some (possibly empty) subset of them. The points of a graph are most commonly known as graph vertices, but may also be called "nodes" or simply "points."

Definitions

A **vertex coloring** is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices. The most common type of vertex coloring seeks to minimize the number of colors for a given graph. Such a coloring is known as a minimum vertex coloring, and the minimum number of colors which with the vertices of a graph G may be colored is called the **chromatic number**, denoted χ' (G).

A **complete graph** is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. A complete digraph is a directed graph in which every pair of distinct vertices is connected by a pair of unique edges.

A **closed walk** consists of a sequence of vertices starting and ending at the same vertex, with each two consecutive vertices in the sequence adjacent to each other in the graph is called cycle.

A **simple graph**, also called a **strict graph** is an unweighted, undirected graph containing no graph loops or multiple edges. A simple graph may be either connected or disconnected.

A ***friendship graph*** Fn is a graph which consists of n triangles with a common vertex.

A **wheel graph** is a graph formed by connecting a single vertex to all vertices of a cycle. A wheel graph with n vertices can also be defined as the 1-skeleton of an (n-1)-gonal pyramid.

The ***fan*** fn (n ≥ 2) is obtained by joining all vertices of Pn (Path of n vertices) to a further vertex called the center and contains n+ 1 vertex and 2n-1 edges. i.e. fn = Pn + K1.

A **snake** is an Eulerian path in the d-hypercube that has no chords (i.e., any hypercube edge joining snake vertices is a snake edge).

A **tree** is an undirected graph in which any two vertices are connected by exactly one path. In other words, any acyclic connected graph is a tree.

A **bipartite graph** is a graph whose vertices can be divided into two disjoint sets and (that is, and are each independent sets) such that every edge connects a vertex in to one in Vertex sets and are usually called the parts of the graph.

II. MAIN RESULTS

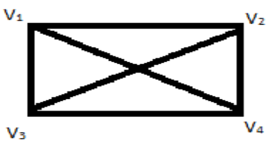
Theorem -2.1

The redundant condition on vertices is not satisfied for complete graph.

Proof

Let G be a complete graph having n vertices. Let $\chi'(G) = n$. where $\chi'(G)$ denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v. Then $\chi'(G\text{-}v) = n-1$ and $\chi'(G) \neq \chi'(G\text{-}v)$ and $\chi'(G) > \chi'(G\text{-}v)$. The redundancy condition is not satisfied for complete graph. This condition can be verified for the complete graphs having 4 vertices.



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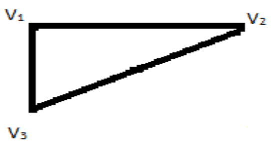


G-V4

Where $\chi'(G)= 4$ and $\chi'(G\text{-}v_4)=3$.Therefore the redundant condition is not satisfied.

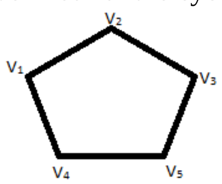
Theorem -2.2

In cycles, when the number of vertices is odd, the redundant condition is not satisfied to these vertices.

Proof

Let G be a cyclic graph having odd number of vertices. Let $\chi'(G) = 3$.where $\chi'(G)$ denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v. Then $\chi'(G\text{-}v) = 2$ and $\chi'(G) \neq \chi'(G\text{-}v)$ and $\chi'(G) > \chi'(G\text{-}v)$. The redundancy condition is not satisfied for graphs having odd cycles. This condition can be verified for the cyclic graph having 5 vertices.



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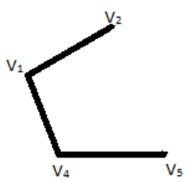


G-v3

Where $\chi'(G) = 3$ and $\chi'(G\text{-}v_3) = 2$. Therefore the redundant condition is not satisfied.

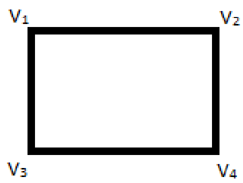
Theorem -2.3

In cycles, when the number of vertices is even, the redundant condition is satisfied to these vertices.

Proof

Let G be a cyclic graph having even number of vertices. Let $\chi'(G) = 2$. Where $\chi'(G)$ denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v. Then $\chi'(G\text{-}v) = 2$ and $\chi'(G) = \chi'(G\text{-}v)$. Therefore redundancy condition is satisfied for graphs having even cycles. This condition can be verified for the cyclic graph having 4 vertices.



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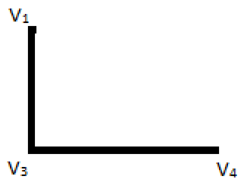


G-v2

Where $\chi'(G)= 2$ and $\chi'(G\text{-}v_2)=2$. Therefore the redundant condition is satisfied.

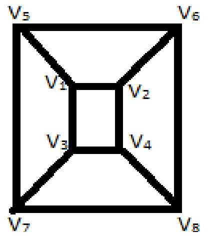
Theorem -2.4

The redundant condition of coloring of vertices is satisfied for simple graph.

Proof

Let G be a simple graph having n vertices. Let $\chi'(G) = n$. where $\chi'(G)$ denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v. Then $\chi'(G\text{-}v) = n$ and $\chi'(G) = \chi'(G\text{-}v)$. The redundancy condition is satisfied for simple graph. This condition can be verified for the simple graph having 8 vertices.



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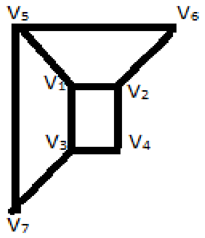
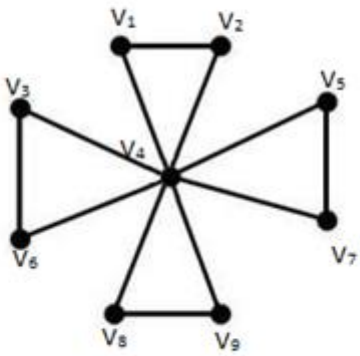


G-V₈

Where χ'(G) = 2 and χ'(G-v_8) = 2. Therefore the redundant condition is satisfied.

Theorem -2.5
The redundant condition of coloring of vertices is satisfied for friendship graph.
Proof
Let G be a friendship graph having n vertices. Let χ'(G) = n. where χ'(G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.
Let G- v be the graph obtained by removing the vertex v.
Then χ'(G-v) = n and

χ'(G) = χ'(G-v) . The redundancy condition is satisfied for friendship graph. This condition can be verified for the friendship graph having 9 vertices.



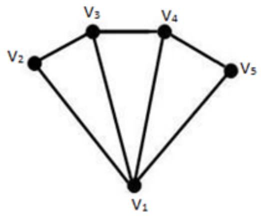
G

Where χ'(G) = 3 and χ'(G-v_9) = 3. Therefore the redundant condition is satisfied.

Theorem -2.6

The redundant condition of coloring of vertices is satisfied for fan graph.

Proof
Let G be a fan graph having n vertices. Let χ'(G) = n. where χ'(G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.
Let G-v be the graph obtained by removing the vertex v.
Then χ'(G-v) = n and χ'(G) = χ'(G-v) . The redundancy condition is satisfied for fan graph. This condition can be verified for the fan graph having 5 vertices.



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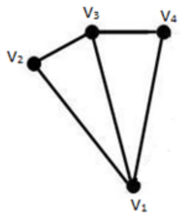
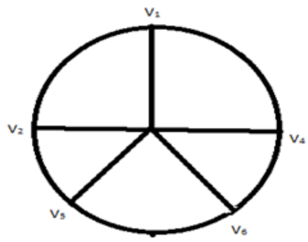


G-V₅

Where χ'(G) = 3 and χ'(G-v_5) = 3. Therefore the redundant condition is satisfied.

Theorem -2.7
The redundant condition on coloring is not satisfied when the number of vertices in the wheel graph is even.
Proof
Let G be a wheel graph having even number of vertices. Let χ'(G) = 4.where χ'(G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.
Let G-v be the graph obtained by removing the vertex v.
Then χ'(G-v) = 3 and χ'(G) ≠ χ'(G-v) and χ'(G) > χ'(G-v). The redundancy condition is not satisfied for graphs having even vertices of wheel. This condition can be verified for the wheel graph having 6 vertices.



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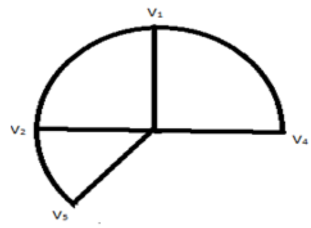


G-V₆

Where χ' (G) = 4 and χ' (G-v_6) = 3. Therefore the redundant condition is not satisfied.

Theorem -2.8

The redundant condition on coloring is satisfied when the number of vertices in the wheel graph is odd.

Proof

Let G be a wheel graph having odd number of vertices.

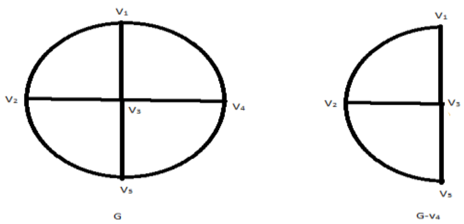
Let χ' (G) = 3. Where

χ' (G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v.

Then χ' (G-v) = 3 and

χ' (G) = χ' (G-v) . Therefore redundancy condition is satisfied for graphs having odd vertices of wheel. This condition can be verified for the wheel graph having 5 vertices.



Where χ' (G) = 3 and χ' (G-v_4) = 3. Therefore the redundant condition is satisfied.

Theorem -2.9

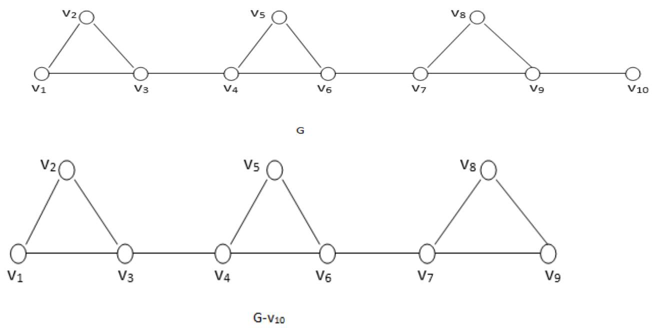
The redundant condition on coloring vertices is satisfied in snake graph.

Proof

Let G be a snake graph having n vertices. Let χ' (G) = n. where χ' (G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v.

Then χ' (G-v) = n and χ' (G) = χ' (G-v) . The redundancy condition is satisfied for snake graph. This condition can be verified for the snake graph having 10 vertices.



Where χ' (G) = 3 and χ' (G-v_{10}) = 3. Therefore the redundant condition is satisfied.

Theorem -2.10

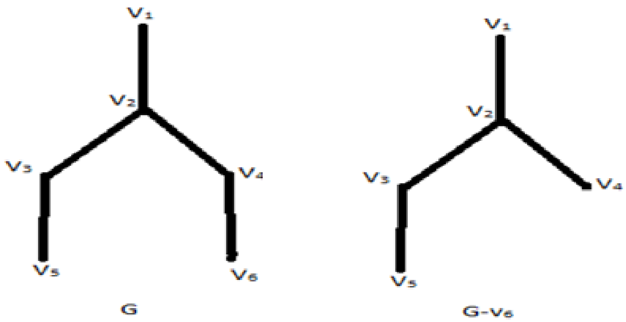
The redundant condition on coloring vertices is satisfied in tree.

Proof

Let G be a Tree having n vertices. Let χ' (G) = n. where χ' (G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v.

Then χ' (G-v) = n and χ' (G) = χ' (G-v). The redundancy condition is satisfied for Tree. This condition can be verified for the Tree having 6 vertices.



Where χ' (G) = 2 and χ' (G-v_6) = 2. Therefore the redundant condition is satisfied.

Theorem -2.11

The redundant condition on coloring vertices is satisfied in bipartite graph.

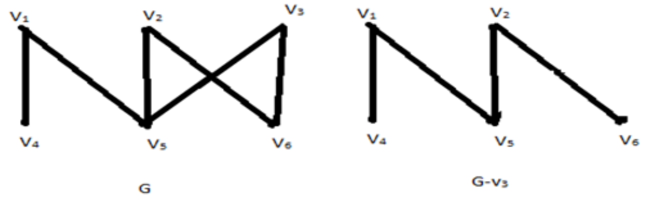
Proof

Let G be a bipartite graph having n vertices. Let χ' (G) = n. where χ' (G) denotes the minimum number of colors needed to color the vertices such that no two adjacent vertices having same color.

Let G-v be the graph obtained by removing the vertex v.

Then χ'(G-v) = n and

χ'(G) = χ'(G-v) . The redundancy condition is satisfied for bipartite graph. This condition can be verified for the bipartite graph having 6 vertices.



Where χ'(G) = 2 and χ'(G-v_3) = 2. Therefore the redundant condition is satisfied.

III. CONCLUSION

In this article, we have shown that the redundant condition χ'(G) = χ'(G-v) of coloring of vertices is satisfied for cycle graph (having even number of vertices), simple graph, friendship graph, fan graph, wheel graph(having odd number of vertices), tree and bipartite graphs. Also we have verified that, this redundant condition is not satisfied for complete graph, cycle graph (having odd number of vertices), wheel graph(having even number of vertices).

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