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Improvement of the Method of Calculation of Belt Gears Drives of Technological Machines and Units

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ABSTRACT

A technique is proposed for calculating belt transfers of drives of technological machines and aggregates, which consists in determining the tension of the belt branches, depending on the properties of its material and the design parameters of transmission in a stationary mode. The obtained analytical solutions allow in the future also establishing the dependence of the current tension of the belt branches on the initial tension of the belt, the speed of rotation and the transmission mechanism and the resistance force on the drive shaft. The proposed technique is demonstrated by examining one of the particular cases of the geometric arrangement in the plane of motion of the belt transmission elements.

Keywords: Belt Drives, Machine Drives, Calculation Technique, Belt Branches Tension.

I. INTRODUCTION

In technological machines and machine units (MU), due to the specificity of a particular process, the working and executive bodies are more often subjected to vibration loads [1] and [2]. Moreover, in a number of cases, the sources of vibration can become not only the processes of interaction of working organs with the object being processed, but also the mechanisms of the drives of the machines themselves. In these cases, with the help of one of the drive mechanisms, vibrations are artificially created to communicate them to the MU workers in order to increase the efficiency of technological processes [3]. For example, in the MU textile industry there have been attempts to apply belt gears with variable gear ratios, which are performed by eccentric pulleys or tension rollers [4], [5]. Due to this, the vibrating movements are communicated to the working body of the machine, and thus the effectiveness of its interaction with the material being processed (in this case, raw cotton) is increased [4]. The same effect is obtained, for example, in the application of belt gears with variable gear ratio in the drives of machines for crushing stones (rock crushers) in the mining and road construction industry where, due to vibrations reported from the drive to the working member, the crushing process will be more productive, [6] and [7]. In this regard, belt drives, which are an integral part of the high-speed drives of many technological MUs, taking into account the prospects of their application for the implementation of various additional functions, require further improvement of the methods of their calculation [8], [9].

As is known, modern computational methods and computer technologies allow the most accurate solution of a wide range of boundary value problems by numerical methods. However, for comparison, the qualitative and quantitative estimation of the error in the results of numerical calculations of physically and geometrically nonlinear problems, taking into account the flexibility and deformation of the material, requires analytical solutions for at least the simplest test cases and the most accurate values of unknown parameters at the boundaries of the region or discontinuity.

II. PROBLEM STATEMENT AND THE RESEARCH PURPOSE

The task of ensuring reliability, durability and increasing the service life of the transmission belt in a new setting can be taken as the definition of rational values of external loads, design parameters in which the tension of the branches of the belt transmission at each time will be distributed more evenly and remain within the allowable for the corresponding material of the belt . The solution of such a problem could be obtained if we apply the method of conditional rotation of the transmission mechanism in a stationary mode, where the values of the time constant of the tension and the speed of movement of the belt branches are within their acceptable values. Meanwhile, it is obvious that it is difficult and practically impossible to ensure rotation of the transmission mechanism in a stationary mode for a long period of time. Therefore, in theoretical and practical studies, one can speak only about small or infinitesimal time intervals, within which the mechanism rotates in stationary modes, having in each individual piece of time piecewise constant values of tension within the permissible limits [10], [11].

The purpose of the research is to demonstrate, on the example of considering one of the special cases of the geometrical arrangement in the plane of motion of the belt transmission elements, a new technique for determining the tension of the belt branches, depending on the properties of its material and the design parameters of transmission in a stationary mode.

The assumed assumption of the stationary mode of rotation of the transmission mechanism is based on the possibilities of dividing the operation time of specific machines into small or infinitely small segments in which the mechanism can rotate at constant speeds and tensions. However, it is difficult to find any real explanations for the assumption that the belt material is not extensible, especially with sufficiently large static and dynamic loads. This assumption is based on the fact that during the operation of the transmission mechanisms, the belts are relatively slightly stretched (relative deformations within ($0.01 \div 0.02$)).

III. DEVELOPMENT OF THE METHODOLOGY AND DISCUSSION OF THE RESULTS

As an object of study, consider a special case of a belt drive consisting of three pulleys, when the second pulley is located to the left of the first and third pulleys (Fig. 1). The carried out theoretical and numerical and experimental studies have shown that in this case the belt movement patterns, the forces acting on the belt and the distribution of belt tension substantially differ from other cases
For example, consider a case that satisfies the condition). $d_1 > d_3$ (Fig.1).

Start the coordinate system (x, y) in the center o_1 of the first pulley. The axis y passes through the centers of the first and third pulleys, and the axis x - perpendicular to the latter.

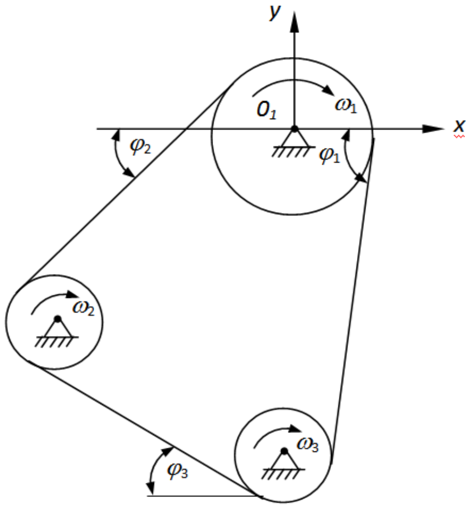


Figure 1. General scheme of the mechanism of belt drive with three pulleys

In the case under consideration, the angle φ_1 depends on the transverse dimensions of the first and third pulleys. An increase in the diameter of the first pulley leads to a decrease in the angle φ_1, and an increase in the diameter of the third pulley - on the contrary, leads to an increase in this angle.
The values of the angles φ_2 and φ_3 depend on the transverse dimensions of all the pulleys and the location of the second pulley (x_2, y_2) in the (x, y) plane:
- An increase in diameter d_1 leads to an increase in the angle φ_2;
- An increase in the diameter d_2 leads to a decrease in the angles φ_2 and φ_3;
- A decrease in the coordinate x_2 at $y_2 = const$ leads to a decrease in the angles φ_2 and φ_3;
- An increase in the coordinate y_2 at $x_2 = const$ leads to an increase in the angle φ_3 and the decrease in angle φ_2.
The force scheme on the surfaces of the first, a second and third pulley is shown in Fig. 2, 3 and 4 respectively.
Forces R_1 and R_3 and the vertical axis y form angles θ_1 and θ_3 respectively (Fig. 2), (Fig. 4):

$$\theta_1 = \frac{\pi}{2} - \frac{\varphi_1 + \varphi_2}{2}; \quad \theta_3 = \frac{\varphi_1 - \varphi_3}{2}.$$

The force R_2 depending on the angles φ_2 and φ_3 can form a positive or negative angle θ_3 with the axis x (Fig. 3 (a), (b))

$$\theta_2 = \begin{cases} \frac{1}{2}(\varphi_3 - \varphi_2) & at \quad \varphi_2 < \varphi_3 \\ 0 & at \quad \varphi_2 = \varphi_3 \\ \frac{1}{2}(\varphi_2 - \varphi_3) & at \quad \varphi_2 > \varphi_3 \end{cases}.$$

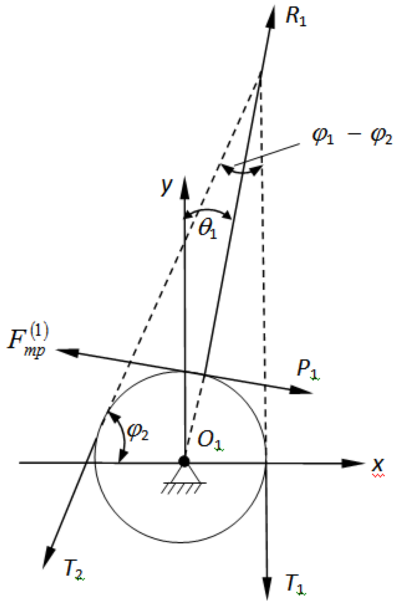
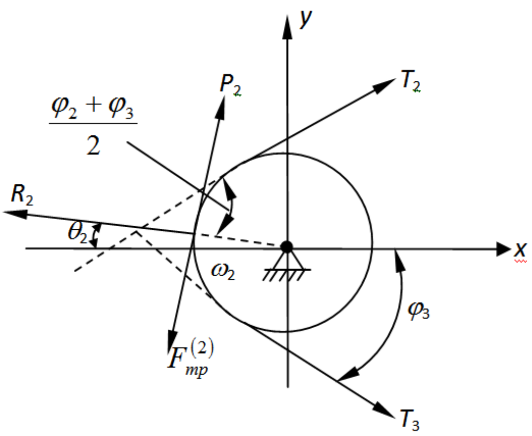


Figure 2. The scheme of the forces on the surface of the first pulley

a)



b)

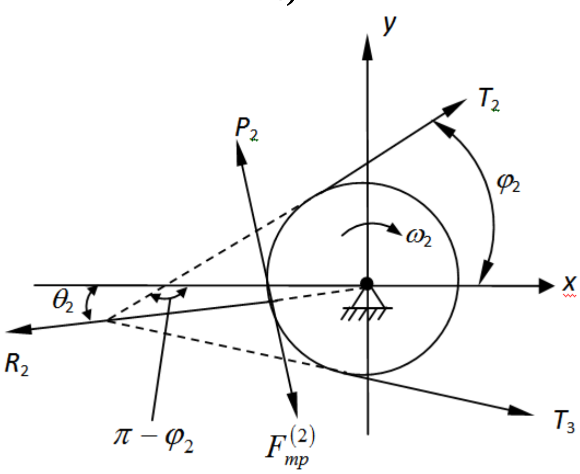


Figure 3. The scheme of forces on the surface of the second pulley

Kinematic conditions - conditions of continuity of motion on the surface of pulleys take the form:

$$x_1^{\bullet}dt=-\cos\varphi_1 ds_1,\qquad y_1^{\bullet}dt=-\sin\varphi_1 ds_1,$$
$$x_2^{\bullet}dt=\cos\varphi_2 ds_2,\qquad y_2^{\bullet}dt=\sin\varphi_2 ds_2,$$
$$x_3^{\bullet}dt=-\cos\varphi_3 ds_3,\qquad y_3^{\bullet}dt=\sin\varphi_3 ds_3,$$

where minus signs mean that the directions of the velocities $x_1^{\bullet}$, $x_3^{\bullet}$ and $y_1^{\bullet}$ are opposite to the directions of the horizontal and vertical axes, respectively.

Equations of the law of conservation of momentum have the form:

- on the surface of the first pulley

$$\rho_1F_1ds_1\left(x_1^{\bullet}-x_2^{\bullet}\right)=\begin{pmatrix}-T_1\cos\varphi_1-T_2\cos\varphi_2+R_1\cos\beta_1-\\-fR_1\sin\beta_1+P_1\sin\beta_1\end{pmatrix}dt,$$
$$\rho_1F_1ds_1\left(y_1^{\bullet}-y_2^{\bullet}\right)=\begin{pmatrix}-T_1\sin\varphi_1-T_2\sin\varphi_2+R_1\sin\beta_1+\\+fR_1\cos\beta_1-P_1\cos\beta_1\end{pmatrix}dt;$$

- on the surface of the second pulley

$$\rho_2F_2ds_2\left(x_2^{\bullet}-x_3^{\bullet}\right)=\begin{pmatrix}T_2\cos\varphi_2+T_3\cos\varphi_3-R_2\cos\beta_2\mp\\\mp fR_2\sin\beta_2\mp P_2\sin\beta_2\end{pmatrix}dt,$$
$$\rho_2F_2ds_2\left(y_2^{\bullet}-y_3^{\bullet}\right)=\begin{pmatrix}T_2\sin\varphi_2-T_3\sin\varphi_3\pm R_2\sin\beta_2-\\-fR_2\cos\beta_2+P_2\cos\beta_2\end{pmatrix}dt;$$

- на on the surface of the third pulley

$$\rho_3F_3ds_3\left(x_3^{\bullet}-x_1^{\bullet}\right)=\begin{pmatrix}T_1\cos\varphi_1-T_3\cos\varphi_3+R_3\cos\beta_3+\\+fR_3\sin\beta_3-P_3\sin\beta_3\end{pmatrix}dt,$$
$$\rho_3F_3ds_3\left(y_3^{\bullet}-y_1^{\bullet}\right)=\begin{pmatrix}T_1\sin\varphi_1+T_3\sin\varphi_3-R_3\sin\beta_3+\\+fR_3\cos\beta_3-P_3\cos\beta_3\end{pmatrix}dt;$$

where the upper signs are taken when the force R_2 is directed, as shown in Fig. 3 (a), and the lower ones in Fig. 3 (б), $\beta_1=\frac{\pi}{2}-\theta_1$, $\beta_2=\theta_2$, $\beta_3=\frac{\pi}{2}-\theta_3$.

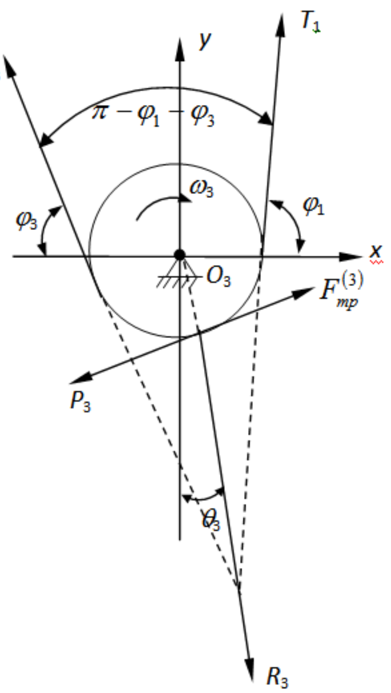


Figure 4. The scheme of the forces acting on the surface of the third pulley

The Riemann integrals have the form:

$$u_1 = a_0\varepsilon_1, \quad u_2 = -a_0\varepsilon_2, \quad u_3 = a_0\varepsilon_3.$$

The equations of the law of conservation of momentum are reduced to the form:

$$\frac{\rho_{00}F_{00}|u_1|}{(1+\varepsilon_{001})(1+\varepsilon_1)}(-u_1\cos\varphi_1 - u_2\cos\varphi_2) = -T_1\cos\varphi_1 - ,$$
$$-T_2\cos\varphi_2 + R_1(\cos\beta_1 - f\sin\beta_1) + P_1\sin\beta_1$$

$$\frac{\rho_{00}F_{00}|u_1|}{(1+\varepsilon_{001})(1+\varepsilon_1)}(-u_1\sin\varphi_1 - u_2\sin\varphi_2) = -T_1\sin\varphi_1 - ,$$
$$-T_2\sin\varphi_2 + R_1(\sin\beta_1 + f\cos\beta_1) - P_1\cos\beta_1$$

$$\frac{\rho_{00}F_{00}|u_2|}{(1+\varepsilon_{002})(1+\varepsilon_2)}(u_2\cos\varphi_2 + u_3\cos\varphi_3) = T_2\cos\varphi_2 + ,$$
$$+T_3\cos\varphi_3 - R_2(\cos\beta_2 \pm f\sin\beta_2) \mp P_2\sin\beta_2$$

$$\frac{\rho_{00}F_{00}|u_2|}{(1+\varepsilon_{002})(1+\varepsilon_2)}(u_2\sin\varphi_2 - u_3\sin\varphi_3) = T_2\sin\varphi_2 - ,$$
$$-T_3\sin\varphi_3 - R_2(\mp\sin\beta_2 + f\cos\beta_2) + P_2\cos\beta_2$$

$$\frac{\rho_{00}F_{00}|u_3|}{(1+\varepsilon_{003})(1+\varepsilon_3)}(-u_3\cos\varphi_3 + u_1\cos\varphi_1) = T_1\cos\varphi_1 - ,$$
$$-T_3\cos\varphi_3 - R_3(-\cos\beta_3 - f\sin\beta_3) - P_3\sin\beta_3$$

$$\frac{\rho_{00}F_{00}|u_3|}{(1+\varepsilon_{003})(1+\varepsilon_3)}(u_3\sin\varphi_3 + u_1\sin\varphi_1) = T_1\sin\varphi_1 + ,$$
$$+T_3\sin\varphi_3 - R_3(\sin\beta_3 - f\cos\beta_3) - P_3\cos\beta_3$$

or

$$\frac{\varepsilon_1}{(1+\varepsilon_{001})(1+\varepsilon_1)}(\varepsilon_1\cos\varphi_1 - \varepsilon_2\cos\varphi_2) = , \quad (1)$$
$$= \varepsilon_1\cos\varphi_1 + \varepsilon_2\cos\varphi_2 - \hat{R}_1\eta_{11} - \hat{P}_1\sin\beta_1$$

$$\frac{\varepsilon_1}{(1+\varepsilon_{001})(1+\varepsilon_1)}(\varepsilon_1\sin\varphi_1 - \varepsilon_2\sin\varphi_2) = , \quad (2)$$
$$= \varepsilon_1\sin\varphi_1 + \varepsilon_2\sin\varphi_2 - \hat{R}_1\eta_{12} + \hat{P}_1\cos\beta_1$$

$$\frac{\varepsilon_2}{(1+\varepsilon_{002})(1+\varepsilon_2)}(-\varepsilon_2\cos\varphi_2 + \varepsilon_3\cos\varphi_3) = , \quad (3)$$
$$= \varepsilon_2\cos\varphi_2 + \varepsilon_3\cos\varphi_3 - \hat{R}_2\eta_{21} \mp \hat{P}_2\sin\beta_2$$

$$\frac{\varepsilon_2}{(1+\varepsilon_{002})(1+\varepsilon_2)}(-\varepsilon_2\sin\varphi_2 - \varepsilon_3\sin\varphi_3) = , \quad (4)$$
$$= \varepsilon_2\sin\varphi_2 - \varepsilon_3\sin\varphi_3 - \hat{R}_2\eta_{22} + \hat{P}_2\cos\beta_2$$

$$\frac{\varepsilon_3}{(1+\varepsilon_{003})(1+\varepsilon_3)}(-\varepsilon_3\cos\varphi_3 + \varepsilon_1\cos\varphi_1) = , \quad (5)$$
$$= \varepsilon_1\cos\varphi_1 - \varepsilon_3\cos\varphi_3 - \hat{R}_3\eta_{31} - \hat{P}_3\sin\beta_3$$

$$\frac{\varepsilon_3}{(1+\varepsilon_{003})(1+\varepsilon_3)}(\varepsilon_3\sin\varphi_3 + \varepsilon_1\sin\varphi_1) = , \quad (6)$$
$$\varepsilon_1\sin\varphi_1 + \varepsilon_3\sin\varphi_3 - \hat{R}_3\eta_{32} - \hat{P}_3\cos\beta_3$$

where

$$\hat{R}_i = \frac{R_i}{\rho_{00}F_{00}a_0^2}, \quad \hat{P}_i = \frac{P_i}{\rho_{00}F_{00}a_0^2}, \quad i = 1, 2, 3,$$

$$\eta_{11} = f\cos\beta_1 - \sin\beta_1, \quad \eta_{12} = f\sin\beta_1 + \cos\beta_1,$$
$$\eta_{21} = \cos\beta_2 \pm f\sin\beta_2, \quad \eta_{22} = \sin\beta_2 \mp f\cos\beta_2,$$
$$\eta_{31} = -\cos\beta_3 - f\sin\beta_3, \quad \eta_{32} = \sin\beta_3 - f\cos\beta_3.$$

Equations (1) ÷ (6) can be reduced to the form

$$\frac{-\varepsilon_1\varepsilon_2\sin(\varphi_1-\varphi_2)}{(1+\varepsilon_{001})(1+\varepsilon_1)} = -\varepsilon_2\sin(\varphi_1-\varphi_2) - \hat{R}_1\lambda_{11} - ,$$
$$-\hat{P}_1\cos(\varphi_1-\beta_1)$$

$$\frac{-\varepsilon_1^2\sin(\varphi_1-\varphi_2)}{(1+\varepsilon_{001})(1+\varepsilon_1)} = -\varepsilon_1\sin(\varphi_1-\varphi_2) - \hat{R}_1\lambda_{12} - ,$$
$$-\hat{P}_1\sin(\varphi_2-\beta_1)$$

$$\frac{\varepsilon_3\varepsilon_2\sin(\varphi_2+\varphi_3)}{(1+\varepsilon_{002})(1+\varepsilon_2)} = \varepsilon_3\sin(\varphi_2+\varphi_3) - \hat{R}_2\lambda_{21} + ,$$
$$+\hat{P}_2\cos(\varphi_2 \pm \beta_2)$$

$$\frac{-\varepsilon_2^2\sin(\varphi_2+\varphi_3)}{(1+\varepsilon_{002})(1+\varepsilon_2)} = \varepsilon_2\sin(\varphi_2+\varphi_3) - \hat{R}_2\lambda_{22} + ,$$
$$+\hat{P}_2\cos(\varphi_3 \pm \beta_2)$$

$$\frac{\varepsilon_1\varepsilon_3\sin(\varphi_3+\varphi_1)}{(1+\varepsilon_{003})(1+\varepsilon_3)} = \varepsilon_1\sin(\varphi_3+\varphi_1) - \hat{R}_3\lambda_{31} - ,$$
$$-\hat{P}_3\cos(\varphi_3-\beta_3)$$

$$\frac{-\varepsilon_3^2\sin(\varphi_3+\varphi_1)}{(1+\varepsilon_{003})(1+\varepsilon_3)} = -\varepsilon_3\sin(\varphi_3+\varphi_1) - \hat{R}_3\lambda_{32} + ,$$
$$+\hat{P}_3\cos(\varphi_1+\beta_3)$$

where

$$\lambda_{11} = \eta_{11}\sin\varphi_1 - \eta_{12}\cos\varphi_1, \quad \lambda_{12} = \eta_{11}\sin\varphi_2 - \eta_{12}\cos\varphi_2,$$
$$\lambda_{21} = \eta_{21}\sin\varphi_2 - \eta_{22}\cos\varphi_2, \quad \lambda_{22} = \eta_{21}\sin\varphi_3 + \eta_{22}\cos\varphi_3,$$
$$\lambda_{31} = \eta_{31}\sin\varphi_3 + \eta_{32}\cos\varphi_3, \quad \lambda_{32} = \eta_{31}\sin\varphi_1 - \eta_{32}\cos\varphi_1.$$

Excluding unknown reactive forces, we obtain

$$\frac{-\varepsilon_2}{\lambda_{11}}\left[\frac{\varepsilon_1}{(1+\varepsilon_{001})(1+\varepsilon_1)} + 1\right] + \hat{P}_1\frac{\sin(\varphi_1-\beta_1)}{\lambda_{11}\sin(\varphi_1-\varphi_2)} = ,$$
$$= \frac{-\varepsilon_1}{\lambda_{12}}\left[\frac{\varepsilon_1}{(1+\varepsilon_{001})(1+\varepsilon_1)} - 1\right] + \hat{P}_1\frac{\cos(\varphi_2-\beta_1)}{\lambda_{12}\sin(\varphi_1-\varphi_2)}$$

$$\frac{\varepsilon_3}{\lambda_{21}}\left[\frac{\varepsilon_2}{(1+\varepsilon_{002})(1+\varepsilon_2)} - 1\right] - \hat{P}_2\frac{\cos(\varphi_2 \pm \beta_2)}{\lambda_{21}\sin(\varphi_2+\varphi_3)} =$$
$$= \frac{-\varepsilon_2}{\lambda_{22}}\left[\frac{\varepsilon_2}{(1+\varepsilon_{002})(1+\varepsilon_2)} + 1\right] - \hat{P}_2\frac{\cos(\varphi_3 \pm \beta_2)}{\lambda_{22}\sin(\varphi_2+\varphi_3)},$$

$$\frac{\varepsilon_1}{\lambda_{31}}\left[\frac{\varepsilon_3}{(1+\varepsilon_{003})(1+\varepsilon_3)}-1\right]+\hat{P}_3\frac{\cos(\varphi_3-\beta_3)}{\lambda_{31}\sin(\varphi_3+\varphi_1)}=$$

$$=\frac{-\varepsilon_3}{\lambda_{32}}\left[\frac{\varepsilon_3}{(1+\varepsilon_{003})(1+\varepsilon_3)}-1\right]-\hat{P}_3\frac{\cos(\varphi_1+\beta_3)}{\lambda_{32}\sin(\varphi_3+\varphi_1)}.$$

Substituting $\varepsilon_{001}=0$, $\varepsilon_{002}=0$, $\varepsilon_{003}=0$ we will have

$$\lambda_{12}\varepsilon_2(1+2\varepsilon_1)+\lambda_{11}\varepsilon_1=(1+\varepsilon_1)\frac{\hat{P}_1}{\sin(\varphi_1-\varphi_2)}\times,$$
$$\times\left[-\lambda_{11}\cos(\varphi_2-\beta_1)+\lambda_{12}\cos(\varphi_1-\beta_1)\right]$$

$$-\lambda_{22}\varepsilon_3+\lambda_{21}\varepsilon_2(1+2\varepsilon_2)=(1+\varepsilon_2)\frac{\hat{P}_2}{\sin(\varphi_2+\varphi_3)}\times,$$
$$\times\left[\lambda_{22}\cos(\varphi_2\pm\beta_2)-\lambda_{21}\cos(\varphi_3\pm\beta_2)\right]$$

$$\lambda_{32}\varepsilon_1+\lambda_{31}\varepsilon_3=(1+\varepsilon_3)\frac{\hat{P}_3}{\sin(\varphi_3+\varphi_1)}\times.$$
$$\times\left[\lambda_{31}\cos(\varphi_1+\beta_3)+\lambda_{32}\cos(\varphi_3-\beta_3)\right]$$

We introduce the notation

$$\gamma_1=\frac{\hat{P}_1}{\sin(\varphi_1-\varphi_2)}\left[-\lambda_{11}\cos(\varphi_2-\beta_1)+\lambda_{12}\cos(\varphi_1-\beta_1)\right],$$

$$\gamma_2=\frac{\hat{P}_2}{\sin(\varphi_2+\varphi_3)}\left[\lambda_{22}\cos(\varphi_2\pm\beta_2)-\lambda_{21}\cos(\varphi_3\pm\beta_2)\right],$$

$$\gamma_3=\frac{\hat{P}_3}{\sin(\varphi_3-\varphi_1)}\left[\lambda_{31}\cos(\varphi_1+\beta_3)+\lambda_{32}\cos(\varphi_3-\beta_3)\right].$$

Substituting these notations, the equations under consideration can be reduced to the form

$$\lambda_{12}\varepsilon_2(1+2\varepsilon_1)+\lambda_{11}\varepsilon_1=(1+\varepsilon_1)\gamma_1, \qquad (7)$$

$$-\lambda_{22}\varepsilon_3+\lambda_{21}\varepsilon_2(1+2\varepsilon_2)=(1+\varepsilon_2)\gamma_2, \qquad (8)$$

$$\lambda_{32}\varepsilon_1+\lambda_{31}\varepsilon_3=(1+\varepsilon_3)\gamma_3. \qquad (9)$$

From the equations (7) and (9) we find

$$\varepsilon_2=\frac{(1+\varepsilon_1)\gamma_1-\lambda_{11}\varepsilon_1)}{\lambda_{12}(1+2\varepsilon_1)}=\frac{\gamma_1+\gamma_{11}\varepsilon_1}{\lambda_{12}(1+2\varepsilon_1)},$$

$$\varepsilon_3=\frac{\gamma_3-\lambda_{32}\varepsilon_1}{\gamma_{33}},$$

where

$$\gamma_{33}=\lambda_{31}-\gamma_3,\quad \gamma_{11}=\gamma_1-\lambda_{11}.$$

Then, taking into account the last expressions, the equation of the system under consideration takes the form

$$\frac{-\lambda_{22}\gamma_3+\lambda_{22}\lambda_{32}\varepsilon_1}{\gamma_{33}}+\frac{\lambda_{21}\gamma_1+\lambda_{21}\gamma_{11}\varepsilon_1}{\lambda_{12}(1+2\varepsilon_1)}\left[1+\frac{2\gamma_1+2\gamma_{11}\varepsilon_1)}{\lambda_{12}(1+2\varepsilon_1)}\right]=$$
$$=\gamma_2\left[1+\frac{\gamma_1+\gamma_{11}\varepsilon_1}{\lambda_{12}(1+2\varepsilon_1)}\right]$$

or

$$\lambda_{12}^2(1+4\varepsilon_1+4\varepsilon_1^2)(-\lambda_{22}\gamma_3+\lambda_{22}\lambda_{32}\varepsilon_1)+$$
$$+(\gamma_{33}\lambda_{21}\gamma_1+\lambda_{21}\gamma_{11}\gamma_{33}\varepsilon_1)(\lambda_{12}+2\lambda_{12}\varepsilon_1+$$
$$+2\gamma_1+2\gamma_{11}\varepsilon_1)=$$
$$=(\lambda_{12}\gamma_2\gamma_{33}+2\lambda_{12}\gamma_2\gamma_{33}\varepsilon_1)(\lambda_{12}+2\lambda_{12}\varepsilon_1+\gamma_1+\gamma_{11}\varepsilon_1).$$

From here

$$-\lambda_{12}^2\lambda_{22}\gamma_3+\lambda_{12}^2\lambda_{22}\lambda_{32}\varepsilon_1-4\lambda_{12}^2\lambda_{22}\gamma_3\varepsilon_1+$$
$$+4\lambda_{12}^2\lambda_{22}\lambda_{32}\varepsilon_1^2-4\lambda_{12}^2\lambda_{22}\gamma_3\varepsilon_1^2+4\lambda_{12}^2\lambda_{22}\lambda_{32}\varepsilon_1^3+$$
$$+(\lambda_{12}+2\gamma_1)\gamma_1\gamma_{33}\lambda_{21}+(\lambda_{12}+2\gamma_1)\gamma_{11}\gamma_{33}\lambda_{21}\varepsilon_1+$$
$$+2(\lambda_{12}+\gamma_{11})\gamma_1\gamma_{33}\lambda_{21}\varepsilon_1+2(\lambda_{12}+\gamma_{11})\gamma_{11}\gamma_{33}\lambda_{21}\varepsilon_1^2=$$
$$=(\lambda_{12}+\gamma_1)\gamma_2\gamma_{33}\lambda_{12}\varepsilon_1^2+2(\lambda_{12}+\gamma_1)\lambda_{12}\gamma_2\gamma_{33}\varepsilon_1+$$
$$+(2\lambda_{12}+\gamma_{11})\gamma_2\gamma_{33}\lambda_{12}\varepsilon_1+(2\lambda_{12}+\gamma_{11})\gamma_2\gamma_{33}\lambda_{12}\varepsilon_1^2$$

or

$$b_3\varepsilon_3^3+b_2\varepsilon_3^2+b_1\varepsilon_3+b_0=0,$$

where

$$b_3=4\lambda_{12}^2\lambda_{22}\lambda_{32},$$

$$b_2=4\lambda_{12}^2\lambda_{22}(\lambda_{32}-\gamma_3)+2\gamma_{33}\begin{bmatrix}(\lambda_{12}+\gamma_{11})\gamma_{11}\lambda_{21}-\\-(2\lambda_{12}+\gamma_{11})\gamma_2\lambda_{12}\end{bmatrix}$$

$$b_1=\lambda_{12}^2\lambda_{22}(\lambda_{32}-4\gamma_3)+(\lambda_{12}+\gamma_1)\gamma_{11}\gamma_{33}\lambda_{21}+$$
$$+2(\lambda_{12}+\gamma_{11})\gamma_1\gamma_{33}\lambda_{21}-2(\lambda_{12}+\gamma_1)\gamma_2\gamma_{33}\lambda_{12}-$$
$$-(2\lambda_{12}+\gamma_{11})\gamma_2\gamma_{33}\lambda_{12},$$

$$b_0=-\lambda_{12}^2\lambda_{22}\gamma_3-(\lambda_{12}+\gamma_1)\gamma_2\gamma_{33}\lambda_{12}.$$

Let us now consider the motion of an inextensible belt. In this case, the equations of the law of conservation of momentum have the form

$$\rho_{01}F_{01}ds_{01}\left(\dot{x}_{01}-\dot{x}_{02}\right)=\begin{pmatrix}-T_{01}\cos\varphi_{01}-T_{02}\cos\varphi_{02}+\\+R_{01}\cos\beta_{01}-fR_{01}\sin\beta_{01}+P_{01}\cos\beta_{01}\end{pmatrix}dt,$$

$$\rho_{01}F_{01}ds_{01}\left(\dot{y}_{01}-\dot{y}_{02}\right)=\begin{pmatrix}-T_{01}\sin\varphi_{01}-T_{02}\sin\varphi_{02}+\\+R_{01}\sin\beta_{01}+fR_{01}\cos\beta_{01}-P_{01}\sin\beta_{01}\end{pmatrix}dt;$$

$$\rho_{02}F_{02}ds_{02}\left(\dot{x}_{02}-\dot{x}_{03}\right)=\begin{pmatrix}T_{02}\cos\varphi_{02}+T_{03}\cos\varphi_{03}-\\-R_{02}\cos\beta_{02}\mp fR_{02}\sin\beta_{02}\mp P_{02}\sin\beta_{02}\end{pmatrix}dt,$$

$$\rho_{02}F_{02}ds_{02}\left(\dot{y}_{02}-\dot{y}_{03}\right)=$$
$$=\begin{pmatrix}T_{02}\sin\varphi_{02}-T_{03}\sin\varphi_{03}\pm R_{02}\sin\beta_{02}-\\-fR_{02}\cos\beta_{02}+P_{02}\cos\beta_{02}\end{pmatrix}dt;$$

$$\rho_{03}F_{03}ds_{03}\left(\dot{x}_{03}-\dot{x}_{01}\right)= \\ =\begin{pmatrix} T_{01}\cos\varphi_{01}-T_{03}\cos\varphi_{03}+R_{03}\cos\beta_{03}+ \\ +fR_{03}\sin\beta_{03}-P_{03}\sin\beta_{03} \end{pmatrix}dt,$$

$$\rho_{03}F_{03}ds_{03}\left(\dot{y}_{03}-\dot{y}_{01}\right)= \\ =\begin{pmatrix} T_{01}\sin\varphi_{01}+T_{03}\sin\varphi_{03}-R_{03}\sin\beta_{03}+ \\ +fR_{03}\cos\beta_{03}-P_{03}\cos\beta_{03} \end{pmatrix}dt,$$

or

$$T_{01}\cos\varphi_{01}+T_{02}\cos\varphi_{02}-R_{01}\eta_{11}=A_1, \qquad (10)$$

$$T_{01}\sin\varphi_{01}+T_{02}\sin\varphi_{02}-R_{01}\eta_{12}=A_2, \qquad (11)$$

$$T_{02}\cos\varphi_{02}+T_{03}\cos\varphi_{03}-R_{02}\eta_{21}=B_1, \qquad (12)$$

$$T_{02}\sin\varphi_{02}-T_{03}\sin\varphi_{03}-R_{02}\eta_{22}=B_2, \qquad (13)$$

$$T_{01}\cos\varphi_{01}-T_{03}\cos\varphi_{03}-R_{03}\eta_{31}=C_1, \qquad (14)$$

$$T_{01}\sin\varphi_{01}+T_{03}\sin\varphi_{03}-R_{03}\eta_{32}=C_2, \qquad (15)$$

where

$$A_1=\rho_{00}F_{00}u_{02}\left(\cos\varphi_{01}+\cos\varphi_{02}\right)-P_{01}\cos\beta_{01},$$
$$A_2=\rho_{00}F_{00}u_0^2\left(\sin\varphi_{01}+\sin\varphi_{02}\right)-P_{01}\sin\beta_{01},$$
$$B_1=\rho_{00}F_{00}u_0^2\left(\cos\varphi_{02}+\cos\varphi_{03}\right)\mp P_{02}\sin\beta_{02},$$
$$B_2=\rho_{00}F_{00}u_0^2\left(\sin\varphi_{02}+\sin\varphi_{03}\right)-P_{02}\cos\beta_{02},$$
$$C_1=\rho_{00}F_{00}u_0^2\left(\cos\varphi_{01}-\cos\varphi_{03}\right)+P_{03}\sin\beta_{03},$$
$$C_2=\rho_{00}F_{00}u_0^2\left(\sin\varphi_{01}+\sin\varphi_{03}\right)+P_{03}\cos\beta_{03},$$

Coefficients η_{ij} are retained in the same form, where $i,j=1,2,3$.

Eliminating the unknown reactive forces, we represent the equations (10) ÷ (15) in the form

$$T_{01}\left(\eta_{12}\cos\varphi_{01}-\eta_{11}\sin\varphi_{01}\right)+ \\ +T_{02}\left(\eta_{12}\cos\varphi_{02}-\eta_{11}\sin\varphi_{02}\right)=A_1\eta_{12}-A_2\eta_{11},$$

$$T_{02}\left(\eta_{22}\cos\varphi_{02}-\eta_{21}\sin\varphi_{02}\right)+ \\ +T_{03}\left(\eta_{22}\cos\varphi_{03}+\eta_{21}\sin\varphi_{03}\right)=B_1\eta_{22}-B_2\eta_{21},$$

$$T_{01}\left(\eta_{32}\cos\varphi_{01}-\eta_{31}\sin\varphi_{01}\right)- \\ -T_{03}\left(\eta_{32}\cos\varphi_{03}+\eta_{31}\sin\varphi_{03}\right)=C_1\eta_{32}-C_2\eta_{31}.$$

From here

$$T_{01}\alpha_{11}+T_{02}\alpha_{12}=A,\quad T_{02}\alpha_{21}+T_{03}\alpha_{22}=B,$$
$$T_{01}\alpha_{31}+T_{03}\alpha_{32}=C, \qquad (16)$$

where

$$\alpha_{11}=\eta_{12}\cos\varphi_{01}-\eta_{11}\sin\varphi_{01},$$
$$\alpha_{12}=\eta_{12}\cos\varphi_{02}-\eta_{11}\sin\varphi_{02},\quad A=A_1\eta_{12}-A_2\eta_{11};$$
$$\alpha_{21}=\eta_{22}\cos\varphi_{02}-\eta_{21}\sin\varphi_{02},$$
$$\alpha_{22}=\eta_{22}\cos\varphi_{03}+\eta_{21}\sin\varphi_{03},\quad B=B_1\eta_{22}-B_2\eta_{21},$$
$$\alpha_{31}=\eta_{32}\cos\varphi_{01}-\eta_{31}\sin\varphi_{01},$$
$$\alpha_{32}=-\eta_{32}\cos\varphi_{03}-\eta_{31}\sin\varphi_{03},\ C=C_1\eta_{32}-C_2\eta_{31}.$$

Equations (16) have a solution

$$T_{01}=\frac{(-B\alpha_{32}+C\alpha_{22})\alpha_{12}+A\alpha_{21}\alpha_{32}}{\alpha_{11}\alpha_{21}\alpha_{32}+\alpha_{12}\alpha_{22}\alpha_{31}},$$
$$T_{02}=\frac{A\alpha_{31}\alpha_{22}-C\alpha_{11}\alpha_{22}+B\alpha_{11}\alpha_{32}}{\alpha_{11}\alpha_{21}\alpha_{32}+\alpha_{12}\alpha_{22}\alpha_{31}},$$
$$T_{03}=\frac{-A\alpha_{21}\alpha_{31}+C\alpha_{11}\alpha_{21}+B\alpha_{12}\alpha_{31}}{\alpha_{32}\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22}\alpha_{31}}.$$

Unknown reactive forces are determined from equations (10) ÷ (15)

$$R_{01}=\frac{T_{01}\sin\left(\varphi_{02}-\varphi_{01}\right)-A_1\sin\varphi_{02}+A_2\cos\varphi_{02}}{\eta_{11}\sin\varphi_{02}-\eta_{12}\cos\varphi_{02}},$$
$$R_{02}=\frac{T_{02}\sin\left(\varphi_{03}+\varphi_{02}\right)-B_1\sin\varphi_{03}-B_2\cos\varphi_{03}}{\eta_{21}\sin\varphi_{03}+\eta_{22}\cos\varphi_{03}},$$
$$R_{03}=\frac{T_{01}\sin\left(\varphi_{01}+\varphi_{03}\right)+C_1\sin\varphi_{01}-C_2\cos\varphi_{01}}{-\eta_{31}\sin\varphi_{01}+\eta_{32}\cos\varphi_{01}}.$$

The conditions for the balance of the belt are obtained from the equation of the law of conservation of momentum:

$$-T_{001}\cos\varphi_{001}-T_{002}\cos\varphi_{002}+R_{001}\sin\beta_{001}- \\ -f\cos\beta_{001}=-P_{001}\cos\beta_{001},$$

$$-T_{001}\sin\varphi_{001}-T_{002}\sin\varphi_{002}+R_{001}\cos\beta_{001}+ \\ +f\sin\beta_{001}=P_{001}\sin\beta_{001},$$

$$-T_{002}\cos\varphi_{002}-T_{003}\cos\varphi_{003}+R_{002}\cos\beta_{002}+ \\ +f\sin\beta_{002}=P_{002}\sin\beta_{002},$$

$$T_{002}\sin\varphi_{002}-T_{003}\sin\varphi_{003}\pm R_{002}\sin\beta_{002}- \\ -f\cos\beta_{002}=-P_{002}\cos\beta_{002},$$

$$T_{001}\cos\varphi_{001}-T_{003}\cos\varphi_{003}+R_{003}\cos\beta_{003}+ \\ +f\sin\beta_{003}=P_{003}\sin\beta_{003},$$

$$T_{001}\sin\varphi_{001}+T_{003}\sin\varphi_{003}-R_{003}\sin\beta_{003}+ \\ +f\cos\beta_{003}=P_{003}\cos\beta_{003}.$$

Excluding unknown reactive forces, we find

$$T_{001}(\eta_{12}\cos\varphi_{001}-\eta_{11}\sin\varphi_{001})+ \\ +T_{002}(\eta_{12}\cos\varphi_{002}-\eta_{11}\sin\varphi_{002})=M, \quad (17)$$

$$T_{002}(\eta_{22}\cos\varphi_{002}-\eta_{21}\sin\varphi_{002})+ \\ +T_{003}(\eta_{22}\cos\varphi_{003}+\eta_{21}\sin\varphi_{003})=N, \quad (18)$$

$$T_{001}(\eta_{32}\cos\varphi_{001}-\eta_{31}\sin\varphi_{001})- \\ -T_{003}(\eta_{32}\cos\varphi_{003}+\eta_{31}\sin\varphi_{003})=K, \quad (19)$$

where

$$M_1=-P_{001}\sin\beta_{001}, \quad M_2=-P_{001}\cos\beta_{001};$$

$$N_1=\mp P_{002}\sin\beta_{002}, \quad N_2=-P_{002}\cos\beta_{002};$$

$$K_1=P_{003}\sin\beta_{003}, \quad K_2=P_{003}\cos\beta_{003},$$

$$M=M_1\eta_{12}-M_2\eta_{11}, \quad N=N_1\eta_{22}-N_2\eta_{21},$$

$$K=K_1\eta_{32}-K_2\eta_{31}.$$

Equations (16) and (17) ÷ (19) can be represented in the form

$$T_{001}\alpha_{11}+T_{002}\alpha_{12}=M, \quad T_{002}\alpha_{21}+T_{003}\alpha_{22}=N,$$

$$T_{001}\alpha_{31}+T_{003}\alpha_{32}=K, \quad (20)$$

where

$$\alpha_{11}=\eta_{12}\cos\varphi_{001}-\eta_{11}\sin\varphi_{001},$$

$$\alpha_{12}=\eta_{12}\cos\varphi_{002}-\eta_{11}\sin\varphi_{002},$$

$$\alpha_{21}=\eta_{22}\cos\varphi_{002}-\eta_{21}\sin\varphi_{002},$$

$$\alpha_{22}=\eta_{22}\cos\varphi_{003}+\eta_{21}\sin\varphi_{003},$$

$$\alpha_{31}=\eta_{32}\cos\varphi_{001}-\eta_{31}\sin\varphi_{001},$$

$$\alpha_{32}=-\eta_{32}\cos\varphi_{003}-\eta_{31}\sin\varphi_{003}.$$

Equations (20) have a solution

$$T_{001}=\frac{(-N\alpha_{32}+K\alpha_{22})\alpha_{12}+M\alpha_{21}\alpha_{32}}{\alpha_{11}\alpha_{21}\alpha_{32}+\alpha_{12}\alpha_{22}\alpha_{31}},$$

$$T_{002}=\frac{(M\alpha_{31}-K\alpha_{11})\alpha_{22}+N\alpha_{11}\alpha_{32}}{\alpha_{11}\alpha_{21}\alpha_{32}+\alpha_{12}\alpha_{22}\alpha_{31}},$$

$$T_{003}=\frac{-M\alpha_{21}\alpha_{31}+K\alpha_{11}\alpha_{21}+N\alpha_{12}\alpha_{31}}{\alpha_{32}\alpha_{11}\alpha_{12}+\alpha_{12}\alpha_{22}\alpha_{31}}.$$

Unknown reactive forces are determined from equations (17) ÷ (19):

$$R_{001}=\frac{T_{001}\sin(\varphi_{002}-\varphi_{001})-M_1\sin\varphi_{002}+M_2\cos\varphi_{002}}{\eta_{11}\sin\varphi_{002}-\eta_{12}\cos\varphi_{002}},$$

$$R_{002}=\frac{T_{002}\sin(\varphi_{003}+\varphi_{002})-N_1\sin\varphi_{003}-N_2\cos\varphi_{003}}{\eta_{21}\sin\varphi_{003}+\eta_{22}\cos\varphi_{003}},$$

$$R_{003}=\frac{T_{003}\sin(\varphi_{003}+\varphi_{001})+K_1\sin\varphi_{001}-K_2\cos\varphi_{001}}{-\eta_{31}\sin\varphi_{001}+\eta_{32}\cos\varphi_{001}}.$$

IV. CONCLUSIONS

The obtained analytical solutions can be used:

- When adjusting and assembling the mechanisms of belt drives of machines;
- When designing new and upgrading existing transmission mechanisms;
- When predicting the causes of various inaccuracies and deficiencies in the work of the belt transmission mechanism and developing measures to eliminate them;
- for comparison and evaluation of the results of approximate solutions of various dynamic problems of mechanisms with flexible connections in a more general setting (belt drives, belt conveyors, hoisting mechanisms of hoisting devices, etc.), taking into account the real physical and mechanical properties of their material, effects on the transmission mechanism.

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