

A New Approach to Solve Fuzzy Assignment Problem Using Dodecagonal Fuzzy Numbers

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ABSTRACT

In this paper, we proposed to solve fuzzy assignment problem using dodecagonal fuzzy numbers. A new approach for solving fuzzy assignment problem using dodecagonal fuzzy numbers with α -cut and Robust Ranking technique are introduced. The balanced assignment problem is formulated to a crisp assignment problem which is solved by Robust Ranking technique.

Keywords: Dodecagonal fuzzy numbers, Fuzzy assignment problem, Fuzzy set, Membership function, Robust ranking technique.

I. INTRODUCTION

The fuzzy assignment problem is a special type of fuzzy linear programming problem and it is a subclass of fuzzy transportation problem. The fuzzy assignment problem can be stated as follows:

Let n number of jobs is performed by number of persons, where the costs a depend on the specific assignments. Each job must be assigned to one and only one worker and each worker has to perform one and only one job.

The problem is to find such an assignment so that the total cost is optimized. The fuzzy assignment problem can be applied to $n \times n$ fuzzy cost matrix (C_{ij}) , where C_{ij} represents the fuzzy cost associated with worker $(i=1,2,3,\dots,n)$ who has performed job $(j=1,2,3,\dots,n)$. The fuzzy assignment problem when costs are fuzzy numbers can also be modeled as 0-1 integer programming problems. The fuzzy balanced assignment problems can be solved by the method proposed for balanced assignment method.

II. METHODS AND MATERIAL

Preliminaries

Assignment Problem:

Assignment Problem is a special case of the transportation problem in which the number of square and destination are the same and the objective is to assign the given job to most appropriate person so as to optimize the objective function like minimize cost.

Balanced Assignment Problem:

When the number of rows equals to the number of columns.

Number of rows = Number of columns.

Row Reduction:

Row reduction subtracts the minimum cost of each row of the cost matrix from all the elements of the respective row of the resulting matrix.

Column Reduction:

Column reduction subtracts the minimum cost of each column of the cost matrix from all the elements of the respective column of the resulting matrix.

Fuzzy Set:

Let x be a non-empty set. A fuzzy set A in x is characterized by its membership function $A \rightarrow [0,1]$ and $A(x)$ is interpreted as the degree of membership of elements x in fuzzy A for each $x \in X$.

The value zero is used to represent complete non-membership; the value one is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A.

Crisp Sets:

A crisp set is a special case of fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

Fuzzy Number:

A fuzzy \tilde{A} is a fuzzy set on the real line R, must satisfy the following conditions.

- (i) There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0)=1$
- (ii) $\mu_{\tilde{A}}(x)$ is piecewise conditions.
- (iii) \tilde{A} Must be normal and convex.

Robust Ranking Technique:

Robust ranking technique which satisfy compensation, linearity, and additivity properties and provides results which are consist human intuition. If \tilde{a} is a fuzzy number then the Robust

Ranking is defined by,

$$R(\tilde{a}) = \int_0^1 0.5(a^L, a^U) d\alpha,$$

Where

$$(a^L, a^U) = [(b-a) \alpha + a, d-(d-c) \alpha] + [(d-c) \alpha + c, e-(e-f) \alpha] + [(h-g) \alpha + g, j-(j-i) \alpha + i] + [(j-i) \alpha + i, l-(l-k) \alpha]$$

Where (a_D^L, a_D^U) is a α -level cut of a fuzzy number \tilde{a} . In this paper we use this method for ranking the objective values. The robust ranking index $R(\tilde{a})$ gives the representative value of the fuzzy number \tilde{a} .

α -cut of fuzzy number:

The α -cut of a fuzzy number A(x) is defined as $A(\alpha) = \{x; \mu(x) \geq \alpha; \alpha \in [0,1]\}$

Mathematical formulation of fuzzy Assignment problem:

Mathematically, the fuzzy assignment problem is,

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to the constraint;

$$\sum_{i=1}^n x_{ij} = 1; i=1,2,\dots,n$$

$$\sum_{j=1}^m x_{ij} = 1; j=1,2,\dots,n$$

$$x_{ij} = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ person is assigned } j^{\text{th}} \text{ work} \\ 0 & \text{otherwise} \end{cases}$$

Where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

Mathematical formulation of Assignment problem:

Consider a problem of assignment of n resources (persons) to n-activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (C_{ij}) is given as follows

Activities		A ₁	A ₂	A ₃	A _n	Available
Resource	R ₁	C ₁₁	C ₁₂	C ₁₃	C _{1n}	1
	R ₂	C ₂₁	C ₂₂	C ₂₃	C _{2n}	1
	R ₃	C ₃₁	C ₃₂	C ₃₃	C _{3n}	.
.
.
Required
	R _n	C _{n1}	C _{n2}	C _{n3}	C _{nn}	1
		1	1	1	1	

This cost matrix is same as that of a transportation problem except that availability at each of the resources and requirements at each of the destination is unity (due to the fact that assignments on a one-to-one basis)

Let x_{ij} denotes the assignment of i^{th} resource to j^{th} activity, such that

$$x_{ij} = \begin{cases} 1, & \text{if resource } i \text{ is assigned to activity } j \\ 0 & \text{otherwise} \end{cases}$$

Then the Mathematical formulation of the Assignment Problem is

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to the constraint;

$$\sum_{i=1}^n x_{ij} = 1; i=1,2,\dots,n$$

$$\sum_{j=1}^m x_{ij} = 1; j=1,2,\dots,n$$

Dodecagonal fuzzy Numbers:

The fuzzy number D is a dodecagonal fuzzy. A_D is a dodecagonal fuzzy number denoted A_D (a, b, c, d, e, f, g, h, I, j, k, l; 1) and its membership function $\mu_{AD}(x)$ is given below:

$$\mu_{AD}(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ \frac{x-d}{e-d} & d \leq x \leq e \\ \frac{f-x}{f-e} & e \leq x \leq f \\ 1 & x \leq f \\ \frac{x-g}{g-f} & f \leq x \leq g \\ \frac{h-x}{h-g} & g \leq x \leq h \\ \frac{x-i}{i-h} & h \leq x \leq i \\ \frac{j-x}{j-i} & i \leq x \leq j \\ \frac{x-k}{k-j} & j \leq x \leq k \\ \frac{l-x}{l-k} & k \leq x \leq l \\ 1 & x \geq l \end{cases}$$

Numerical Method

Let us consider a fuzzy balanced assignment problem with rows representing S_1, S_2, S_3 and columns representing A_1, A_2 and A_3 .

	A_1	A_2	A_3
S_1	(2,4,6,8,10,12,14,16,18,20,22,24)	(1,2,3,4,5,6,7,8,9,10,11,12)	(1,3,5,7,9,11,13,15,17,19)
S_2	(0,1,2,3,4,5,6,7,8,9,10,11)	(-1,0,1,2,3,4,5,6,7,8,9,10)	(5,7,9,11,13,15,17,19,21,23,25,27)
S_3	(4,6,8,10,12,14,16,18,20,22,24,26)	(7,9,11,13,15,17,19,21,23,25,27,29)	(2,3,4,5,6,7,8,9,10,11,12,13)

Solution:

The fuzzy balanced assignment problem can be formulated in the following mathematical programming form.

$$\begin{aligned} & \text{Min} \{ R(2,4,6,8,10,12,14,16,18,20,22,24) x_{11} + R(1,2,3,4,5,6,7,8,9,10,11,12) x_{12} + \\ & R(1,3,5,7,9,11,13,15,17,19) x_{13} + R(0,1,2,3,4,5,6,7,8,9,10,11) x_{21} + R(-1,0,1,2,3,4,5,6,7,8,9,10) x_{22} + \\ & R(5,7,9,11,13,15,17,19,21,23,25,27) x_{23} + R(4,6,8,10,12,14,16,18,20,22,24,26) x_{31} + \\ & R(7,9,11,13,15,17,19,21,23,25,27,29) x_{32} + R(2,3,4,5,6,7,8,9,10,11,12,13) x_{33} \} \\ & \text{(i) } [2+8] + [6+10] + [14+20] + [18+24] \cdot [0.5] \\ & = [10] + [16] + [34] + [42] \cdot [0.5] \\ & = [51] \end{aligned}$$

Initial Table:

$$\begin{pmatrix} 51 & 25.5 & 47 \\ 21.5 & 17.5 & 63 \\ 59 & 71 & 29.5 \end{pmatrix}$$

Step-1:-

Row Reduction

$$\begin{pmatrix} 25.5 & 0 & 21.5 \\ 4 & 0 & 45.5 \\ 29.5 & 41.5 & 0 \end{pmatrix}$$

Step-2:-

Column Reduction

$$\begin{pmatrix} 21.5 & 0 & 21.5 \\ 0 & 0 & 45.5 \\ 25.5 & 41.5 & 0 \end{pmatrix}$$

Step-3:-

$$\begin{pmatrix} 21.5 & (0) & 21.5 \\ (0) & 0 & 45.5 \\ 25.5 & 41.5 & (0) \end{pmatrix}$$

The optimal Assignment

$$S_1 \rightarrow A_2, S_2 \rightarrow A_1, S_3 \rightarrow A_3$$

$$\begin{aligned} \text{The optimal cost} &= 25.5 + 21.5 + 29.5 \\ &= 76.5 \end{aligned}$$

III. CONCLUSION

In this paper the fuzzy balanced assignment problem has been transformed into crisp assignment problem using dodecagonal fuzzy number. Numerical example shows that by using this method we can have the optimal assignment as well as the fuzzy optimal total cost. By using dodecagon fuzzy number methods, we have shown that the total cost obtained is optimal moreover; one can conclude that the solution of fuzzy problems can be obtained by dodecagonal fuzzy number method effectively.

IV. REFERENCES

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