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Membership Abelian Group of Fuzzy Sets

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ABSTRACT

In this paper, we discussed about membership function satisfying the abelian group. Also some basic definition have been discussed. Fuzzy set provide a methametical notation for representing real world concepts with are essentially vague.
Keywords : Fuzzy sets, Membership function, Abelian group, Membership abelian group

I. INTRODUCTION

Fuzzy sets were introduced by Zadeh as a means of representing and manipulating data that was not precise, but rather fuzzy.

In the mathematics fuzzy sets are sets whose elements have been degrees of membership. This article introduces the fuzzy theory. At first, the definition of fuzzy set characterized by membership function is described. Next, definitions of empty fuzzy set and universal fuzzy set and basic operations of these fuzzy sets are shown.

Some basic definition of group in definition 1. Then the definition of membership group in the definition 2. The example for membership group.

II. Some basic definition

Definition:1

An abelian group is a set, A, together with an operation " $*$" that combines any two elements a and$b$ to form anotherelement denoted $a * b$. The symbol " $*$" is a general placeholder for a concretely given operation. An abelian group, the set and operation, $(A, *)$, must satisfy five requirements known as the *abelian group axioms*:

Group Closure

For all a, b in A, the result of the operation $a * b$ is also in A.

Associativity
For all a, b and c in A, the equation $(a * b) * c = a * (b * c)$ holds.

Identity element
There exists an element e in A, such that for all elements $a$in A, the equation $e * a = a * e = a$ holds.

Inverse element
For each $a$in A, there exists an element b in A such that $a * b = b * a = e$, where e is the identity element.

Abelian Group

Commutativity: For all a, b in A, $a * b = b * a$.

More compactly, an abelian group is a commutative group. A group in which the group operation is not commutative is called a "non-abelian group" or "non-commutative group".

Definition: 2

Membership Function

For any set X, a membership function on X is any function from X to the real unit interval [0,1].

Membership function on X represent fuzzy subsets of X. The membership function which represents a fuzzy set $\tilde{A}$ is usually denoted by μ_A

$$\mu_G : X \rightarrow [0,1]$$

Where μ_G is membership function

Membership Group

Membership Closure

For all μ_A,$\mu_B$in μ_G, the result of the operation $\mu_A * \mu_B$is also inμ_G.

Membership Associative

For all μ_A, $\mu_B$and $\mu_C$in μ_G, the equation $(\mu_A * \mu_B) * \mu_C$= $\mu_A *(\mu_B * \mu_C)$ holds.

Membership Identity

There exists an element $\mu_e$in μ_G, such that for all elements $\mu_A$in μ_G, the equation $\mu_e * \mu_A$= $\mu_A * \mu_e$= $\mu_G$holds.

Membership Inverse

For each $\mu_A$in μ_G, there exists an element $\mu_B$in $\mu_G$such that $\mu_A * \mu_B$= $\mu_B * \mu_A$= μ_e, where $\mu_e$is the identity element.

Where μ_B is the inverse of μ_A.

AbelianGroup:

Membership Commutativity:

For all μ_A, $\mu_B$in μ_G, $\mu_A * \mu_B$= $\mu_B * \mu_A$.

More compactly, an membershipabelian group is a membership commutative group. A membership group in which the membership group operation is not membership commutative is called a "non-abelian group" or "non- membership commutative group".

Examples:

For any membership group,X is a set , μ_G be a membership function of triangle membership function then to prove that in a Triangle Membership Function

(i) Membership Closure: $\mu_A, \mu_B \in \mu_G$ then $\mu_A * \mu_B \in \mu_G$

(ii) Membership Associative:$\mu_A, \mu_A, \mu_A \in \mu_G$ then $(\mu_A * \mu_B) * \mu_C$ =$\mu_A * (\mu_B * \mu_C)$

(iii) Membership Identity: $\mu_A, \mu_e \in \mu_G$ then $\mu_A * \mu_I$ =$\mu_I * \mu_A$ =$\mu_A \in \mu_G$

(iv) Membership inverse: $\mu_A, \mu_B \in \mu_G$ where μ_B is inverse of μ_A then $\mu_A *(\mu_A)^{-1}$ = $(\mu_A)^{-1} * \mu_A$ =$\mu_I \in \mu_G$

(v) Membership Abelian group: $\mu_A, \mu_B \in \mu_G$ then $\mu_A * \mu_B$ = $\mu_B * \mu_A \in \mu_G$

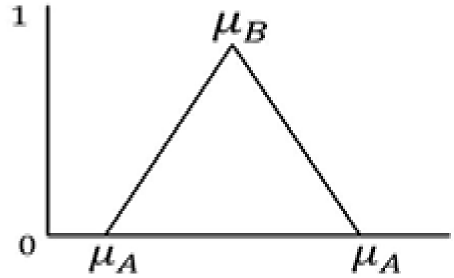
Proof:

Given: μ_G be a membership group

To prove :Triangle membership function

(i) Membership Closure:

Let x∈[0,1] ∀ x ∈ X be a fuzzy set and Let $\mu_A, \mu_B \in \mu_G$



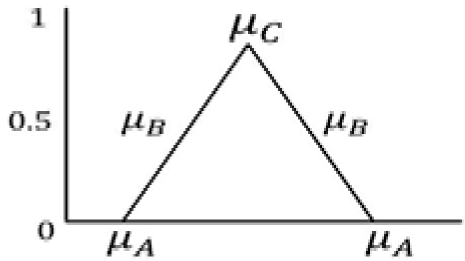
Let $\mu_A, \mu_B \in \mu_G$

μ_A(x) $* \mu_B$(x) = $\mu_A(0) * \mu_B(1) \in \mu_G$

$\therefore \mu_A * \mu_B \in \mu_G$

(ii) Membership Associative:

Let x ∈[0,1] ∀ x ∈ X be a fuzzy set and Let $\mu_A, \mu_B, \mu_C \in \mu_G$



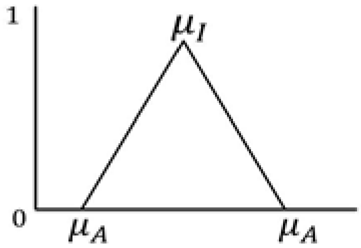
$(\mu_A(x) * \mu_B(x)) * \mu_C$(x) = $\mu_A(x) * (\mu_B(x) * \mu_C(x))$

$(\mu_A(0) * \mu_B(0.5)) * \mu_C(1)$ = $\mu_A(0) * (\mu_B(0.5) * \mu_C(1))$

$\therefore (\mu_A * \mu_B) * \mu_C$= $\mu_A * (\mu_B * \mu_C) \in \mu_G$

(iii) Membership Identity:

Let x ∈[0,1] ∀ x ∈ X be a fuzzy set and Let $\mu_A, \mu_I \in \mu_G$



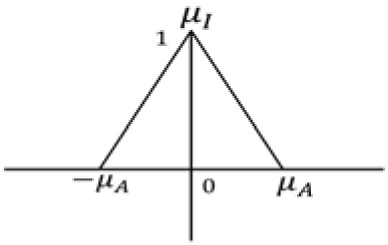
$\mu_A(x) * \mu_I$(x) = $\mu_I(x) * \mu_A$(x)

$\mu_A(0) * \mu_I(1) = \mu_I(1) * \mu_A(0)$

$\therefore \mu_A * \mu_I = \mu_I * \mu_A \in \mu_G$

(iv) Membership Inverse:

Let x ∈[0,1] ∀ x ∈ X be a fuzzy set and Let $\mu_A, (\mu_A)^{-1} \in \mu_G$



μ_A(x) $* (\mu_A)^{-1}$(x) = $(\mu_A)^{-1}$(x) $* \mu_A$(x)

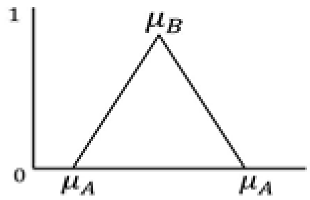
$\mu_A(0) * (\mu_A)^{-1}(1) = (\mu_A)^{-1}(1) * \mu_A(0)$

$\therefore \mu_A * (\mu_A)^{-1} = (\mu_A)^{-1} * \mu_A \in \mu_G$

∴ Membership Group is satisfied in Triangle Membership function

(v)Membership Abelian Group:

Let x ∈[0,1] ∀ x ∈ X be a fuzzy set and Let $\mu_A, \mu_B \in \mu_G$



μ_A(x) $* \mu_B$(x) = μ_B(x) $* \mu_A$(x)

μ_A(x) $* \mu_B$(x) = μ_B(x) $* \mu_A$(x)

$\mu_A(0) * \mu_B(1) = \mu_B(1) * \mu_A(0)$

$\therefore \mu_A * \mu_B = \mu_B * \mu_A \in \mu_G$

∴ Membership Abelian Group is satisfied in Triangle Membership function

III. CONCLUSION

Fuzzy sets are powerful mathematical tools for modeling and controlling uncertain system in industry, humanity and nature ; they are facilitaters for approximate reasoning on decision making in the absence of complete and precies information. Their role is significant when applied to complex phenomena not easily described by traditional mathematic. Membership function are used in fuzzificationand defuzzification. The output of a membership function, this value is always limited to between ‘0’ and ‘1’.

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