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Study of b-Chromatic Number of Wheel Graph

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ABSTRACT

In this paper we have generalized some of basic result on chromatic number.The b-chromatic number of a graph G is the largest integer k such that G admits a proper k-coloring in which every color class contains at least one vertex that has a neighbor in each of the other color classes. All graph considered here are simple, undirected and finite. For a graph G, we denote by V(G)its vertex set and by E(G)its edge set;|V(G)|is the order and x(G)is the chromatic number of G. for a graph G and a vertex x of G. Let G=(V,E) be an undirected and loopless graph.The b-chromatic number of a graph G is the largest iteger k such that G admits a proper k-colouring in which every colour class contains atleast one vertex adjacent to some vertex in all the other colour classes. A proper k-colouring of a graph G=(V(G),E(G)) is a mapping f:V(G)->N such that every two adjacent vertices receive different colors. The chromatic number of a graph G is denoted by X(G), is the minimum number foe which G has a proper k-colouring. The set of vertices with a specific colour is called a colour class. The b-chromatic number φ(G)is the largest integer k such that G admits a b-colouring with k colour.

Keywords: b-Chromatic Number, Line Graph, Wheel Graph, Complete Graph and Line Graph of a Wheel Graph.

I. INTRODUCTION

The b-chromatic number '(G) of a graph G is the largest integer k such that G admits a proper k-coloring in which every color class has a vertex adjacent to at least one vertex in each of the other color classes. Such a coloring is called a b-coloring. Let G be a graph and c a b-coloring of G. If x 2 V (G) has a neighbor in each other color class we will say that x realizes color c(x) and that c(x) is realized on x. All graphs considered here are simple, undirected and finite. For a graph G, we denote by V(G) its vertex set and by E(G) its edge set; |V(G)| is the order and x(G) is the chromatic number of G. Let G=(V,E) be an undirected graph with loopless and multiple edges. A colouring of vertices of graph G is a mapping c:V(G)->{1,2,…..k} for every vertex . A colouring is said to be proper if any two adjacent vertices of a graph have different colors. The chromatic number X(G)of a graph G is the smallest integer k which admits a proper coloring. A proper k-chromatic c of a graph G is a b-chromatic if for every color class c_i. The Four-Color Conjecture have been originated by Francis Guthrie. Let G be a simple graph with vertex set V (G) and edge set E (G). A colouring of the vertices of G is a mapping f: V (G)->N where N=1, 2,……k. For every vertex v V(G), f (v) is called the colour of v. The chromatic number (G) is the smallest integer k such that G admits a proper colouring using k colors. A b-colouring by k colors is a proper colouring of the vertices of G such that in each colour class there exists a vertex that has neighbours in all the other k-1 colour classes. If v is a colour predominating vertex of a colour class c then we write cdv(c) = v. The b-chromatic number (G) is the largest integer k such that G admits a b-colouring with k colors.

In accordance with faik the graph is b-continuous if b-colouring exists for every integer k satisfying x(G)≤k≤ φ(G).

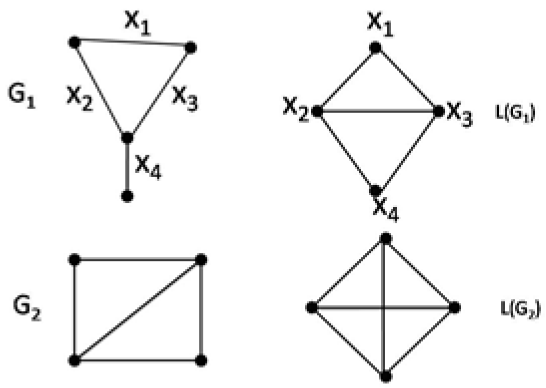
II. PRELIMINARIES

Definition.1.1 LINE GRAPH:
Consider the set X of lines of a graph G with at least one line as a family of 2- point members of V(G). The line graph of G, denoted L(G), is the intersection graph

Ω(X). Thus the points of L(G) are the lines of G, with two points of L(G) adjacent whenever the corresponding line of G. If x=uv is a line of G, then the degree of x in L(G) is clearly deg u+deg v to example of graphs and their line graphs. Note that in this fig G_2 =L(G_1), so that L(G_2) =L(L(G_1)). We write L^1(G)=L(G), L^2(G)=L(L(G)), and in general the iterated line graph is L^n(G)=(L(L^{n-1}(G)).

As a immediate consequence of definition of L(G), we know that every cut point of L(G) is a bridge of G which is not an end line, and conversely.

When defining any class of graph, it is desirable to know the number of points and lines in each; this is easy to determine for line graphs.



Definition 1.2. COMPLETE GRAPH
A simple graph in which each pair of distinct vertices joined by on edge is called a complete graph. Up to isomorphism there is just one complete graph of n vertices and its an denoted by k_n

In complete graph every vertices is connected to every other vertices.

Definition 1.3 WHEEL GRAPH

A wheel graph Wn is a graph with n vertices (n ≥ 4), formed by connecting a single vertex to all vertices of an (n-1)-cycle

Definition 1.4. LINE GRAPH OF A WHEEL GRAPH
A wheel graph and line graph of wheel graph by following figures.

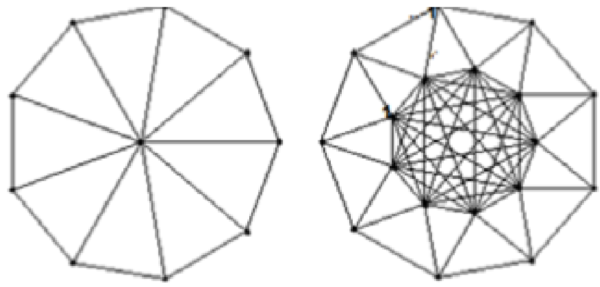


Figure 2.a Figure.2.b

Figure 2: The wheel graph W_{10} and its line graph $L(W_{10})$

Proposition 2.1. For any graph G, ϕ(G) ≤ Δ (G)+1, where Δ(G) is the maximum degree of the graph G.

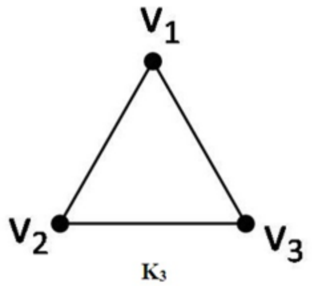
Proposition 2.2. If a graph G admits a b-colouring with m-colours then G must have at least m vertices with degree at least m - 1

Lemma 2.3 If Kn be a complete graph, then $\phi(K_n) = n$, for all n.

Proof: Let K_n be a complete graph with n vertices. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ be vertex set of the complete graph K_n.Then |V (G)| = n and |E(G)| =n(n-1)/2. To determined proper colouring we consider the following cases

Case 1: when n=3, $V(K_3)=\{v_1, v_2, v_3\}$, | V(G)| =3, |E(G)|=n(n-1)/2 where (n=3) |E(G)|=3(3-1)/2= 3
In this case, G has three vertices of degree 2. Maximum degree is 2.
Using Preposition 2.1, φ (K_3)≤3
If φ (K_3) = 3, as determined by preposition 2.2 the graph G must have three vertices of degree 2 which is possible. To assign proper colouring and for b-colouring consider the colour set C={1,2,3} and define the colour function f:V->C such that $f(V_1)$ =1, $f(V_2)$=2, $f(V_3)$=3.

The above proper colouring enables cdv (1) = V_1 cdv (2) =V_2; cdv (3) = V_3.
Therefore φ (K_3)=3.

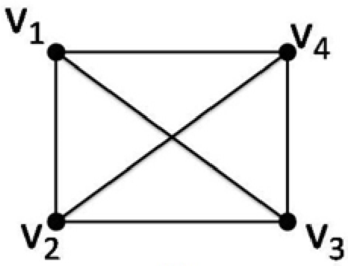


K_3

Case 2: when n=4, $V(K_3)=\{v_1, v_2, v_3, v_4\}$, $|V(G)| = 4$, $|E(G)|=n(n-1)/2$ where (n=4) $|E(G)|=4(4-1)/2= 6$
In this case, G has four vertices of degree three. Maximum degree is 3.

Using Preposition 2.1, $\varphi(K_4)\leq 4$
If $\varphi(K_4) = 4$, as determined by preposition 2.2 the graph G must have four vertices of degree 3 which is possible. To assign proper colouring and for b-colouring consider the colour set C={1,2,3,4} and define the colour function f:V->C such that $f(V_1) =1$, $f(V_2)=2$, $f(V_3)=3$, $f(V_4)=4$.

The above proper colouring enables cdv (1) = V_1 ; cdv (2) =V_2; cdv (3) = V_3; cdv (4) = V_4.
Therefore $\varphi(K_4)=4$

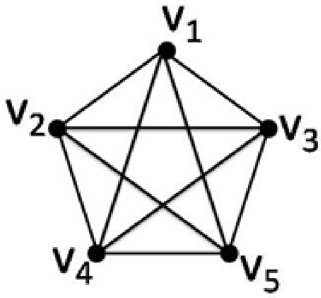


K_4

Case 3: when n=5, $V(K_3)=\{v_1,v_2,v_3,v_4,v_5\}$, $|V(G)| =5$, $|E(G)|=n(n-1)/2$ where (n=5) $|E(G)|=5(5-1)/2= 10$
In this case, G has five vertices of degree four. Maximum degree is 4.

Using Preposition 2.1, $\varphi(K_5)\leq 5$
If $\varphi(K_5) = 5$, as determined by preposition 2.2 the graph G must have five vertices of degree 4 which is possible. To assign proper colouring and for b-colouring consider the colour set C={1,2,3,4,5} and define the colour function f:V->C such that $f(V_1) =1$, $f(V_2)=2$, $f(V_3)=3$, $f(V_4)=4$, $f(V_5)=5$.

The above proper colouring enables cdv (1) = V_1 ; cdv (2) =V_2; cdv (3) = V_3; cdv (4) = V_4; cdv (5) = V_5
Therefore $\varphi(K_5)=5$

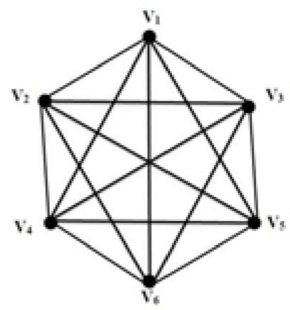


Case 4: when n=6, $V(K_3)=\{v_1,v_2,v_3,v_4,v_5,v_6\}$, $|V(G)| =6$, $|E(G)|=n(n-1)/2$ where (n=6) $|E(G)|=6(6-1)/2= 15$
In this case, G has six vertices of degree five. Maximum degree is 5.

Using Preposition 2.1, $\varphi(K_6)\leq 6$
If $\varphi(K_6) = 6$, as determined by preposition 2.2 the graph G must have six vertices of degree 5 which is possible.

To assign proper colouring and for b-colouring consider the colour set C={1,2,3,4,5,6} and define the colour function f:V->C such that $f(V_1) =1$, $f(V_2)=2$, $f(V_3)=3$, $f(V_4)=4$, $f(V_5)=5$, $f(V_6)=6$.
The above proper colouring enables cdv (1) = V_1 ; cdv (2) =V_2; cdv (3) = V_3; cdv (4) = V_4; cdv (5) = V_5 cdv (6) = V_6
Therefore $\varphi(K_6)=6$

From the above case we concluded that the b-chromatic number of the complete graph with n vertices in n
Hence $\varphi(K_n)= n$.



III. MAIN RESULT

Theorem: 3.1

If L (Wn) be the line graph of the wheel graph, then ϕ (L (Wn) = n-1.

Proof:

Let Wn be a wheel graph with n vertices with vertex set V (wn) = {w1, w2,…………..wn} with wn as the hub. Now L (Wn) contains a complete graph Kn-1 with V (Kn-1) ={u1,u2,……….un-1} and a cycle C with V(C) = {v1,v2,………vn-1}. To assign proper colouring and for b-colouring consider the colour set C= {1, 2,….n} and define the colour function f: V→C. Assign the colour ci to the vertex set V(kn-1) =ui for i =1, 2 …n-1. Next we have to colour the vertices of the cycle. If one more colour is introduced, say cn it should be colored to any one of the vertex of the cycle C. In the outer cycle C,v1 is. adjacent with v2,vn-1 ,u1 , u2 . vn-1 is adjacent with v1,vn-2,un-1, u1. In general each vertex vi is adjacent with vi+1,vi-1 for i=2,3…..n-1 and ui , ui+1 for i=1,2,3…..n-1. Every vertex in the cycle has degree four. When the colour cn is assigned to any vertex in the cycle which cannot harmonises its colour as cn.Hence we can assign the n-1 colours which are assigned to the complete graph. Next the vertices in the cycle vi for i=3,4…..n-1 should assigned by the colour ci for i=1, 2,……..n-3 and to the vertex v1 should assigned by the colour cn-2 and v2 by cn-1. So, the new colours cannot be introduced. The above proper colouring enables cdv (1) = u1; cdv (2) =u2; cdv (3) = u3; ; cdv (4) =u4 ; cdv (5) =u5; cdv (n-1) = un-1. Hence ϕ (L(Wn) =n

IV. CONCLUSION

In this paper we established the b-chromatic number of central graph of some special graph. Graph theory has wild applications in biochemistry, electrical engineering, computer science and operations research. Here we have obtained the b chromatic num-ber of line graph of wheel graph. This work can be extended to identify the various graph.

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