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Method for Solving Unbalanced Assignment Problem using Hexgonal Fuzzy Numbers

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ABSTRACT

This paper presents solution methodology for unbalanced assignment problem with fuzzy cost. The fuzzy costs are considered as hexgonal fuzzy numbers. Robust's ranking method has been used for ranking the hexgonal fuzzy numbers. Hungarian method is extended to solve this type of fuzzy un balanced assignment problem. Numerical examples show that the fuzzy ranking method offers an effective tool for handling the fuzzy unbalanced assignment problem.
Keywords: Fuzzy sets, Fuzzy unbalanced assignment problem, hexagonal fuzzy numbers, Hungarian method, Robust's ranking method.

I. INTRODUCTION

The unbalanced assignment problem is a special type of linear programming problem in which our objective is to assign number of salesman's to number of areas at a minimum cost(time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate all the algorithms developed to find optimal solution of transportation problem are applicable to unbalanced assignment problem. Widely known as Hungarian method proposed by Kuhn (1) is used for its solution.

In this paper, we investigate more realistic problem and namely the unbalanced assignment problem, with fuzzy costs. Subject to some crisp constraints, the objective function is considered also as a fuzzy number. The methods are to rank the fuzzy objective values of the objective function by some ranking method for fuzzy number to find the best alternative. On the basic of idea the Robust's ranking methods (3) has been adopted to transform the fuzzy unbalanced assignment problem to a crisp one so that the conventional solution methods may be applied to solve unbalanced assignment problem by R.PanneerSelvam (2).

Lin and Wen solved the unbalanced assignment with fuzzy interval number costs by a labeling algorithm (4) in the paper by sakawa et.al (2), the authors dealts with actual problems on production and work force assignment in a housing material manufacturer and a sub construct firm and formulated two kinds of two level programming problems. Chen (5) proved same theorems and proposed a fuzzy unbalanced assignment model that considers all individuals to have some skills. Wang (6) solved a similar model by graph theory. Dominance of fuzzy numbers can be explained by many ranking methods (8,9,10,11) of these, Robust's ranking method (3) which satisfies the properties of compensation, linearity and additive. In this paper we have applied Robust's ranking technique (3).

II. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed. Zadeh (7) in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

2.1. Fuzzy Set:[7]

A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1] i.e. {(x ,μ_A (x));x∈ X }.

Here μ_A : X → [0, 1] is a mapping called the degree of membership function of the fuzzy set A and μ_A (x) is called the membership value of x∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2. Normal fuzzy set:

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that μ_A (x) =1.

2.3 Fuzzy number

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function μ_A : R →[0,1] has the following properties

i) A must be a normal fuzzy set.

ii) A α_A must be a closed interval for every $\alpha \in$ (0,1].

iii) The support of A,$o +_A$ must be bounded.

2.4 Hexagonal Fuzzy Number

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H$=(a_1, a_2, a_3, a_4, $a_{5,}a_6$) a_1, a_2, a_3, a_4, a_5 and a_6 where are real numbers and its membership function is given below,

$$\mu\tilde{A}_H(x)=\begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), for\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), for\ a_2 \le x \le a_3 \\ 1, for\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), for\ a_5 \le x \le a_6 \\ 0, otherwise \end{cases}$$

2.5 Operations of Hexagonal Fuzzy numbers

The two operations that can be performed on hexagonal fuzzy numbers, suppose $\tilde{A}_H$ =($a_1, a_2, a_{3,}a_4, a_5, a_6$) and $\tilde{B}_H$ =($b_1, b_2, b_3, b_4, b_5, b_6$) are two hexagonal fuzzy numbers then

(i)Addition**:** $\tilde{A}_H + \tilde{B}_H = (a_1 + b_1\ ,\ a_2 + b_2\ ,\ a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$

(ii)Subtraction: $\tilde{A}_H - \tilde{B}_H = (a_1 - b_1\ ,\ a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$

2.6 Robust Ranking Technique[3]

Robust Ranking Technique satisfies the following properties,

i. Compensation

ii. Linearity

iii. Additivity

It provides results which are consist human intuition . If $A^{\tilde{}}$ is a fuzzy number then the Robust ranking is defined by

R($\tilde{A}_H$)=$\int_0^1 0.5(a_{h\alpha}^L, a_{h\alpha}^U)$dk where $(a_{h\alpha}^L, a_{h\alpha}^U)$

Is the α level cut of the fuzzy number $\tilde{A}_H$. In this paper we find the rank
of the objective

III. Unbalanced Assignment problem to change into balanced Assignment Problem

The number of rows (areas) is not equal to the number of columns (salesman's) then the problem is termed as unbalanced assignment problem then this problem into change balanced assignment problem as follows necessary number of dummy row (s) / column(s) are added such that the cost matrix is a square matrix the values for the entries in the dummy row (s) / column(s) are assumed to be zero.

IV. Robust's Ranking Techniques Algorithms

The assignment Problem can be stated in the form of n$\times n$ cost matrix $[C_{ij}]$ of real numbers as given in the following

	Salesman 1	Salesman2	-------------	Salesman j	--------------	SalesmanN
Area 1	C_{11}	C_{12}	………	C_{1j}	…….	C_{1n}
Area 2	C_{21}	C_{22}	…….	C_{11}	……	C_{2n}
	⋮	⋮	⋮	⋮	⋮	⋮
Area N	C_{n1}	C_{n2}	⋮	C_{nj}	⋮	C_{nn}

Minimize

$Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to $\sum_{i=1}^{n} x_{ij}$=1

$\sum_{j=1}^{n} x_{ij}$=1

x_{ij} ∈[0,1]→(1)

where

$$x_{ij} = \begin{cases} 1; \text{ if the } i^{th} \text{ area is assigned the } j^{th} \text{ saleman} \\ 0;\ otherwise \end{cases}$$

ie the decision variable denoting the assignment of the area i to job j. $(\tilde{C}_{ij})$ is the cost of assigning the j^{th} salesman to the i^{th} area. The objective is to minimize the total cost of assigning all the salesman's to the available persons (one salesman to one area)

When the costs or time . $(\tilde{C}_{ij})$are fuzzy numbers, then the total costs becomes a fuzzy number.

$Z^{v^*}=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Hence it cannot be minimized directly for solving the problem. We defined by the fuzzy cost Co-efficient into crisp ones by a fuzzy number ranking method. Robust's ranking technique (3) which satisfies compensation linearity, and additivity properties and provides results which are consistence with human intuition. Given a convex fuzzy number ,$\tilde{c}$ the Robust's Ranking index is defined by

R($\tilde{c}$)=$\int_0^1 0.5(C_K^L, C_K^U)$dk where $(C_K^L C_K^U)$ is the K-level cut of the fuzzy number($\tilde{c}$)

In this paper we use this method for ranking the objective values. The Robust's ranking index R($\tilde{c}$) gives the representative value of the fuzzy number ($\tilde{c}$) it satisfies the linearity and additive property:

If $\hat{P}$ =$l\widehat{E}$+m$\hat{y}$ and $\hat{Q}$ =$\widehat{SC}$-t$\widehat{N}$ where l,m,s,t are constant then we have R$\hat{p}$=lR($\hat{E}$)+n R($\hat{y}$) and R($\hat{\varphi}$)=SR($\hat{c}$)- tR($\widehat{N}$) on the basis of this property the fuzzy assignment problem can be transformed into a crisp assignment problem linear programming problem from. The ranking technique of the Robust's is

If R($\tilde{G}$)≤R($\widetilde{H}$) then $\tilde{G}$≤H^V
ie min{$\tilde{G}$,H^V}=$\tilde{G}$ from the assignment problem (1) with fuzzy objective function.

Min Z^{v^*}-$\sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{C}_{ij})\, X_{ij}$

Subject to

$\sum_{i=1}^{n} x_{ij}$=1

$\sum_{j=1}^{n} x_{ij}$=1

x_{ij} ∈[0,1]→(2)

Where

$$x_{ij} = \begin{cases} 1; \text{ if the } i^{th} \text{ area is assigned the } j^{th} \text{ saleman} \\ 0;\ otherwise \end{cases}$$

Is the decision variable denoting the assignment of the area i to j^{th} salesman $(\tilde{C}_{ij})$ is the cost of designing the j^{th} job to the i^{th}area. The objective is to minimize the total cost of assigning all the salesman to the available areas.

Since R($\tilde{C}_{ij}$) are crisp values, this problem (2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional methods, namely the Hungarian method or simplex method to solve the linear programming problem form of the problem. Once the optimal solution x^* of model (2) is found the optimal fuzzy objective value Z^{v^*} of the original problem can be calculated as

$$(Z^{v^*})=\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{C}_{ij}){x^*}_{ij}$$

Numerical Example

Let us consider a fuzzy unbalanced assignment problem with rows representing 4 area A, B, C, D and columns representing the salesman's $S_1, S_2, S_3, S_4, S_{5,}S_6$ the cost matrix $(\tilde{C}_{ij})$ is given whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

$$(\tilde{C}_{ij})=\begin{pmatrix} (1,2,3,4,5,6) & (1,3,4,6,7,8) & (8,9,7,6,5,4) & (2,6,5,4,3,2) \\ (3,6,5,4,3,2) & (2,3,5,6,7,5) & (4,7,6,5,2,1) & (3,4,5,6,7,5) \\ (1,5,6,7,6,2) & (1,8,7,6,5,6) & (5,9,4,6,7,6) & (8,7,1,0,6,5) \end{pmatrix}$$

Solution:

The given problem is an fuzzy unbalanced assignment problem so to change into the fuzzy balanced assignment problem as follows

$$(\tilde{C}_{ij})=\begin{pmatrix} (1,2,3,4,5,6) & (1,3,4,6,7,8) & (8,9,7,6,5,4) & (2,6,5,4,3,2) \\ (3,6,5,4,3,2) & (2,3,5,6,7,5) & (4,7,6,5,2,1) & (3,4,5,6,7,5) \\ (1,5,6,7,6,2) & (1,8,7,6,5,6) & (5,9,4,6,7,6) & (8,7,1,0,6,5) \\ (0,0,0,0,0,0) & (0,0,0,0,0,0) & (0,0,0,0,0,0) & (0,0,0,0,0,0) \end{pmatrix}$$

In conformation to model (2) the fuzzy balanced assignment problem can be formulated in the following mathematical programming form

Min

{R $(1,2,3,4,5,6)x_{11}$ +R $(1,3,4,6,7,8)x_{12} + R(8,9,7,6,5,4)x_{13} + R\ (2,6,5,4,3,2)x_{14} + R(3,6,5,4,3,2)x_{21} + R\ (2,3,5,6,7,5)x_{22} + R\ (4,7,6,5,2,1)x_{23} + R(3,4,5,6,7,5)x_{24} + R(1,5,6,7,6,2)x_{31} + R\ (1,8,7,6,5,6)\ x_{32} + R(5,9,4,6,7,6)x_{33} + +R\ (8,7,1,0,6,5)x_{34+}(0,0,0,0,0,0)x_{41} + R\ (0,0,0,0,0,0)x_{42} + R(0,0,0,0,0,0)x_{43} + R(0,0,0,0,0,0)x_{44}$}

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{ij} \in [0,1]$$

$R(\tilde{C}_{11})$= R(1,2,3,4,5,6)=$\int_0^1 0.5(C_K^L, C_K^U)$dk

$= \int_0^1 0.5(\alpha + 1 + 4 - \alpha + \alpha + 3 + 6 - \alpha)$dk

$=\int_0^1 0.5(14)$dk =7

Proceeding similarly the Robust's ranking indices for the fuzzy costs $(\tilde{C}_{ij})$ are calculated as:

$R(\tilde{C}_{12}) = 9.75, R(\tilde{C}_{13}) = 13, R(\tilde{C}_{14}) = 7.75$

$R(\tilde{C}_{21}) = 8, R(\tilde{C}_{22}) = 9.75, R(\tilde{C}_{23}) = 9, R(\tilde{C}_{24} =)10.25$

$R(\tilde{C}_{31}) = 10, R(\tilde{C}_{32}) = 11.5, R(\tilde{C}_{33}) = 11.25, R(\tilde{C}_{34}) = 7$

$R(\tilde{C}_{41}) = 0, R(\tilde{C}_{42}) = 0, R(\tilde{C}_{43}) = 0, R(\tilde{C}_{44}) = 0.$

We replace these values for their corresponding $(\tilde{C}_{ij})$ in (3) which results in a convenient assignment problem in the linear programming problem. We solve it by Hungarian method to get the following optimal solution

$$(\tilde{C}_{ij}) = \begin{pmatrix} 7 & 9.75 & 13 & 7.75 \\ 8 & 9.75 & 9 & 10.25 \\ 10 & 11.5 & 11.25 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Solution by Hungarian method

Step:1 we subtract the row minimum for each row giving the follow matrix

$$\begin{pmatrix} 0 & 2.75 & 6 & 0.75 \\ 0 & 1.75 & 1 & 2.25 \\ 3 & 4.5 & 4.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step:2 we subtract the column minimum for each column

$$\begin{pmatrix} 0 & 2.75 & 6 & 0.75 \\ 0 & 1.75 & 1 & 2.25 \\ 3 & 4.5 & 4.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step:3 Now we determine the minimum number of lines required to cover all zeros in the matrix. If the number of lines exactly equal to 4 then the complete assignment is obtained. while if the number of draw lines less than 4.

$$\begin{pmatrix} 0 & 2.75 & 6 & 0.75 \\ 0 & 1.75 & 1 & 2.25 \\ 3 & 4.5 & 4.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step:4 identify the minimum value of undeleted cell values. odd the minimum undeleted cell value at the intersection point of the present matrix and then subtract the minimum undeleted cell value from all then subtract the minimum undeleted cell value from all the undeleted cell values all the other entires remain same.

$$\begin{pmatrix} (0) & 1.75 & 5 & 0.75 \\ \cancel{0} & 0.75 & (0) & 2.25 \\ 3 & 3.5 & 3.25 & (0) \\ 1 & (0) & \cancel{0} & 1 \end{pmatrix}$$

The optimal assignment

$$A\rightarrow S_1,\ B\rightarrow S_3,\ C\rightarrow S_4,\ D\rightarrow S_2$$

The optimum total minimum cost=Rs.7+9+7+0

=23

The fuzzy optimal assignment $=\tilde{C}_{11}+\tilde{C}_{23}+\tilde{C}_{34}+\tilde{C}_{42}$

$= R\,(1,2,3,4,5,6) + R\,(4,7,6,5,2,1) + R\,(8,7,1,0,6,5) + R(0,0,0,0,0,0)$

$= R(13,16,10,9,13,12)$

Also we find that R (Z^{v^*})=24

V. CONCLUSION

In this paper, the fuzzy unbalanced assignment problem with hexagonal fuzzy number has been transformed into crisp assignment problem using Robust's ranking indices (3). Numerical example show that by using method we can have the optimal assignment as well as the crisp and fuzzy optimal total cost. Moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust's ranking methods effectively, this technique can also be used in solving other types of problems like, project schedules, transportation problems and network flow problems.

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