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Solving Fuzzy Assignment Problem for Heptagonal Fuzzy Number using Ones Assignment Method

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ABSTRACT

In this paper, revised ones assignment method is used to solve the fuzzy assignment problem to find minimum objective function using heptagonal fuzzy number. Robust's ranking method has been used for ranking the heptagonal fuzzy numbers.Finally, example are given to explain this method. We find the minimum total cost.
Keywords : Fuzzy Assignment Problem, Revised Ones Assignment Algorithm And Heptagonal Fuzzy Number, Robust's Ranking Method.

I. INTRODUCTION

Fuzzy sets, introduced by zadeh in 1965. A new mathematical tool to deal with uncertainly of information we used basic concepts of fuzzy sets, fuzzy number and fuzzy linear programming. The fuzzy assignment problem is a special type of linear programming problems. Revised ones assignment method is used to solve fuzzy assignment problem to find minimum objective function. Then such as fuzzy number, heptagonal have been introduced in this method. Seven persons are to be performed by seven jobs depending in on their jobs. Examples are given to explain this method and find minimum total cost.

II. Preliminaries

Definition 2.1(Fuzzy set)

A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1] i.e. {(x , μ_A (x));x∈ X }.

Here μ_A : X → [0, 1] is a mapping called the degree of membership function of the fuzzy set A and μ_A (x) is called the membership value of x∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2(Normal fuzzy set)

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that μ_A (x) =1.

Definition 2.3(Fuzzy number)

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function μ_A : R →[0,1] has the following properties

i) A must be a normal fuzzy set.

ii) A α_A must be a closed interval for everyα ∈ (0,1].

iii) The support of A, $o_A{}^+$ must be bounded

III. Heptagonal Fuzzy Number (HFN)

In this section, a new form of fuzzy number called as Heptagonal Fuzzy number (HFN) is introduced which can be effectively used in solving many decision making problem. A fuzzy number $\widetilde{H}$ in R is said to be a heptagonal fuzzy number if its membership function μ_H: R[0,1] has the following characteristics.

We denote the heptagonal fuzzy number by $\widetilde{H}$=($h_1, h_2, h_3, h_4, h_5, h_6, h_7$)

$$\mu_{\tilde{H}}(x)=\begin{cases} 0 \; for \; x < h_1 \\ \frac{1}{2}\left(\frac{x-h_1}{h_2-h_1}\right) \; for \; h_1 \le x \le h_2 \\ \frac{1}{2} \; for \; h_2 \le x \le h_3 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-h_3}{h_4-h_3}\right) \; for \; h_3 \le x \le h_4 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{h_5-x}{h_5-h_4}\right) \; for \; h_4 \le x \le h_5 \\ \frac{1}{2} \; for \; h_5 \le x \le h_6 \\ \frac{1}{2}\left(\frac{h_7-x}{h_7-h_6}\right) \; for \; h_6 \le x \le h_7 \\ 0 \; for \; x \ge h_7 \end{cases}$$

IV. Revised Ones Assignment Using Hexagonal Fuzzy Number

4.1 Mathematical Formulation of Assignment Problem's

The mathematical formulation of the assignment problem is, associated to each assignment problem there is a matrix called cost or effectiveness matrix $[\boldsymbol{c_{ij}}]$ where $\boldsymbol{c_{ij}}$ is the cost of assigning i^{th} persons to j^{th} jobs. In this paper we call it assignment matrix and represent it as follows:

$$\begin{matrix} & 1 \ldots n \\ \begin{matrix} 1 \\ \vdots \\ n \end{matrix} & \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{pmatrix} \end{matrix}$$

The mathematical formulation of the assignment problem is,

Minimize Z=$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} X_{ij}$

Subject to $\sum_{i=1}^{n} X_{ij}$ =1, i=1,…….,n

$\sum_{j=1}^{n} X_{ij}$ =1, j=1,……,n; X_{ij}=0 or 1

V. ROBUST'S RANKING

Robust's ranking which satisfy compensation, linearity, and additively properties and provides results which are consistent with human intuition. If $\tilde{A}_H$ is a fuzzy number then the ranking is defined by

R($\tilde{A}_H$)=$\int_0^1 0.5(a_{h\alpha}^L, a_{h\alpha}^U)d\alpha$

Where $(a_{h\alpha}^L, a_{h\alpha}^U)$ is the α level cut of fuzzy number $\tilde{A}_H$.

VI. Algorithm

Revised Ones Assignment Algorithm for using Heptagonal Fuzzy Number

Step 1: In a minimization or maximization case, find the minimum or maximum element of each row in the assignment matrix and write it on the right hand side of the matrix. Then divide each element of 7 row of the matrix by $a_1$to a_7.These operations create at least one ones in each rows. In term of ones for each row and column. Otherwise go to Step 2.

$$\begin{matrix} & 1\;2\;3\;4\;5\;6\;7 & \\ \begin{matrix} 1\\2\\3\\4\\5\\6\\7 \end{matrix} & \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & c_{27} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & c_{37} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & c_{47} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & c_{57} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & c_{67} \\ c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & c_{77} \end{pmatrix} & \begin{matrix} a_1\\a_2\\a_3\\a_4\\a_5\\a_6\\a_7 \end{matrix} \end{matrix}$$

Step 2: Find the minimum or maximum element of each column in assignment matrix and write it below 7 columns. Then divide each element of 7 column of the matrix by $b_1$to b_7, these operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from Step 1 and 2 then go to Step 3.

$$\begin{matrix} & 1\;2\;3\;4\;5\;6\;7 & \\ \begin{matrix} 1\\2\\3\\4\\5\\6\\7 \end{matrix} & \begin{pmatrix} c_{11}/a_1 & c_{12}/a_1 & c_{13}/a_1 & c_{14}/a_1 & c_{15}/a_1 & c_{16}/a_1 & c_{17}/a_1 \\ c_{21}/a_2 & c_{22}/a_2 & c_{23}/a_2 & c_{24}/a_2 & c_{25}/a_2 & c_{26}/a_2 & c_{27}/a_2 \\ c_{31}/a_3 & c_{32}/a_3 & c_{33}/a_3 & c_{34}/a_3 & c_{35}/a_3 & c_{36}/a_3 & c_{37}/a_3 \\ c_{41}/a_4 & c_{42}/a_4 & c_{43}/a_4 & c_{44}/a_4 & c_{45}/a_4 & c_{46}/a_4 & c_{47}/a_4 \\ c_{51}/a_5 & c_{52}/a_5 & c_{53}/a_5 & c_{54}/a_5 & c_{55}/a_5 & c_{56}/a_5 & c_{57}/a_5 \\ c_{61}/a_6 & c_{62}/a_6 & c_{63}/a_6 & c_{64}/a_6 & c_{65}/a_6 & c_{66}/a_6 & c_{67}/a_6 \\ c_{71}/a_7 & c_{72}/a_7 & c_{73}/a_7 & c_{74}/a_7 & c_{75}/a_7 & c_{76}/a_7 & c_{77}/a_7 \end{pmatrix} & \begin{matrix} a_1\\a_2\\a_3\\a_4\\a_5\\a_6\\a_7 \end{matrix} \end{matrix}$$

$b_1\; b_2\; b_3\; b_4\; b_5\; b_6\; b_7$

Step 3: Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than 7,then the complete assignment is not possible, while if the number of lines is exactly equal to 7, then the complete assignment is obtained.

Step 4: If a complete assignment program is not possible in Step 3, then select the smallest or largest element (sayd_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij}each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new ones to this row or column.

If still a complete optimal assignment is not achieved in this new matrix, then use Step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained. (To assign one we have add Step 5 which is mentioned below.)

Step 5:

i). For minimization problem select maximum number from calculated matrix and write it on right hand side as well as bottom side.
* To assign one, start from minimum number of columns (bottom side) and select ones.
* If there are more than one ones in any column then ignore temporarily, and give last priority to that column.
* If still there are identical ones in column then give the priority to maximum number of rows (right hand side).
(or)
ii). For maximization problem select minimum number from calculated matrix and write it on right hand side as well as bottom side.
* To assign one, start from maximum number of columns (bottom side) and select ones.
* If there are more than one ones in any column then ignore temporarily, and give last priority to that column.
* If still there are identical ones in column then give the priority to minimum number of rows (right hand side).

VII. Numerical Example

To illustrate the proposed method a fuzzy assignment problem is solved by
using the proposed method.

Example 7.1

Seven persons are available to do three different jobs. From past records, the cost that each person takes to do each job is known and is represented by heptagonal fuzzy numbers and find the assignment of persons to jobs that will minimize the total fuzzy cost.

Persons→ Jobs↓	A	B	C
1	(3,6,2,1,5,0,4)	(2,3,1,4,3,6,5)	(2,4,3,1,6,5,2)
2	(2,7,7,6,3,2,1)	(1,3,5,7,9,11,13)	(0,1,2,4,6,0,5)
3	(3,6,3,2,1,8,7)	(3,4,3,2,1,1,0)	(2,4,6,8,10,12,14)

Solution:
In conformation to model (2) the fuzzy balanced assignment problem can be formulated in the following mathematical programming form

Min{R (3,6,2,1,5,0,4)x_{11} +R (2,3,1,4,3,6,5)x_{12} + R(2,4,3,1,6,5,2)x_{13} + R(2,7,7,6,3,2,1)x_{21} + R (1,3,5,7,9,11,13)x_{22} + R (0,1,2,4,6,0,5)x_{23} + R(3,6,3,2,1,8,7)$x_{31}$$R$ (3,4,3,2,1,1,0) x_{32} + R(2,4,6,8,10,12,14)x_{33}

Subject to

x_{11} +x_{12} +x_{13}=1
x_{11} +x_{21} +x_{31}=1
x_{21} +x_{22} +x_{23}=1
x_{12} +x_{22} +x_{32}=1
x_{13} +x_{23} +x_{33}=1
x_{14} +x_{24} +x_{34} =1
x_{31} +x_{32} +x_{33} =1
x_{ij} ∈[0,1]

Now we calculate R (3,6,2,1,5,0,4) by applying Robust's ranking method. The membership function of the heptagonal fuzzy number (3,6,2,1,5,0,4) is

$R(\tilde{C}_{11})$= R(3,6,2,1,5,0,4)=$\int_0^1 0.5(C_K^L, C_K^U)$dk
=$\int_0^1 0.5(3\alpha + 3, 1 - 1\alpha, -5\alpha + 5, 4 + 1\alpha)$dk
=$\int_0^1 0.5(12)$dk =6

Proceeding similarly the Robust's ranking indices for the fuzzy costs ($\tilde{C}_{ij}$) are calculated as:

$$R(\tilde{C}_{12}) = 6.75, R(\tilde{C}_{13}) = 7.25, R(\tilde{C}_{21}) = 8.25, R(\tilde{C}_{22}) = 14.5, R(\tilde{C}_{23}) = 7$$
$$R(\tilde{C}_{31}) = 7.75, R(\tilde{C}_{32}) = 16.5, R(\tilde{C}_{33}) = 10.5$$

We replace these values for their corresponding $\tilde{a}_{ij}$in which result in a convenient
assignment problem in the linear programming problem.

6	6.75	7.25
8.25	14.5	7
7.75	16.5	10.5

We solve it by ones assignment method to get the following optimal solution.

Step 1

In a minimization case, find the minimum element of each row in the assignment matrix (saya_i) and write it on the right hand side of the matrix. Then divide each element of i^{th}row of the matrix by a_i . These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 2.

min

$$=\begin{pmatrix} 6\ 6.75\ 7.25 \\ 8.25\ 14.5\ 7 \\ 7.75\ 16.5\ 10.5 \end{pmatrix} \begin{matrix} 6 \\ 7 \\ 7.5 \end{matrix}$$

$$=\begin{pmatrix} 1\ 1.125\ 1.21 \\ 1.78\ 2.071\ 1 \\ 1\ 2.13\ 1.35 \end{pmatrix}$$

Step 2: Find the minimum element of each column in assignment matrix (b_j),
and write it below j^{th} column. Then divide each element of j^{th} column of the
matrix by b_j .These operations create at least one ones in each columns.

$$=\begin{pmatrix} 1\ 1.125\ 1.21 \\ 1.78\ 2.071\ 1 \\ 1\ 2.13\ 1.35 \end{pmatrix}$$

min 1 1.125 1

$$=\begin{pmatrix} 1\ 1\ 1.21 \\ 1.78\ 1.841\ 1 \\ 1\ 1.89\ 1.35 \end{pmatrix}$$

Step 3: Make assignment in terms of ones

$$=\begin{pmatrix} 1\ (1)\ 1.21 \\ 1.78\ 1.841\ (1) \\ (1)\ 1.89\ 1.35 \end{pmatrix}$$

We can assign the ones and the solution is (1, 2), (2, 3) and (3, 1)

The fuzzy optimal total cost $\tilde{a}_{12} + \tilde{a}_{23} + \tilde{a}_{31}$=R(2,3,1,4,3,6,5)+R(0,1,2,4,6,0,5)+R(3,6,3,2,1,8,7) =R(5,10,6,10,10,14,17)

VIII. CONCLUSION

In this paper the method was introduced to solve fuzzy assignment problem by using robust's ranking of heptagonal fuzzy numbers. This method can be used for all kinds of fuzzy assignment problem. The new method is a systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function.

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