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K_n-Dominating Function in Graph Theory

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ABSTRACT

Graph theory is a branch of Discrete Mathematics. Graph theory is the study of graphs which are mathematical structures used to model pair wise relations between objects. This paper deals with the basic definition of dominating sets in graph theory and theorem has been explained regarding dominating function.
Keywords: Graph theory, K_n-Dominating set, Minimal K_n-Dominating set, Dominating Function.

I. INTRODUCTION

Graph theory is a branch of Discrete Mathematics. Graph theory is the study of graphs which are mathematical structures used to model pair wise relations between objects. A graph is made up of vertices V (nodes) and edges E (lines) that connect them. A graph is an ordered pair G= (V, E) consisting a set of vertices V with a set of edges E. Graph theory is originated with the problem of Koinsberg bridge, in 1735.

Mathematical study of domination in graphs began around 1960. There are some references to domination related problems about 100 years prior.

The study of domination in graphs was further developed in the late 1950's and 1960's, beginning with Claude Berge in 1958. Berge wrote a book on graph theory, in which he introduced the "coefficient of external stability," which is now known as the domination number of a graph. Oystein Ore introduced the terms "dominating set" and "domination number" in his book on graph theory which was published in 1962. The study of domination in graphs came about partially as a result of the study of games and recreational mathematics.

Dominating Set in Graphs

A set $S \subseteq V$ of vertices in a graph G = (V,E) is called a dominating set if every $v \in V$ is either an element of S or is adjacent to an element of S.

Minimal Dominating Set

A dominating set S is minimal dominating set if no proper subset $S'' \subset S$ is a dominating set. The set of all minimal dominating sets of a graph g is denoted by MDS(G).

K_n-Dominating set

Let G = (V,E) be a graph. A subset S of V is said to be a K_n-dominating set of G if for every vertex $v \in (V-S)$ is K_n-adjacent to at least one vertex in S.

Minimal K_n-dominating function

A K_n-dominating function f is called a minimal if for any K_n-dominating function g with $g \leq f$, we have g = f.

Theorem 1

A k_n-dominating function f is minimal k_n-dominating function if and only if for every $u \in V$ with f(u) > 0, there exist $v \in N_{k_n}[u]$ such that $f(N_{k_n}[v])=1$.

Proof

Let f be a k_n-dominating function satisfying for every u ∈ v such that f(u) > 0, there exist v ∈ N_{k_n}[u] such that f(N_{k_n}[v])=1.

To prove that: f is a minimal k_n-domination function.

For that,
Suppose g is a k_n-domination function such that g ≤ f

(To prove that g = f)

Suppose g < f. Then there exist a point u such that g(u) < f(u). But 0 ≤ g(u).

Therefore f(u) > 0.

Therefore there exist v ∈ N_{k_n} [u] such that f(N_{k_n}[v])=1. ………..(1)

Now g(N_{k_n}[v]) < f(N_{k_n}[v]) therefore g(u) < f(u) < 1
Which is a contradiction [therefore g is a k_n-dominating function then g(N_{k_n}[v]) ≥ 1.]

Therefore g =f. Therefore f is a minimal k_n-dominating function.

Suppose f is a minimal k_n-dominating function. Let u∈v satisfy f(u) > 0.

To show that there exist v ∈ N_{k_n} [u] such that f(N_{k_n}[v])=1

Suppose for every v ∈ N_{k_n}[u] then f(N_{k_n}[v]) > 1. (by definition)

Define g : v→[0,1] by g(w) = f(w); for all w≠u.
g(u) = f(u)-min(f(N_{k_n}[v])-1,f(u))

Therefore 0 ≤ g(u) < f(u) ………….(*)

Let x ∈ V. Consider N_{k_n} [x], g(N_{k_n} [x])= f ∉ N_{k_n}[x] $g(N_{k_n}[x]) \Longrightarrow$ x ∈ N_{k_n}[u]

Therefore $f(N_{k_n}[x]) > 1$
Therefore $g(N_{k_n}[x]) = f(N_{k_n}[x])$ – f(u) + g(u)
= f(N_{k_n}[x])- f(u) + f(u) – min{f(N_{k_n}[x])-1,f(u)}

$$= \begin{cases} f(N_{k_n}[x]) - (f(N_{k_n}[x]) - 1) & \text{if } f(u) \geq f(N_{k_n}[x]) - 1 \\ f(N_{k_n}[x]) - f(u) & \text{if } f(u) < f(N_{k_n}[x]) - 1 \end{cases}$$

$$= \begin{cases} 1 & if \quad f(u) \geq f(N_{k_n}[x]) - 1 \\ > 1 & if \quad f(u) < f(N_{k_n}[x]) - 1 \end{cases}$$

Therefore, $g(N_{k_n}[x]) \geq 1$.

Therefore k_n-dominating function f is minimal k_n-dominating function if and only if for every u ∈ V with f(u) > 0, there exist v ∈ N_{k_n}[u] such that f(N_{k_n}[v])=1.

II. CONCLUSION

The basic definition of dominating sets in graph theory has been discussed and the theorem regarding dominating function has been proved.

III. REFERENCES

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