

A Step Towards Gravitoelectromagnetisation

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ABSTRACT

Gravitoelectromagnetic equations have been compared with electromagnetic equations in the same way as used by some workers. Then considering invariance of Maxwell's equation study of the effect of gravity on these equations are made. Lastly, relations among E, H and G as proposed by some earlier workers have been considered. Some characteristics satisfied by the field vectors used in these relations are also studied here. Although the works included here show certain type of mathematical relationships of the field vectors yet experimental verification for these relationships is important.

Keywords : Gravitoelectromagnetism, Classical Field Theory, Unification.

I. INTRODUCTION

Einstein was of strong opinion that all forces of nature have their roots at gravity [1]. Gravitation exists everywhere if there be some space [2] and gets over any substance. Space and gravitation could not be separated. It is well known that an electric charge placed in vacuum produces an electromagnetic field surrounding the charge. An observer having motion relative to the charge will observe current with respect to him. He will, also, observe a magnetic field due to this current. This magnetic field will indicate magnetic energy. Again, speed of a body is always relative and kinetic energy [3,4] is also relative. Therefore, when an observer is moving relative to a mass then the relative speed of the mass will present relative kinetic energy. Thus, magnetic and kinetic energy are analogous for the fact that both exist only when there is relative motion between a charge and an observer.

According to the conjectures [1,5] there were unification of electricity and magnetism by Maxwell and that of earth's gravity and universal gravitation by Newton. Before sufficient development of quantum mechanics we know about the classical field theories which are separated as gravitation and electromagnetism. Also we know that charge is an electric phenomenon and mass is due to gravitation. General theory of relativity implies that gravity and

electromagnetism may be related which is the main root of interrelation.

In this dissertation discussions about some works, (as example), would show some steps towards unification. Here, the gravitoelectromagnetic equations and Maxwell's electromagnetic equations are shown side by side. Trial has been made to compare them in the first part. In the second part, considerations have been made about transformation of field vectors E and H due to the effect of gravity. As the form of Maxwell's equation are unaltered with or without gravitational field, the field equations in the two cases, are shown.

Lastly, electromagnetic and gravitational interactions have been studied. Some characteristics satisfied by the field vectors, appearing in the equations, are included in the work here.

II. GRAVITOELECTROMAGNETISATION:

Here gravity and the force of electromagnetism have been considered so that the behavior of gravity could be expressed in a way similar to that of magnetism. In this section formal analogy between Maxwell's equation of electromagnetism and those of relativistic gravity have been shown [6,7]. We know from general relativity that the gravitational field produced by a rotating mass can be described by equations of the same form as classical

electromagnetism. Let us assume a weak gravitational field such that the space time is reasonably flat when Maxwell's equations of electromagnetism and gravitoelectromagnetisation (GEM) equations could be written as

Maxwell's equations

$$\left. \begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \frac{1}{\epsilon_0 c^2} \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \end{aligned} \right\} \dots(a)$$

GEM equations

$$\left. \begin{aligned} \nabla \cdot \mathbf{E}_g &= -4\pi G_c \rho_g \\ \nabla \cdot \mathbf{B}_g &= 0 \\ \nabla \times \mathbf{E}_g &= -\frac{\partial \mathbf{B}_g}{\partial t} \\ \nabla \times \mathbf{B}_g &= -\frac{4\pi G_c}{c^2} \mathbf{J}_g + \frac{1}{c^2} \frac{\partial \mathbf{E}_g}{\partial t} \end{aligned} \right\} \dots(b)$$

(1)

where the parameters with suffix g correspond to gravitational fields, G_c is the gravitational constant. It is seen from (1) that the equations under the two headings are of the same form but magnitude of the parameters like \mathbf{E} , \mathbf{B} , \mathbf{J} , ρ of general electromagnetism change due to the effect of gravity. Another point to note is that ϵ_0 in general electromagnetism is equivalent to $(-4\pi G_c)^{-1}$ in GEM as is evident from the equations. This may be considered as a connection between electromagnetism and gravity.

Again, Newton's gravitation is based on the fact that fields are produced by objects (masses). Also, the fields are assumed to be produced by gradient of scalar parameters and the equation of similar types in the two cases have been proposed [8,9] as shown below

$$\mathbf{E} = -\nabla \phi_e \quad \text{and} \quad \mathbf{g} = -\nabla \phi_g \quad (2)$$

which leads to Poisson's equations

$$\nabla^2 \phi_e = \frac{\rho_e}{\epsilon_0} \quad \text{and} \quad \nabla^2 \phi_g = 4\pi G_c \rho_g \quad (3)$$

Here, also, ϵ_0 and G_c are related by the same relation as that in (1)

III. GRAVITATION AND ELECTRO - MAGNETISM

For the field strength of gravity, we have the same definition as for the field strength of electric field [9]. It has been assumed that the form of Maxwell's equations of electromagnetism does not change under the effect of gravity [10]. Now, in absence of gravity Maxwell's equations are given by (1a). But, in presence of gravity the parameters \mathbf{E} , \mathbf{B} , \mathbf{J} , ρ will change (are to be primed which would represent the transformed forms due to the effect of gravity) but the form of the equations will remain the same. Let us suppose that the forms of the electric field vectors in the two cases, up to third order approximation, be of the form

$$E_{x\mu} = \sum_{\nu=0}^3 A_\nu (x_\mu)^\nu \quad (4)$$

and
$$E_{x'\mu} = \sum_{\nu=0}^3 A'_\nu (x'_\mu)^\nu \quad (5)$$

where, $\mu=0,1,2,3$. Hence, x'_μ could be evaluated from the transformation equations

$$(dx'_\mu)^2 = g_{\mu\nu} (dx_\nu)^2 \quad (6)$$

where, x_μ corresponds to t, x, y and z coordinates for the values of μ . Again, A_ν and the primed magnitudes of them could be evaluated from the boundary conditions. Of course, at infinity the boundary conditions remain unchanged and we may have $A_\nu = A'_\nu$. So, at infinity (6) becomes important. It must, also, be mentioned that the metric $g_{\mu\nu}$ could be obtained from Schwarzschild line element [11] for negligible effect of cosmological constant. Hence,

$$g_{\mu\nu} = \begin{pmatrix} -(1-\frac{2m}{r}) & 0 & 0 & 0 \\ 0 & (1-\frac{2m}{r})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (7)$$

for the Schwarzschild line element

$$ds^2 = -(1-\frac{2m}{r})dt^2 + (1-\frac{2m}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (8)$$

at a distance r from a mass point m .

IV. RELATION BETWEEN ELECTROMAGNETISM AND GRAVITY

It was assumed that change of both electric and gravitational field results in creation of a magnetic field in the region of space time which has electrogravitational nature. Again, change of magnetic field results in the creation of both electric and gravitational fields. On the basis of the above assumptions Kosyev [2] proposed some relations connecting electromagnetic and gravitational effects as shown below.

$$\begin{aligned} \text{(i) } \operatorname{div} \mathbf{E} &= 0, & \text{(ii) } \operatorname{div} \mathbf{G} &= 0, \\ \text{(iii) } \operatorname{div} \mathbf{H} &= 0, & \text{(iv) } \operatorname{curl} \mathbf{E} &= -\mu_{GO} \frac{\partial \mathbf{H}}{\partial t}, \\ \text{(v) } \operatorname{curl} \mathbf{G} &= -\mu_{EO} \frac{\partial \mathbf{H}}{\partial t}, \\ \text{(vi) } \operatorname{curl} \mathbf{H} &= \varepsilon_{GO} \frac{\partial \mathbf{E}}{\partial t} - \varepsilon_{EO} \frac{\partial \mathbf{G}}{\partial t} \end{aligned} \quad (9)$$

\mathbf{E} , \mathbf{H} , \mathbf{G} are respectively the strength of electrical, magnetic and gravitational fields; μ_{GO} , μ_{EO} and ε_{GO} , ε_{EO} are respectively the magnetic constants and electrical constants of gravitational and electrical space-time respectively.

The equations show that interrelationship between gravitational and electromagnetic fields have been assumed. They also reveal that in presence of electric potential a portion of magnetic energy is spent to create

gravitational alternating fields. Also, it is evident that all the equations are dependent on gravitational field.

Let us study some characteristics assuming plane waves in free space. Taking curl of both sides of (iv), (v) and (vi) of (9) we respectively get

$$\nabla^2 \mathbf{E} = \mu_{GO} \varepsilon_{GO} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_{GO} \varepsilon_{EO} \frac{\partial^2 \mathbf{G}}{\partial t^2} \quad (10)$$

$$\nabla^2 \mathbf{G} = \mu_{EO} \varepsilon_{GO} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_{EO} \varepsilon_{EO} \frac{\partial^2 \mathbf{G}}{\partial t^2} \quad (11)$$

$$\nabla^2 \mathbf{H} = (\mu_{GO} \varepsilon_{GO} - \mu_{EO} \varepsilon_{EO}) \frac{\partial^2 \mathbf{H}}{\partial t^2} \quad (12)$$

It is seen that the second term of (10) and first term of (11) prominently contain the interaction of the electrical and gravitational characteristics of the medium. Again, (10) and (11) reveal that electrical and gravitational waves depend on the oscillation of both electric and gravitational fields. If there be variations of electric field due to relative movement of electric charge then there would be flow of current. This produces magnetic field. Since, there is variation of gravitation also hence, the magnetic field depends upon the variation of both gravitational and electric fields. Now, we may assume the solution of (10), (11) and (12) to be respectively

$$\mathbf{E} = E_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) \quad (13)$$

$$\mathbf{G} = G_0 \exp(i\mathbf{k}' \cdot \mathbf{r} - i\omega' t) \quad (14)$$

$$\mathbf{H} = H_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) \quad (15)$$

Here, (13) and (15) will signify the electromagnetic oscillation while (14) is for the oscillation of gravitational field; \mathbf{k} and \mathbf{k}' used in the equation are complicated due to the interaction between \mathbf{E} , \mathbf{G} and \mathbf{H} . It is also evident that frequency of oscillation of electromagnetic and gravitational waves are different. Using (13), (14) and (15) equation (10), (11) and (12) respectively becomes

$$k^2 \mathbf{E} = \frac{\omega^2}{V_G^2} \mathbf{E} - \frac{\omega'^2}{V_{GE}^2} \mathbf{G} \quad (16)$$

$$k'^2 \mathbf{G} = \frac{\omega^2}{V_{EG}^2} \mathbf{E} - \frac{\omega'^2}{V_E^2} \mathbf{G} \quad (17)$$

$$\text{and } k^2 = \left(\frac{1}{V_{EO}^2} - \frac{1}{V_{EO}^2} \right) \omega^2 \quad (18)$$

It is seen that V_E and V_G are respectively the velocities of the electric and gravitational disturbances while V_{GE} and V_{EG} are those for interaction between gravitational and electric field oscillations. Under certain boundary conditions V_{GE} and V_{EG} would be equal.

Now, using (18) we shall have from (16)

$$\omega' = \sqrt{\frac{E}{G} \cdot \frac{V_{GE}}{V_E}} \cdot \omega \quad (19)$$

It is evident that ω and ω' are different and the values depend upon E, G, V_{GE} and V_E . Substituting (19) in (17) we obtain for the case $V_{GE} = V_{EG}$

$$k' = \sqrt{\left(\frac{1}{V_{EG}^2} - \frac{V_{EG}^2}{V_E^4} \right)} \sqrt{\frac{E}{G}} \cdot \omega \quad (20)$$

Again, from (18) and (20) the relation between k and k' could be obtained as

$$\frac{k'}{k} = \frac{V_G}{V_{EG} V_E} \sqrt{\left(\frac{V_E^4 - V_{EG}^4}{V_E^2 - V_G^2} \right)} \sqrt{\frac{E}{G}} \quad (21)$$

We can, also, arrive at the relation between k and k' directly from (16) and (17) by simple substitution (of $\omega'^2 G$ from (17) in (16)). Thus,

$$k'^2 = \left[k^2 \frac{V_{GE}^2}{V_E^2} + \left(\frac{1}{V_{GE}^2} - \frac{V_{GE}^2}{V_E^2 V_G^2} \right) \omega^2 \right] \frac{E}{G} \quad (22)$$

It is to be mentioned that we may use any one of (21) and (22) to show the relation between k and k' . Now, the solutions (13), (14) and (15) will yield

$$\mathbf{k} \cdot \mathbf{E} = 0, \mathbf{k} \cdot \mathbf{G} = 0 \text{ and } \mathbf{k} \cdot \mathbf{H} = 0 \quad (23)$$

which show that \mathbf{E}, \mathbf{G} and \mathbf{H} are normal to corresponding propagation vector. Again, considering the curl we have by using (13), (14) and (15)

$$(a) \mathbf{k} \times \mathbf{E} = \mu_{GO} \omega \mathbf{H},$$

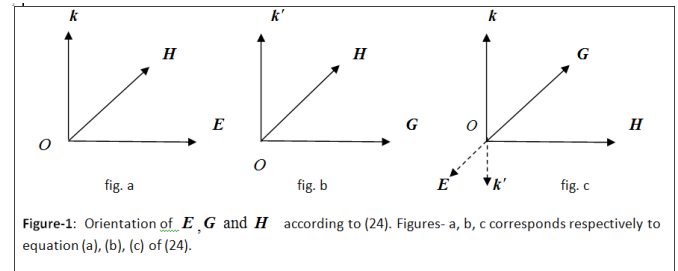
$$(b) \mathbf{k}' \times \mathbf{G} = \mu_{EO} \omega \mathbf{H},$$

$$(c) \mathbf{k} \times \mathbf{H} = -\epsilon_{GO} \omega \mathbf{E} + \epsilon_{EO} \omega' \mathbf{G} \quad (24)$$

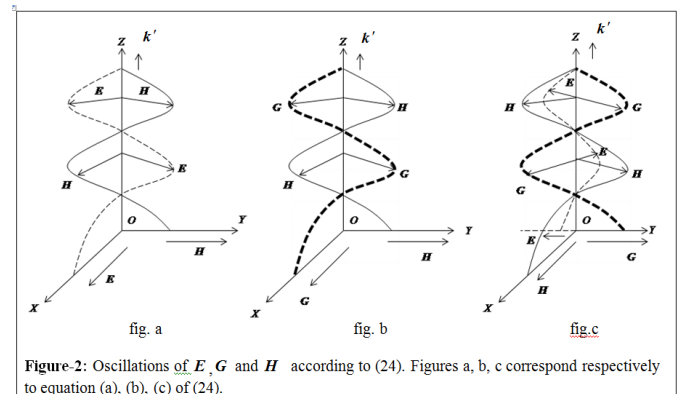
From (a) and (b) of (24) it is observed that \mathbf{H} may be obtained from \mathbf{E} or \mathbf{G} while from (24c) we see that \mathbf{H} could be obtained from simultaneous variation of \mathbf{E} and \mathbf{G} . Also, from (a) and (b) of (24) we have

$$\frac{\mathbf{k} \times \mathbf{E}}{\mathbf{k}' \times \mathbf{G}} = \frac{\mu_{GO} \omega}{\mu_{EO} \omega'} \quad (25)$$

which shows the relation between \mathbf{E} and \mathbf{G} under certain conditions to be applied to the system. Again, (24) gives the orientation of \mathbf{E}, \mathbf{G} and \mathbf{H} for any point O in space as shown below.



From (a) and (b) of (24) it is observed that the propagating waves may be called the electromagnetic wave and gravitational wave respectively. Relation (24a) shows that the wave is produced due to the action of \mathbf{E}, \mathbf{G} and \mathbf{H} simultaneously. So, it may be termed as gravitoelectromagnetic wave propagating along \mathbf{k}' . Accordingly, the waves in these cases may be represented as in figure-2 assuming $\omega = \omega'$.



Now, from (24a) we can write

$$\left| \frac{\mathbf{E}}{\mathbf{H}} \right| = \frac{E_0}{H_0} = \mu_{GO} c \quad (26)$$

Similarly from (24b) we get

$$\left| \frac{\mathbf{G}}{\mathbf{H}} \right| = \frac{G_0}{H_0} = \mu_{EO} C \quad (27)$$

It is seen that \mathbf{E} , \mathbf{G} and \mathbf{H} are in phase although the values of propagation constants for oscillations of these fields are different. Again, it could be shown that (26) and (27) satisfy (24). So, (24) reveals that magnetic field produces electric and gravitational fields and vice versa.

V. CONCLUSION

The work reveals that different workers have tried to proceed towards unification step by step which is evident from a few number of works considered here. At the first place, under certain assumptions, it is seen that there is analogy between two sets of equations (Maxwell's and GEM equations) as discussed in section-2. Again, we see that Poisson's equations in the two cases are also similar. In section-3 the effects of gravitation on electromagnetism have been included and transformations of different parameters for the systems have been used. We notice that the solutions for the fields at infinity become simpler. It is observed that the transformation equations, the metric $g_{\mu\nu}$ and Schwarzschild line element ds has the same form after transformation.

In section-4 some equations, connecting \mathbf{E} , \mathbf{G} and \mathbf{H} , proposed by Kosyev [2] have been taken into account. Using these equations several characteristics of waves have been studied mathematically. Thus, it appears that (10) may be used to study the interaction between electromagnetism and gravity under a lower order approximation.

All the assumptions made in this work are debatable for the assumptions made. Of course, confirmation regarding the validity of the said relations could not be achieved unless they are verified experimentally.

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