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Adomain Decomposition Method for Initial Value Problem

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ABSTRACT

The Adomian decomposition method (ADM) is a powerful method which considers the approximate solution of a non-linear equation as an infinite series which usually converges to the exact solution. In this paper, this method is proposed to solve some initial value problems. It is shown that the series solutions converges to the exact solution for each problem.

Keywords : Adomain Decomposition Method; Initial value problems; Non-linear; Differential equations

I. INTRODUCTION

The Adomain Decomposition Method (ADM) is a powerful tool for solving linear or nonlinear functional equation . Adomain Decomposition Method is extended to the calculations of the non – differential function. Recently considerable attention has been given to ADM for solving nonlinear initial value problem. The ADM was introduced by Adomain in the early year to solve nonlinear ordinary and partial Differential Equation. In this paper, we extend the famous ADM to deal with the initial problems.

II. The Adomain Decomposition method

Consider the differential equation

$$Ly + Ry + Ny = g(x) \qquad (1)$$

Where N is a non-linear operator , L is the highest order derivative which is assumed to be invertible and R is a linear differential operator of order less than. Making Ly subject of the formula, we get

$$Ly = g(x) - Ry - Ny \qquad (2)$$

By solving (2) for Ly, since L is invertible , we can write

$$L^{-1}Ly = L^{-1}g(x) - L^{-1}Ry - L^{-1}Ny \qquad (3)$$

For initial value problems we conveniently define L^{-1} for $L=\frac{d^n}{dx^n}$ as the n-fold definite integration from 0 to x. If L is a two fold integral and so by solving (3) for y, we get

$$y = A + Bx + L^{-1}g(x) - L^{-1}Ry - L^{-1}Ny \qquad (4)$$

Where A and B are constants of integration and can be found from the initial or boundary conditions.

The Adomian method consists of approximating the solution of (1) as an infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \qquad (5)$$

and decomposing the non-linear operator N as

$$N(y) = \sum_{n=0}^{\infty} A_n, \qquad (6)$$

Where A_n are Adomain polynomials [4,5] of $y_0, y_1, y_3, \ldots\ldots y_n$ given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i y_i\right)\right]_{\lambda=0}, \quad n=0,1,2,\ldots\ldots$$

Substituting (5) and (6)into (4) yields

$$\sum_{n=0}^{\infty} y_n = A + Bx + L^{-1}g(x) - L^{-1}R\left(\sum_{n=0}^{\infty} y_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} y_n\right)$$

The recursive relationship is found to be

$$y_0 = g(x)$$
$$y_{n+1} = -L^{-1}Ry_n - L^{-1}A_n$$

Non-linear

A nonlinear system of equations is a set of equation where one or more terms have a variable of degree two higher and there is a product of variables in one of the equations. Most real-life physical systems are non-linear systems.

Initial value problem

In mathematics, in field of differential equations, an initial value problem (also called the Cauchy problem by some authors) is an ordinary differential equation together with a specified value, called the initial condition, of the unknown function at a given point in the domain of the solution.

Numerical problem

We consider the nonlinear singular initial value problem:

$$y''+\frac{2}{3}y'+y^3=6+x^6 \qquad (7)$$

$$y(0)=0\ ,\ y'(0)=0$$

In an operator form, Equation (7) becomes

$$Ly= 6+x^6-y^3 \qquad (8)$$

Applying L^{-1} on both sides of (8) we find where $L^{-1}(.)=x^{-1}\int_0^x\int_0^x x(.)dxdx$

$$y= L^{-1}(6+x^6) - L^{-1}y^3$$

Therefore,

$$y= x^2+\frac{x^8}{72} - L^{-1}y^3$$

By adomain decomposition method [6]

We divided $x^2+\frac{x^8}{72}$ into two points and we using the polynomial series for the nonlinear term, we obtain the recursive relationship .

$$y_0=x^2$$

$$y_{k+1}=\frac{x^8}{72} - L^{-1}(A_k) \qquad (9)$$

$$y_0=x^2$$

$$y_1=\frac{x^8}{72} - \frac{1}{x}\int_0^x\int_0^x x(x^2)^3 dx\,dx=0$$

$$y_{k+1}=0,\ k\geq 0.$$

In view of (9), the exact solution is given by

$$y=x^2.$$

Example 1

Consider the equation

$$y'-y = x\cos x-x\sin x,\quad y(0) = 0 \qquad (10)$$

Solving as the previous problem,

Applying $L^{-1}$to both sides yields

$$y(x) = x\sin x - x\cos x-x\sin x + L^{-1}y(x)$$

The recursive relationship

$$y_0 = x\sin x-x\cos x- \sin x$$

$$y_{n+1}=L^{-1}y_n,\ y\geq 0 \qquad (11)$$

So

$$y_1=L^{-1}y_0 =-x\cos x+ \sin x+x\sin x +2(\cos x-1)$$

Other components can be evaluated in a similar manner. It is easily observed that the terms such as (cos x) and (-sin *x*) appear in y_0 and y_1 with opposite signs. Canceling these terms from y_0 gives the exact solution y(x) =sin *x* that can be justified through substitution. It is worth noting that other noise terms that appear in other components vanish in the limit.

Example 2

Consider the equation

$$y'' + xy' = y-e^{5x} \qquad (12)$$

$$y(0)=0\quad y'(0) = 0$$

Equation (12) becomes

$$Ly = e^{5x} - y \qquad (13)$$

Apply L^{-1} on the both sides of (13) we get

$$y = L^{-1}(e^{5x}) - L^{-1}y'$$

By domain decomposition method we drive

$$x^2 + \frac{x^2}{7^2}\dots\dots$$

$$y_0 = x^2$$

$$y_{k+1} = \frac{x^2}{7^2} - L^{-1}(A_k) \qquad (14)$$

$$y_1 = \frac{x^2}{7^2} - \frac{1}{x}\int_0^x\int_0^x e^{2x}.e^{2x}.e^x\,dx\,dx$$

$$y_{k+1} = 0\quad k = 0$$

$$y = e^{2x}$$

Example 3

Consider the equation

$$y'' + y' + y^2 = sinx \qquad (15)$$

$$y(0) = 0\ y'(0) = 0$$

Becomes (15)

$$Ly = sinx - y^2 \qquad (16)$$

Apply L^{-1} both sides

$$y = y^{-1}(sinx) - L^{-1}y^2$$

$$y = x^2 + \frac{x^2}{7^2} - L^{-1}y^2$$

By adomain decomposition method we derive

$$y = x^2$$

$$y_{k+1} = \frac{x^2}{7^2} - L^{-1}(A_k) \qquad (17)$$

$$y_1 = \frac{x^2}{7^2} - \frac{1}{x}\int_0^x\int_0^x x\ x'\,dx\,dx = 0$$

$$y_{k+1} = 0\quad ,\quad k = 0$$

$$y = x$$

III. Conclusion

The main objective of this work is to obtain a solution for initial value problem. We observe the ADM is a powerful method to solve nonlinear initial value problem. To show the applicability and efficiency of the proposed method, the method is applied to obtain the solution of several examples. It is worth mentioning that the proposed technique is capable of reducing the volume of the computational work as compared to the classical method.

IV. REFERENCES

[1]. Adomian G (1988) A Review of the Decomposition Method in Applied Mathematics. J Math Anal Appl 135: 501-544.

[2]. Somali S, Gokmen G (2007)Adomian Decomposition Method for Non-Linear Sturm-Liouville Problems. Surveys in Mathematics and its Applications 2: 11-20.

[3]. Mustafa I (2005) On Numerical Solution of One-Dimensional Non-Linear Burger's Equation and Convergence of the Decomposition Method. Appl Math Comput 170: 76-85

[4]. Biazar J, Shafiof SM (2007) A Simple Algorithm for Calculating Adomian Polynomials. Int J Contemp Math Sciences 2: 975-982.

[5]. Biazar J, Pourabd M (2006) A Maple Program for Computing Adomian Polynomials. International Mathematical Forum 1: 1919-1924.

[6]. A. M.Wazwaz, A reliable modi_cation of Adomian decomposition method, Appl. Math. Comput. 102 (1999), 77-86. MR1682855(99m:65156). Zbl 0928.65083.