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τ*- Nano Generalized Closed Sets in Nano Topological Space

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ABSTRACT

In this paper, we introduce a new class of sets called τ*-Nano generalized closed sets and τ*- Nano generalized open sets in Nano topological spaces and study some of their properties.
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I. INTRODUCTION

In 1970, Levine [6] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Later on N. Palaniappan [7] studied the concept of regular generalized closed set in a topological space. Maki et al [1] introduced the concepts of generalized pre closed sets and pre generalized closed sets in an analogous manner. In 1977, Y. Gnanambal [10] have introduce the concept of generalized pre regular closed sets in topological spaces. In 2011, Sharmistha Bhattacharya [9] have introduced the notation of generalized regular closed sets in topological space. The notation of Nano topology was introduced by LellisThivagar [4], which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano interior and Nano closure. Moreover in this paper we defined subset A of a Nano topological space X, Nint(A), Ncl(A), Ncl*(A), Nscl(A), Nspcl(A), $Ncl_{\alpha}(A)$ and A^C denote the Nano interior, Nano closure, Nano closure*,Nano semi-closure, Nano semi pre closure, Nano α-closure and complement of A respectively.

II. PRELIMINARIES

Definition 2.1 [5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the **approximation space**. Let X ⊆U.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$.

$$L_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}$$

where R(X) denotes the equivalence class determined by x ∈U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.

$$U_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \cap X = \phi\}.$$

3. The boundary region of X with respect to R is the set of all objects, which can be possibly classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.

$$B_R(X) = U_R(X) – L_R(X).$$

Property 2.2 [5] If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$.
2. $L_R(\phi) = U_R(X) = \phi \,\&\, L_R(U) = U_R(U) = U$.
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
4. $U_R(X \cap Y) \subseteq U_R(X) \bigcap U_R(Y)$.
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
6. $L_R(X \cap Y) \subseteq L_R(X) \bigcap L_R(Y)$.

7. $L_R(X) \subseteq L_R(Y) \& U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
8. $U_R(X^C) = [L_R(X)]^C \& L_R(X^C) = [U_R(X)]^C$
   $U_R U_R(X) = L_R U_R(X) = U_R(X)$.
9. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 [4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X)$ = {U, ϕ, L_R(X), U_R(X), B_R(X)} where X ⊆U. Then by property $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the **Nano topology** on U with respect to X. (U, $\tau_R(X)$) as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets and complement of Nano open sets is called Nano closed.

Definition 2.4 [4]If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X ⊆U and if A ⊆U, then the **Nano interior** of A is defined as the union of all Nano-open subsets contained in A and it is denoted by NInt(A). That is, NInt(A) is the largest Nano-open subset of A. The **Nano closure** of A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). NCl(A) is the smallest Nano closed set containing A.

Definition 2.5 [8] Let (U, $\tau_R(X)$)be a Nano topological space and A ⊆U. Then A is said to be

1. Nano Pre-open if A ⊆NInt(NCl(A)).
2. Nano Pre-closed if NCl(NInt(A)) ⊆A.
3. Nano Regular open if A ⊆NInt(NCl(A)).
4. Nano Regular closed if NCl(NInt(A)) ⊆A.

Definition 2.6 [3] Let (U, $\tau_R(X)$) be a Nano topological space. A subset A of (U, $\tau_R(X)$) is called **Nano generalized closed set** (briefly Ng-closed) if NCl(A) ⊆V where A ⊆V and V is Nano open.

Definition 2.7 [2] If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X ⊆U and if A ⊆U, then the Nano pre interior of A is defined as the union of all Nano pre open subsets of A contained in A and it is denoted by NpInt(A). NpInt(A) is the largest Nano pre open subset of A. The Nano pre closure of A is defined as the intersection of all Nano pre closed sets containing A and it is denoted by Npcl(A). That is, Npcl(A) is the smallest Nano pre closed set containing A.

Definition 2.8 [2] A subset A of (U, $\tau_R(X)$) is called **Nano generalized pre closed set** (briefly Ngp-closed) if Npcl(A) ⊆V whenever A ⊆V and V is Nano open in (U, $\tau_R(X)$).

Definition 2.9 [12] A subset A of a topological space (U, $\tau_R(X)$) is Called (i)Nano α-closed[8] if Ncl(Nint(Ncl(A))) ⊆A . (ii) Nano α-generalized closed (briefly Nαg-closed)[9] if Nclα(A)⊆G whenever A ⊆G and G is Nano open in X. (iii) Nano Generalized α-closed (briefly Ngα-closed)[10] if Nspcl (A)⊆G whenever A ⊆G and G is Nano open in X. (iv) Nano Generalized semi-preclosed (briefly Ngsp-closed)[2] if Nscl(A) ⊆G whenever A ⊆G and G is Nano open in X.

III. τ*-Generalized Closed Sets in Topological Spaces

In this section, we introduce the concept of τ*- Nano generalized closed sets in topological spaces.

Definition 3.1.A subset A of a nano topological space (U, $\tau_R(X)$) is called τ*-nano generalized closed set (briefly τ*-nano g-closed) if Ncl*(A)⊆G whenever A ⊆ G and G is τ*-nano open. The complement of *-nano generalized closed set is called the τ*-nano generalized open set (briefly τ*-nano g-open).

Example: Let U = {a,b,c,d} with U/R = { {a}, {c}, {b,d}} and X = {a,b}. Then $\tau_R(X)$ ={ U,Φ, {a}, {a,b,d}, {b,d}} which are open sets.

1. The Nano closed sets = { U,Φ, {b,c,d}, {c}, {a,c}} .
2. The Nano generalized closed sets are { Φ,U,{c}, {a,c}, {b,c} , {c,d} , {a,b,c} , {b,c,d}, {a,c,d}}
3. τ*- Nano Generalized Closed Set { Φ,U,{a},{c},{a,b},{b,c}}

Theorem 3.2. Every closed set in (U, $\tau_R(X)$) is τ*-nano g-closed.

Proof: Let A be a closed set. Let A ⊆ G. Since A is nano closed, Ncl(A) = A ⊆ G. But Ncl*(A) ⊆ Ncl(A). Thus, we have Ncl*(A) ⊆ G whenever A ⊆ G and G is τ*-nano open. Therefore A is *-nano g-closed.

Theorem 3.3. Every τ*-nano closed set in (U, $\tau_R(X)$) is τ*-nano g-closed.

Proof. Let A be a τ*-nano closed set. Let A ⊆ G where G is τ*-nano open. Since A is τ*-nano closed, Ncl*(A) = A ⊆G. Thus, we have Ncl*(A) ⊆ G whenever A ⊆ G and G is τ*-nano open. Therefore A is τ*-nano g-closed.

Theorem 3.4. Every nano g-closed set in (U, $\tau_R(X)$) is a τ*-nano g-closed set but not conversely.

Proof : Let A be a nano g-closed set. Assume that A ⊆G, G is τ*-nano open in X. Then Ncl(A) ⊆G, since A is nano g-closed. But Ncl*(A)Ncl(A). Therefore Ncl*(A) ⊆G. Hence A is τ*-nano g-closed.

Theorem 3.5. For any two sets A and B, N**c**l*(A ∪ B) =Ncl*(A) ∪ Ncl*(B)

Proof: Since A ⊆A ∪ B, we have Ncl*(A) ⊆Ncl*(A ∪ B)and since B ⊆A ∪ B , we have Ncl*(B) ⊆Ncl*(A ∪ B).Therefore Ncl*(A) ∪ Ncl*(B) ⊆Ncl*(A ∪ B). Also, Ncl*(A) and Ncl*(B) are the closed sets Therefore Ncl*(A) ∪ Ncl*(B) is also a closed set. Again, A ⊆Ncl*(A) and B ⊆Ncl*(B) implies A∪B ⊆Ncl*(A)∪Ncl*(B).Thus, Ncl*(A) ∪ Ncl*(B) is a closed set containing A ∪ B. Since Ncl*(A ∪ B) is the smallest closed set containing A∪B we have Ncl*(A ∪ B) ⊆Ncl*(A) ∪ Ncl*(B). Thus, Ncl*(A ∪B) = Ncl*(A) ∪ Ncl*(B)

Theorem 3.6. Union of two τ* nano g-closed sets in (U, $\tau_R(X)$) is a τ*-nano g-closed set in (U, $\tau_R(X)$).

Proof : Let A and B be two τ* nanog-closed sets. Let A ∪ B ⊆G, where G is τ*-nano open. Since A and B are τ*-nano g-closed sets, Ncl*(A) ∪ Ncl*(B) ⊆ G. But by Theorem 3.5., Ncl*(A) ∪ Ncl*(B) =N**c**l*(A ∪ B). Therefore N**c**l*(A ∪ B) ⊆ G. Hence A ∪ B is a τ*-nano g-closed set.

Theorem 3.7.A subset A of (U, $\tau_R(X)$) is τ*-nano g-closed if and only if Ncl*(A) – A contains no non-empty τ* -nano closed set in (U, $\tau_R(X)$).

Proof: Let A be a τ*-nano g-closed set. Suppose that F is a non emptyτ*-nano closed subset of Ncl*(A) – A. Now F ⊆ Ncl*(A) – A. Then F ⊆Ncl*(A) ∩ A^C, since Ncl*(A) – A = Ncl*(A) ∩ A^C. Therefore F ⊆Ncl*(A) and F ⊆ A^C. Since Fc is a τ*-nano open set and A is a τ*-nanog-closed, Ncl*(A) ⊆ F^C. That is F⊆ [Ncl*(A)]^c Hence F⊆Ncl*(A) ∩[Ncl*(A)]^c = φ. That is F = φ, a contradiction. Thus Ncl*(A) – A contain no non-emptyτ*-nano closed set in X Conversely, assume that Ncl*(A) – A contains no nonempty τ*-nano closed set. Let A ⊆G, G is τ*-nano open. Suppose that Ncl*(A) is not contained in G, then Ncl*(A) ∩ G^C is a non-empty τ*-Nano closed set of Ncl*(A) – A which is a contradiction. Therefore Ncl*(A) ⊆G and hence A is τ*-nano g-closed.

Corollary 3.8.A subset A of (U, $\tau_R(X)$) is τ* nanog-closed if and only if Ncl*(A) – A contain no non-empty closed set in X.

Proof : The proof follows from the Theorem 3.7. and the fact that every closed set is τ*- nanoclosed set in (U, $\tau_R(X)$).

Corollary 3.9.A subset A of (U, $\tau_R(X)$) is τ*-nano g-closed if and only if Ncl*(A) – A contain no non-empty nano open set in X.

Proof: The proof follows from the Theorem 3.7. and the fact that every nano open set is τ*- Nano open set in (U, $\tau_R(X)$).

Theorem 3.10. If a subset A of (U, $\tau_R(X)$) is τ*-nano g-closed and A ⊆B ⊆Ncl*(A), then B is τ*-nano g-closed set in (U, $\tau_R(X)$).

Proof : Let A be a τ*-nano g-closed set such that A ⊆B⊆Ncl*(A). Let U be a τ*-nano open set of X such that B⊆U. Since A is τ*-nano g-closed, we have Ncl*(A) ⊆U. Now Ncl*(A)⊆Ncl*(B)⊆Ncl*[Ncl*(A)] = Ncl*(A) ⊆U. That is Ncl*(B) ⊆U, U is τ*-nano open. Therefore B is τ*-nanog-closed set in (U, $\tau_R(X)$).

Theorem 3.11. Let A be a τ*-nano g-closed in (U, $\tau_R(X)$). Then A is nano g-closed if and only if Ncl*(A) – A is τ*-nano open.

Proof : Suppose A is nano g-closed in (U, $\tau_R(X)$). Then Ncl*(A) = A and so Ncl*(A) – A = φ which is τ*-nano open in (U, $\tau_R(X)$). Conversely, suppose Ncl*(A)

– A is τ*-Nano open in (U, $\tau_R(X)$). Since A is τ*-nanog-closed, by the Theorem 3.10, Ncl*(A) – A contains no non-emptyτ*-nano closed set in X. Then Ncl*(A) – A = φ Hence A is nanog-closed.

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