

# Analytic Study on the Relationship between the Transferred Charge and Tribo-Charge in a Rotary Freestanding TENG (RF-TENG)

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## ABSTRACT

In this study, an analytical equation for a metal-to-dielectric rotary freestanding triboelectric nanogenerator with a grating structure is derived and discussed. The equation-based analysis is performed to understand the relationship between the transferred charge and the triboelectric charge in this system. As a result, it is found that the transferred charge is directly proportional to the triboelectric charge, which indicates that the triboelectric charge is the main factor that influences the transferred charge.

**Keywords:** Rotary freestanding TENG, transferred charge, tribo-charge

## I. INTRODUCTION

Triboelectrification [1-12], which involves continuous rubbing between a metal and a dielectric leading to the accumulation of triboelectric charge on the dielectric surface in a metal-to-dielectric rotary freestanding TENG, has not been sufficiently described in terms of device parameters. Normally, we consider the short-circuit current and open-circuit voltage curves after the tribo-charge reaches the saturation point of stable states. Therefore, it is important to derive the analytical equations in a rotary freestanding TENG system to understand the relationship between the tribo-charge (associated with the charge accumulation on the dielectric surface) and the transferred charge (associated with the induced currents through the external circuit) during repeated triboelectrification. In this paper, it is found that the transferred charge is directly proportional to the tribo-charge.

## II. Derivation of relationship between tribo-charge and transferred charge in a metal RF-TENG system [13].

The governing differential equation that describes the metal RF-TENG (as shown in Fig. 1) is given below [13].

$$R \frac{dQ}{dt} = -\frac{Q}{C} + V_{oc} \quad \dots\dots (1)$$

During the first half cycle ( $0 \leq t \leq \frac{\theta_0}{\omega}$ ), the solution of Eq. (1) with the boundary condition  $Q(0)=Q_0$  and angular velocity  $\omega$ , which is constant, can be stated as

$$Q = \left(\frac{a-t}{t-b}\right)^{-A_1} \cdot \left\{ \int_0^t \frac{A_2 \cdot t}{(a-t)(t-b)} \left(\frac{a-t}{t-b}\right)^{A_1} dt + \left(\frac{a}{-b}\right)^{A_1} \cdot Q_0 \right\} \quad (2)$$

where

$$a = \frac{\theta_0}{2\omega} + \Delta \quad (3)$$

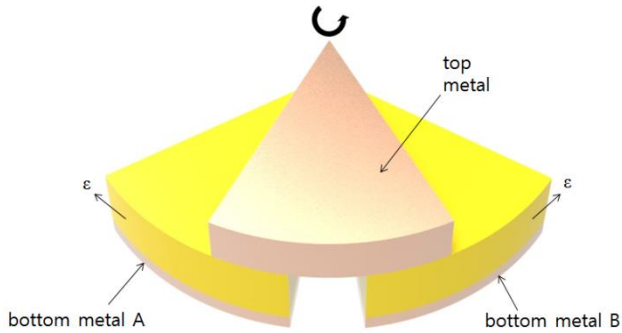
$$b = \frac{\theta_0}{2\omega} - \Delta \quad (4)$$

$$\Delta = \frac{1}{2} \sqrt{\frac{\theta_0^2}{\omega^2} + \frac{8d\theta_0 C_p}{N\epsilon_0 \epsilon_r \omega^2 (r_2^2 - r_1^2)}} \quad (5)$$

$$A_1 \cong 0 \quad (6)$$

$$A_2 = \frac{2d\theta_0 \sigma}{R\epsilon_0 \epsilon_r \omega} \quad (7)$$

Note that the resistance,  $R$ , is assumed to be very large; therefore,  $A_1$  is approximated to zero. However,  $A_2$  remains as it is, because it contains  $\sigma$ .



**Figure 1 :** Schematic of metal rotary freestanding TENG

Thus, Q can be calculated as follows.

$$Q = \int_0^t \frac{A_2 \cdot t \, dt}{(a-t)(t-b)} + Q_0 \quad (8)$$

As  $Q_0 = Q(t=0)$  with  $A_1 = 0$

$$Q_0 = N\sigma\theta_0(r_2^2 - r_1^2) - \int_0^{\frac{\theta_0}{\omega}} \frac{A_2 \cdot t \, dt}{(a-t)(t-b)} \quad (9)$$

Substituting  $Q_0$  into Eq. (8), Q can be written as follows.

$$Q = A_2 \int_0^t \frac{t \, dt}{(a-t)(t-b)} + N\sigma\theta_0(r_2^2 - r_1^2) - A_2 \int_0^{\frac{\theta_0}{\omega}} \frac{t \, dt}{(a-t)(t-b)} \quad \dots\dots (10)$$

$$= A_2 \left( \int_0^t - \int_0^{\frac{\theta_0}{\omega}} \right) \frac{t \, dt}{(a-t)(t-b)} + N\sigma\theta_0(r_2^2 - r_1^2) \quad \dots\dots (11)$$

$$= \frac{2d\theta_0\sigma}{R\epsilon_0\epsilon_r\omega} \left( \int_0^t - \int_0^{\frac{\theta_0}{\omega}} \right) \frac{t \, dt}{(a-t)(t-b)} + N\sigma\theta_0(r_2^2 - r_1^2) \quad \dots\dots (12)$$

$$= \sigma \left[ \frac{2d\theta_0}{R\epsilon_0\epsilon_r\omega} \left( \int_0^t - \int_0^{\frac{\theta_0}{\omega}} \right) \frac{t \, dt}{(a-t)(t-b)} + N\theta_0(r_2^2 - r_1^2) \right] \quad \dots\dots (13)$$

with  $(0 \leq t \leq \frac{\theta_0}{\omega})$ ,

In order to calculate the integral parts, we used the following relationships.

$$\int_0^t \frac{t \, dt}{(a-t)(t-b)} = \frac{a \cdot \log(-a) - b \cdot \log(-b) - a \cdot \log(t-a) + b \cdot \log(t-b)}{a-b} \quad \dots\dots (14)$$

$$\int_0^{\frac{\theta_0}{\omega}} \frac{t \, dt}{(a-t)(t-b)} = \frac{a \cdot \log(-a) - b \cdot \log(-b) - a \cdot \log\left(\frac{\theta_0}{\omega} - a\right) + b \cdot \log\left(\frac{\theta_0}{\omega} - b\right)}{a-b} \quad \dots\dots (15)$$

Therefore, Q can be written as

$$Q = \sigma \left[ \frac{2d\theta_0}{R\epsilon_0\epsilon_r\omega} \left( \frac{-a \log(t-a) + b \log(t-b) + a \log\left(\frac{\theta_0}{\omega} - a\right) - b \log\left(\frac{\theta_0}{\omega} - b\right)}{a-b} \right) + N\theta_0(r_2^2 - r_1^2) \right] \quad \dots\dots (16)$$

For the second half cycle ( $\frac{\theta_0}{\omega} \leq t \leq \frac{2\theta_0}{\omega}$ ), the solution of Eq. (1) with the boundary condition  $Q(\frac{\theta_0}{\omega})=Q_1$  and  $\omega$  can be written as

$$Q = N\sigma\theta_0(r_2^2 - r_1^2) - \int_0^{t-\frac{\theta_0}{\omega}} \frac{A_2 \cdot t dt}{(a-t)(t-b)} + [Q_1 - N\sigma\theta_0(r_2^2 - r_1^2)] \quad \dots\dots (17)$$

with ( $\frac{\theta_0}{\omega} \leq t \leq \frac{2\theta_0}{\omega}$ )

$$Q_1 = \int_0^{\frac{\theta_0}{\omega}} \frac{A_2 \cdot t dt}{(a-t)(t-b)} + Q_0 \quad \dots\dots (18)$$

$$= \int_0^{\frac{\theta_0}{\omega}} \frac{A_2 \cdot t dt}{(a-t)(t-b)} + N\sigma\theta_0(r_2^2 - r_1^2) - \int_0^{\frac{\theta_0}{\omega}} \frac{A_2 \cdot t dt}{(a-t)(t-b)} \quad \dots\dots (19)$$

$$= N\sigma\theta_0(r_2^2 - r_1^2) \quad \dots\dots (20)$$

Therefore, Q can be calculated as

$$Q = N\sigma\theta_0(r_2^2 - r_1^2) - \int_0^{t-\frac{\theta_0}{\omega}} \frac{A_2 \cdot t dt}{(a-t)(t-b)} \quad \dots\dots (21)$$

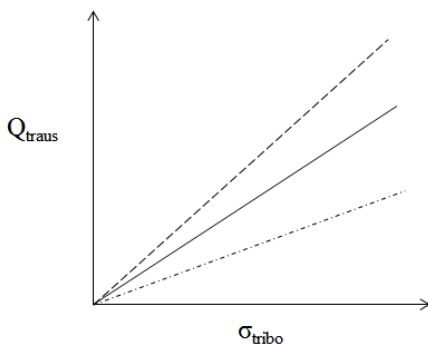
$$= N\sigma\theta_0(r_2^2 - r_1^2) - \frac{2d\theta_0\sigma}{R\epsilon_0\epsilon_r\omega} \left[ \frac{a \log(-a) - b \log(-b) - a \log\left(t - \frac{\theta_0}{\omega} - a\right) - b \log\left(t - \frac{\theta_0}{\omega} - b\right)}{a - b} \right]$$

with ( $\frac{\theta_0}{\omega} \leq t \leq \frac{2\theta_0}{\omega}$ ) ..... (22)

Eq. (22) is the same as Eq. (16).

In any case, the transferred charge, Q, is directly proportional to  $\sigma$ , as shown in Fig.2.

$$Q = \sigma \cdot f \text{ (structural parameters, material properties)} \quad \dots\dots (23)$$



**Figure 2 :** Plot of  $Q_{\text{trans}}$  (transferred charge) vs.  $\sigma_{\text{tribo}}$  (tribo-charge).  $Q_{\text{trans}}$  is proportional to  $\sigma_{\text{tribo}}$

In other words, the main contributor for Q is  $\sigma$ , which can shape Q.

### III. CONCLUSION

In this work, an analytical equation for the metal RF-TENG is derived and discussed. The analytic study was carried out to investigate the relationship between Q and  $\sigma$ . It was found that Q is controlled by  $\sigma$ , the structural parameters, and the material properties. Among them,  $\sigma$  is directly proportional to Q. These results can provide useful theoretical information on the RF-TENG in energy harvesting applications.

### IV. ACKNOWLEDGMENTS

This work was supported by the mathematical modeling & analysis group in Korea (NRF-2017R1A2B4010642).

## V. REFERENCES

- [1]. F. R. Fan, Z. Q. Tian, Z. L. Wang: Flexible triboelectric generator. *Nano energy*. 2012;1: 328-334.
- [2]. Y. Yang, H. L. Zhang, J. Chen, Q. S. Jing, Y. S. Zhou, X. N. Wen, Z. L. Wang : Single-Electrode-Based Sliding Triboelectric Nanogenerator for Self-Powered Displacement Vector Sensor System. *ACS Nano*. 2013;7: 7342-7351.
- [3]. C. Zhang, T. Zhou, W. Tang, C. B. Han, L. M. Zhang, Z. L. Wang : Rotating-Disk-Based Direct-Current Triboelectric Nanogenerator. *Adv. Energy Mater*. 2014;4: 1301798.
- [4]. Y. J. Su, X. N. Wen, G. Zhu, J. Yang, J. Chen, P. Bai, Z. M. Wu, Y. D. Jiang, Z. L. Wang : Hybrid triboelectric nanogenerator for harvesting water wave energy and as a self-powered distress signal emitter. *Nano Energy*. 2014;9: 186-195.
- [5]. G. Zhu, Y. S. Zhou, P. Bai, X. S. Meng, Q. S. Jing, J. Chen, Z. L. Wang : A Shape-Adaptive Thin-Film-Based Approach for 50% High-Efficiency Energy Generation Through Micro-Grating Sliding Electrification. *Adv. Mater*. 2014;26: 3788-96.
- [6]. S. M. Niu, Y. S. Zhou, S. H. Wang, Y. Liu, L. Lin, Y. Bando, Z. L. Wang : Simulation method for optimizing the performance of an integrated triboelectric nanogenerator energy harvesting system. *Nano Energy*. 2014;8: 150-156.
- [7]. C. Zhang, W. Tang, C. B. Han, F. R. Fan, Z. L. Wang : Theoretical comparison, equivalent transformation, and conjunction operations of electromagnetic induction generator and triboelectric nanogenerator for harvesting mechanical energy. *Adv. Mater*. 2014;26: 3580-91.
- [8]. Z. L. Wang, Triboelectric Nanogenerators as New Energy Technology for Self-Powered Systems and as Active Mechanical and Chemical Sensors. *ACS Nano*. 2013;7: 9533-9557.
- [9]. C. B. Han, W. M. Du, C. Zhang, W. Tang, L. M. Zhang, Z. L. Wang: Harvesting energy from automobile brake in contact and non-contact mode by conjunction of triboelectrication and electrostatic-induction processes. *Nano Energy*. 2014;6: 59-65.
- [10]. S. H. Wang, L. Lin, Y. N. Xie, Q. S. Jing, S. M. Niu, Z. L. Wang : Sliding-Triboelectric Nanogenerators Based on In-Plane Charge-Separation Mechanism. *Nano Lett*. 2013;13: 2226-2233.
- [11]. F. R. Fan, L. Lin, G. Zhu, W. Z. Wu, R. Zhang, Z. L. Wang: Transparent Triboelectric Nanogenerators and Self-Powered Pressure Sensors Based on Micropatterned Plastic Films. *Nano Lett*. 2012;12: 3109-3114.
- [12]. S. M. Niu, S. H. Wang, L. Lin, Y. Liu, Y. S. Zhou, Y. F. Hu, Z. L. Wang : Theoretical study of contact-mode triboelectric nanogenerators as an effective power source. *Energy Environ. Sci*. 2013;6: 3576-3583.
- [13]. T. Jiang, X. Chen, C. B. Han, W. Tang, Z. L. Wang : Theoretical Study of Rotary Freestanding Triboelectric Nanogenerators. *Adv. Funct. Mater*. 2015;25: 2928-2938.