© 2017 IJSRSET | Volume 3 | Issue 6 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section: Engineering and Technology

Method for Solving Octagonal Fuzzy Assignment Problem Using Magnitude Ranking Technique

S. Menakadevi, **K. M. Manikandan**
Department of Mathematics, Dr. SNS. Rajalakshmi College of Arts and Science, Coimbatore, Tamilnadu, India

ABSTRACT

In this paper, the octagonal fuzzy assignment problem using magnitude ranking technique and the Hungarian method has been applied to find an optimal solution. The numerical examples show that the octagonal fuzzy ranking magnitude ranking method offers an effective tool for handling the octagonal fuzzy assignment problem over Robust Ranking Method.

Keywords: Octagonal Fuzzy Number, Fuzzy Set, Magnitude Ranking, Magnitude Ranking Method. Hungarian Algorithm

I. INTRODUCTION

The assignment model is the special case of transportation problem. In 1965, Lotfi Zadeh has introduced fuzzy sets which provide as a new mathematical tool to deal with uncertainty of information. In this paper we provided a Hungarian and Magnitude ranking method to solve assignment problem with fuzzy cost. Lin and Wen solved the assignment problem with fuzzy interval number cost by a labeling algorithm [2004]. Chen proved some theorems are proposed a fuzzy assignment model that con-siders all individuals to have same skills [1985]. Dominance of fuzzy numbers can be explained by many ranking methods. Robust ranking method satisfies the properties of compensation, linearity and additivity. Zimmermann showed that solutions are obtained by fuzzy linear programming are always efficient [1978]. R.Nagarajan and A. Solairaju [2010] presented an algorithm for solving fuzzy assignment problems using Robust ranking technique with fixed fuzzy numbers. In this paper the octagonal fuzzy assignment problem has been converted into crisp assignment problem using Method of Magnitude and Hungarian assignment has been applied to find an optimal solution.

1.1. Fuzzy Set

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function A→[0,1] and A(x) is interpreted as the degree of membership of element x in fuzzy A for each x∈ X.

The Value zero is used to represent complete non-membership; the value one is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A.

1.2. Crisp set

Crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

1.3. Fuzzy number

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) There exist atleast one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$
(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.
(iii) $\tilde{A}$ Must be normal and convex.

2. Octagonal fuzzy numbers

Definition 2.1.

An octagonal fuzzy number denoted by $\tilde{A}_w$ is defined to be the ordered quadruple

$\tilde{A}_w$ = ($l_1(r)$, $s_1(t)$, $s_2(t)$, $l_2(r)$) for r ∈ [0,k] and t ∈ [k,w] where

1. $l_1(r)$ is a bounded left continuous non-decreasing function over $[0,w_1]$, $[0 \le w_1 \le k]$
2. $s_1(t)$ is a bounded left continuous non-decreasing function over $[k,w_2]$, $[k \le w_2 \le w]$
3. $s_2(t)$ is a bounded left continuous non-increasing function over $[k,w_2]$, $[k \le w_2 \le w]$
4. $l_2(r)$ is a bounded left continuous non-increasing function over $[0,w_1]$, $[0 \le w_1 \le k]$

Definition 2.2.

A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ k\left(\dfrac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(1-k)\left(\dfrac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ 1 & for\ a_4 \le x \le a_5 \\ k+(1-k)\left(\dfrac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k\left(\dfrac{a_8-x}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 0 & for\ x \ge a_8 \end{cases}$$

Where 0 < k < 1.

2.3 α -cut of an octagonal fuzzy number

The α -cut of an octagonal fuzzy number $\tilde{A}$ = $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ is

$$[\tilde{A}]_\alpha = \begin{cases} \left[a_1+\left(\frac{\alpha}{k}\right)(a_2-a_1)\ ,a_8-\left(\frac{\alpha}{k}\right)(a_8-a_7)\right] & for\ \alpha\epsilon[0,k] \\ \left[a_3+\left(\frac{\alpha-k}{1-k}\right)(a_4-a_3)\ ,a_6-\left(\frac{\alpha-k}{1-k}\right)(a_6-a_5)\right] & for\ \alpha\epsilon[k,1] \end{cases}$$

II. Ranking of octagonal fuzzy numbers

A magnitude of fuzzy number $\tilde{A}_w$ is a function M_α : $R_\omega(I) \rightarrow R^+$ which assigns a nonnegative real number $M_\alpha(\tilde{A}_w)$ that expresses the measure of $\tilde{A}_w$

$$M_\alpha(\tilde{A}_w) = \frac{1}{2}\int_\alpha^k (l_1(r)+l_2(r))\,dr + \frac{1}{2}\int_k^\omega (s_1(t)+s_2(t))\,dt \text{ where } 0 \le \alpha \le 1$$

Definition 3.1. The magnitude of an octagonal fuzzy number is obtained by the average of the two fuzzy side areas, left side area and right side area, from membership function to $\alpha -$ axis.

3.2. Magnitude ranking of octagonal fuzzy numbers

Let $\tilde{A}$ be a normal octagonal fuzzy number. The value $M_0^{Oct}(\tilde{A})$, called the magnitude of $\tilde{A}$ is calculated as follows:

$$M_O^{oct}(\tilde{A}) = \frac{1}{2}\int_\alpha^k (l_1(r)+l_2(r))\,dr + \frac{1}{2}\int_k^\omega (s_1(t)+s_2(t))\,dt \text{ where } 0 \le k \le 1$$

$$= \frac{1}{4}[(a_1+a_2+a_7+a_8)\,k + (a_3+a_4+a_5+a_6)\,(1-k)]$$

where $0 \le k \le 1$

3.3 Algorithm to Solve octagonal Fuzzy Assignment Problem:

Step 1: First test whether the given fuzzy cost matrix of a octagonal fuzzy assignment problem is a balanced one or not. If not change this unbalanced assignment problem into balanced one by adding the number of dummy row(s) / column(s) and the values for the entries are zero. If it is a balanced one (i.e. number of persons are equal to the number of works) then go to step 2.

Step 2: Defuzzify the fuzzy cost by using Magnitude ranking method.

Step 3: Apply Hungarian Algorithm to determine the best combination to produce the lowest total costs, where each machine should be assigned to only one job and each job requires only one machine.

III. Numerical Example:

Example1 : Here we are going to solve octagonal fuzzy Assignment problem using Magnitude Ranking Technique: To allocate 3 jobs to 3 different machines, the fuzzy assignment cost C_{ij} is given below:

$$\begin{pmatrix} (2,4,5,6,7,8,9,11) & (2,3,4,5,6,7,8,9) & (-1,0,1,2,3,4,5,6) \\ (-1,0,1,2,3,4,5,6) & (-3,-2,-1,0,1,2,3,4) & (4,5,6,7,8,9,10,11) \\ (8,9,10,11,12,13,14,15) & (0,1,2,3,4,5,6,7) & (2,3,4,5,6,7,8,9) \end{pmatrix}$$

Min Mag $(2,4,5,6,7,8,9,11)x_{11}$ + Mag $(2,3,4,5,6,7,8,9)x_{12}$ + Mag$(-1,0,1,2,3,4,5,6)x_{13}$ +

Mag$(-1,0,1,2,3,4,5,6)x_{21}$ +
Mag $(-3,-2,-1,0,1,2,3,4)x_{22}$ +
Mag $(4,5,6,7,8,9,10,11)x_{23}$ +
Mag $(8,9,10,11,12,13,14,15)x_{31}$ +
Mag $(0,1,2,3,4,5,6,7)x_{31}$ + Mag $(2,3,4,5,6,7,8,9)$ x_{33}

Subject to

$x_{11}+x_{12}+x_{13}=1$ $\quad x_{11}+x_{21}+x_{31}=1$
$x_{21}+x_{22}+x_{23}=1$ $\quad x_{12}+x_{22}+x_{32}=1$
$x_{31}+x_{32}+x_{33}=1$ $\quad x_{13}+x_{23}+x_{33}=1$ where $x_{ij}\in[0,1]$

Now we calculate Mag (2,4,5,6,7,8,9,11) by applying method of magnitude. The membership function of the octagonal fuzzy number (2,4,5,6,7,8,9,11) is

$$(x) = \begin{cases} 0 & for\ x < 2 \\ 0.4\left(\frac{x-2}{4-2}\right) & for\ 2 \le x \le 4 \\ 0.4 & for\ 4 \le x \le 5 \\ 0.4+(1-0.4)\left(\frac{x-5}{6-5}\right) & for\ 5 \le x \le 6 \\ 1 & for\ 6 \le x \le 7 \\ 0.4+(1-0.4)\left(\frac{8-x}{8-7}\right) & for\ 7 \le x \le 8 \\ 0.4 & for\ 8 \le x \le 9 \\ 0.4\left(\frac{11-x}{11-9}\right) & for\ 9 \le x \le 11 \\ 0 & for\ x \ge 11 \end{cases}$$

$M_O^{oct}(\tilde{A}) = \frac{1}{4}[(a_1+a_2+a_7+a_8)\ k + (a_3+a_4+a_5+a_6)\ (1-k)]$ where $0 \le k \le 1$

$a_{11} = M_O^{oct}(2,4,5,6,7,8,9,11) = \frac{1}{4}[(2+4+9+11)(0.4)+(5+6+7+8)(1-0.4)]$
$= \frac{1}{4}[26(0.4)+26(0.6)] =$ **6.5**

$a_{12} = M_O^{oct}(2,3,4,5,6,7,8,9) = \frac{1}{4}[(2+3+8+9)(0.4)+(4+5+6+7)(1-0.4)]$
$= \frac{1}{4}[22(0.4)+22(0.6)] =$ **5.5**

$a_{13} = M_O^{oct}(-1,0,1,2,3,4,5,6) = 2.5$
$a_{21} = M_O^{oct}(-1,0,1,2,3,4,5,6) = 2.5$
$a_{22} = M_O^{oct}(-3,-2,-1,0,1,2,3,4) = 0.5$
$a_{23} = M_O^{oct}(4,5,6,7,8,9,10,11) = 7.5$
$a_{31} = M_O^{oct}(8,9,10,11,12,13,14,15) = 11.5$
$a_{32} = M_O^{oct}(0,1,2,3,4,5,6,7) = 3.5$
$a_{33} = M_O^{oct}(2,3,4,5,6,7,8,9) = 5.5$

We replace these values for this corresponding C_{ij} . We get a convenient assignment problem

$$\begin{pmatrix} 6.5 & 5.5 & 2.5 \\ 2.5 & 0.5 & 7.5 \\ 11.5 & 3.5 & 5.5 \end{pmatrix}$$

We solve it by using Hungarian methods to get the following optimal solution.

Step 1: Row reduction. Subtract the minimum element of each row from all elements of that row

$$\begin{pmatrix} 4 & 3 & 0 \\ 2 & 0 & 7 \\ 8 & 0 & 2 \end{pmatrix}$$

Step 2: Column reduction. Subtract the minimum element of each column from all elements of that column

$$\begin{pmatrix} 2 & 3 & 0 \\ 0 & 0 & 7 \\ 6 & 0 & 2 \end{pmatrix}$$

Step 3: Making assignments

$$\begin{pmatrix} 2 & 3 & (0) \\ (0) & 0 & 7 \\ 6 & (0) & 2 \end{pmatrix}$$

The fuzzy optimal total cost

$a_{13}+a_{21}+a_{32} = (-1,0,1,2,3,4,5,6) + (-1,0,1,2,3,4,5,6) + (0,1,2,3,4,5,6,7)$
= (-2,1,4,7,10,13,16,19)

Mag(-2,1,4,7,10,13,16,19) $= \frac{1}{4}[(-2+1+16+19)(0.4)+(4+7+10+13)(1-0.4)]$
$= \frac{1}{4}[34(0.4)+34(0.6)]$
$= \frac{1}{4}[13.6+20.4]$
$= \frac{34}{4}$

Mag(-2,1,4,7,10,13,16,19 = 8.5

IV. CONCLUSION

In this paper, a method of solving fuzzy assignment problem using magnitude ranking of octagonal fuzzy numbers has been considered. The parameter k can be modified suitably by the decision maker to get the desired suitably by the decision maker to get the desired result. We may get different fuzzy assignment value for different values of k for the same assignment problem.

.

V. REFERENCES

[1]. D. Selvi, R. Queen Mary, 2017, Method for solving fuzzy assignment problem using magnitude ranking technique, International Journal of Applied and Advanced Scientific Research.

[2]. Dubois, D. & Fortempts, P. (1999), Computing improved optimal solutions to max-min flexible constraint satisfaction problems, European J. Op. R. 118, 95-126.

[3]. Gani, A.N. &Assarudeen, S.N.M. (2012), A new operation on triangular fuzzy number for solving fuzzy linear programming problem, Appl. Math. Sci. 6, 525-532.

[4]. Hajjari T, Abbasbandy S. A promoter operator for defuzzification methods. Australian Journal of Basic and Applied Sciences. 2011; 5(10): 1096-1105.

[5]. Huang, L. & Xu, G. (2005), Solution of assignment problem of restriction of qualification, Operations Research and Management Science 14, 28-31.

[6]. Jatinder Pal Singh & Neha Ishesh Thakur (2015), A Novel Method to Solve Assignment Problem in Fuzzy Environment, Industrial Engineering Letters 1, ISSN 2224-6096 (Paper) ISSN 2225-0581 (online).