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Bianchi type-III Cosmological Model with Linear Equation of State in f (R) Gravity

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ABSTRACT

In this context, we study Bianchi type-III cosmological model with linear equation of state in the metric version of $f(R)$ gravity. The field equations solved by considering an expansion scalar is proportional to the shear scalar (relation between the metric coefficients) and the law of variation of Hubble's parameter that yields the constant value of deceleration parameter. The physical and kinematical parameters of the models have discussed. In addition, the function of the Ricci scalar evaluated for each model.

Keywords: Bianchi type models; linear equation of state; f (R) gravity.
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I. INTRODUCTION

The prediction about expansion of the universe is flat, past decelerated and present accelerating can classified into two categories [1, 2]. One can measure due to mysterious energy with negative pressure dubbed as Dark Energy (DE) and other observations such as cosmic microwave background anisotropies measured with WMAP satellite [3] and large-scale structure (LSS) [4] suggest that nearly two-third of our universe consists of negative pressure dubbed as DE, and the remaining consists of relativistic dark matter and baryons [5]. A very important parameter for the DE investigation is that of the equation of state (EoS) parameter which is usually parameterized of the form $\omega = p/\rho$, where p and ρ be the pressure and density respectively. One can see that the value of EoS parameter $\omega < -1/3$ is required for accelerated cosmic expansion. The primary candidates in this category are scalar field models such as Quintessence [6, 7] and K-essence [8]. In quintessence models, the range of EoS parameter is $-1 < \omega < -1/3$, and the DE density decreases by a scale factor $a(t)$ as $\rho \propto a^{-3(1+\omega)}$ [9]. A specific exotic form of DE denoted phantom energy, with $\omega < -1$ [10]. Many authors have scrutinized DE models for different context such as Ray *et al.* [11] investigated the variable EoS for generalized DE models. Yadav and Yadav [12] obtained Bianchi type-II anisotropic DE models with a constant deceleration parameter. Recently, Pradhan and Amirhashchi [13] have investigated a new anisotropic Bianchi type-III DE model in general relativity with the EoS parameter without assuming a constant deceleration parameter. Saha and Yadav [14] have obtained exact solutions of Einstein's field equations, which for some suitable choices of problem parameters yield time-dependent EoS and deceleration parameters, representing a model that generates a transition of the universe from early decelerating phase to the present accelerating phase. Accelerating and decelerating cosmological models with perfect fluid and DE for kasner type metric had examined by Katore *et al.* [15]. Chirde and Shekh [16, 17] investigated an anisotropic and homogenous Bianchi Type VI0 space-time under the assumption of anisotropy of the fluid within the frame work of Lyra manifold in the presence and absence of magnetism using special form of deceleration parameter which gives an early deceleration and late time accelerating cosmological model, also the same author discussed two fluids viscous DE cosmological models with linearly varying deceleration parameter in self creation cosmology.

Another alternative way to explain cosmic acceleration is a classical generalization of general relativity (GR). Various modified gravity theories such as $f(R)$, $f(T)$, $f(R,T)$ etc has proposed which provides successful gravitational alternatives.

Harko*et al.* [18] proposed a maximal extension of the Hilbert-Einstein action (GR) by assuming that the gravitational Lagrangian is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T). Several authors have investigated the aspect of cosmological models in this gravity [19-25]. Another modification goes to Einstein attempt, Einstein [26] has presented another form of gravity called Teleparallel gravity, namely $f(T)$ gravity to explain the current accelerating expansion without introducing DE, which allows one to say gravity is not due to curvature, but due to torsion. In this theory some authors Rodrigues *et al.* [27], Jamil *et al.* [28], Sharif *et al.* [29], Setare*et al.* [30], Chirde&Shekh [31-33], Shekh*et al.* [34] have discussed several features of cosmological models.
Among the various modifications of Einstein's GR, the $f(R)$ gravity treated most seriously during the last decade. It provides a natural gravitationalalternative to DE. It has been suggested that cosmic accelerationcan be achieved by replacing the Einstein-Hilbert action of GR with ageneral function Ricci scalar $f(R)$. Viable $f(R)$ gravity models [35] havebeen proposed which show the unification of early-time inflation and latetimeacceleration. The explanation of cosmic acceleration is obtained just by introducingthe term 1/R, which is essential at small curvatures. $f(R)$gravity has been shown [36]equivalent to scalar-tensor theory of gravity that is incompatible with solar system tests of GR. From a power law f(R) cosmological modeldust matter and dark energy phases can be achieved [37]. Azadi *et al.* [38] studied vacuum solution in cylindrically symmetric space-time. Recently, Bianchi type-III cosmological model with bulk viscosity in$f(R)$gravity investigated by katore and Shaikh [39]. Sharif and Yousaf [40] studied the impact of DE and dark matter models on the dynamical evolution of collapsing self-gravitating systems in this gravity and very recently, Bhoyar*et al.* [41] examined Bianchi type-I space-time with quadratic equation of state in the $f(R)$ gravity by applying volumetric power law and exponential law of expansion. Thus,$f(R)$gravity seems attractive and a reasonable amount of work has been done in different contexts.

II. Basics of $f(R)$ gravity

The $f(R)$ theory of gravity is the generalization of General Relativity. The three main approaches in $f(R)$ gravity are "Metric Approach", "Palatine formalism" and "affine $f(R)$ gravity". In metric approach, the connection is the Levi-Civita connection and variation of the action done with respect to the metric tensor. While, in Palatine formalism, the metric and the connection is independent of each other. The variation is done for the two mentioned parameters independently. In metric-affine $f(R)$ gravity, both the metric tensor and connection are treating independently and assuming the matter action depends on the connection as well.
The action for this theory given by

$$S=\int \sqrt{-g}\left(f(R)+L_m\right)d^4x\,. \tag{2.1}$$

Here $f(R)$ is a general function of the Ricci Scalar, g is the determinant of the metric $g_{\mu\nu}$ and L_m is the matter Lagrangian.
It noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein–Hilbert action.
The corresponding field equation from this action are found

$$F(R)\,R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\nabla_\mu\nabla_\nu F(R)+g_{\mu\nu}\,\Box F(R)=T_{\mu\nu}, \tag{2.2}$$

where $\Box\equiv\nabla^\mu\nabla_\mu$, $F(R)\equiv\frac{df(R)}{dR}$, ∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

III. Metric and energy Momentum tensor

We consider a spatially homogeneous Bianchi type III metric of the form

$$ds^2=-dt^2+A^2dx^2+B^2e^{-2x}dy^2+C^2dz^2, \tag{3.1}$$

where the metric potentials A,B,C are the functions of time *t*.
Some geometrical parameters related with the metric potential for the metrics (3.1) are defined as,
The average scale factor a and the volume scale factor V is

$a = \sqrt[3]{ABC}, \qquad V = a^3 = ABC, \qquad (3.2)$

The anisotropy parameter of the expansion expressed in terms of mean and directional Hubbleparameters as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \qquad (3.3)$$

where

$$H = (\ln a)^{\cdot} = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3), H_1 = \frac{\dot{A}}{A},$$

$H_2 = \frac{\dot{B}}{B}, \; H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters of *x, y* and *z* axes respectively.

The average Hubble parameter H given by

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \qquad (3.4)$$

The expansion scalar θ, shear scalar σ and deceleration parameter q are defined as

$$\theta = u^m_{;m} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \qquad (3.5)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left\{\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right\} - \frac{\theta^2}{6}, \qquad (3.6)$$

and

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \qquad (3.7)$$

Let us consider that the matter content is a perfect fluid such that the energy momentum tensor T^ν_μ taken as

$$T^\nu_\mu = (p + \rho)u^\nu u_\mu + p g^\nu_\mu, \qquad (3.8)$$

satisfying the equation of state

$$p = \varepsilon\rho - \gamma, \qquad (3.9)$$

where ε and γ are the constants, together with commoving coordinates

$$u^\nu = (0,0,0,1) \text{ and } u^\nu u_\nu = -1, \qquad (3.10)$$

where u^ν is the four-velocity vector of the fluid, p and ρ be the pressure and energy density of the fluid respectively.

IV. Field equations and their Solutions

In the presence of perfect fluid source given in equation (3.8), the field equations (2.2) corresponding to the metric (3.1) lead to the following set of linearly independent differential equations

$$\left[H^2(4 - q) + \frac{2(h^2 + h + 1)}{3A^2}\right]F + \frac{1}{2}f(R) + 2H\dot{F} + \ddot{F} = -p, \qquad (4.1)$$

$$\left(3\dot{H} + 3H^2 + 2\sigma^2\right)F + \frac{1}{2}f(R) - 3H\dot{F} = \rho, \qquad (4.2)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0. \qquad (4.3)$$

Here the overhead dot denotes differentiation with respect to *t*.

Solution of the field equations:

In this section, we discussed the acts of Bianchi type-III cosmological model with linear equation of state in the metric version of $f(R)$ gravity using power and exponential law of expansion with the changing aspects of physical behaviour of universe.

Since, the set of field equations (4.1) - (4.3) are coupled system of highly nonlinear differential equations. Thus, we can introduce conditions to obtain unique solutions of the field equations. The solutions to the field equations generated by using two different forms of volumetric expansion laws

i) Power law and ii) Exponential law [41]

$$V = c_1 t^{3k}, \qquad (4.4)$$

$$V = c_2 e^{3m_1 t}, \qquad (4.5)$$

where c_1, c_2, k and m_1 are constants.

The model describes accelerating volumetric expansion with power law and exponential law for $k > 1$, and for $k < 1$ the model exhibit a decelerating volumetric expansion.

The field equation (4.3) gives

$$A = B, \qquad (4.6)$$

(without loss of generality we consider integration constant is one).

According to Collin *et al.* for spatially homogeneous metric; the normal congruence to the homogeneous expansion satisfies that the condition (σ/θ) is constant i.e. the expansion scalar is proportional to the shear scalar, which gives the relation between metric potentials as

$$B = C^n, \qquad (4.7)$$

where *n* be any real constant.

4a. Model with power law expansion:

Using equations (4.4), (2.2) and (3.1), corresponding metric coefficients A , B and C comes out to be

$$A = B = c_1^{\frac{n}{2n+1}} t^{\frac{3nk}{2n+1}}, \qquad (4.8)$$

$$C = c_1^{\frac{1}{2n+1}} t^{\frac{3k}{2n+1}}. \qquad (4.9)$$

With the values of metric coefficients given in equations (4.8) and (4.9), model defined as

$$ds^2 = -dt^2 + c_1^{\frac{2n}{2n+1}} t^{\frac{6nk}{(2n+1)}} dx^2 + c_1^{\frac{2n}{2n+1}} t^{\frac{6nk}{(2n+1)}} e^{-2x} dy^2 + c_1^{\frac{2}{2n+1}} t^{\frac{6k}{(2n+1)}} e^{-2hx} dz^2. \qquad (4.10)$$

Energy density and Pressure of the universe is comes out to be Energy density,

$$\rho = \frac{1}{2}\alpha_1 t^{-2} - \alpha_2 t^{2k} - \frac{1}{2} t^{-6k}. \qquad (4.11)$$

In power law expansion of the universe, it is observed that the energy density is always positive and decreasing function of time *t*. At the initial stage $t \to 0$ the universe has infinitaly large energy density $\rho \to \infty$ but with the expansion of the universe it declines and at large $t \to \infty$ it is null $\rho \to 0$. Thus, our derived universe is free from big rip.

Pressure,

$$p = \alpha_3 t^{-2m} - \alpha_4 t^{-2} - \frac{t^{-6k}}{2}. \qquad (4.12)$$

From equation (4.12), it is observe that, in the power law expansion of the universe, at the initial epoch $t \to 0$ when universe start to expand the fluid pressure is infinitely large $p \to \infty$ throughout the universe and decreases with the expansion of the universe and at $t \to \infty$ which is same as that of the behavior of the energy density.

4b. Model with exponential law expansion:

Using equations (4.5), (2.2) and (3.1), corresponding metric coefficients A , B and C comes out to be

$$A = B = c_2^{\frac{n}{2n+1}} e^{\frac{3nm_1 t}{2n+1}}, \qquad (4.13)$$

$$C = c_2^{\frac{1}{2n+1}} e^{\frac{3nm_1 t}{2n+1}}. \qquad (4.14)$$

With the values of metric coefficients given in equations (4.13) and (4.14), the model defined as

$$ds^2 = -dt^2 + c_2^{\frac{2n}{2n+1}} e^{\frac{6nm_1 t}{2n+1}} dx^2 + c_2^{\frac{2n}{2n+1}} e^{\frac{6nm_1 t}{2n+1}} e^{-2x} dy^2 + c_2^{\frac{2}{2n+1}} e^{\frac{6m_1 t}{2n+1}} e^{-2hx} dz^2. \qquad (4.15)$$

Energy density,

$$\rho = \alpha_5 - \frac{1}{2} e^{-6m_1 t}. \qquad (4.16)$$

In an exponential expansion of the universe, it is observed that the energy density of DE is always positive and decreasing function of time *t*. From equation (4.16) it is conclude that at the initial stage of the universe the energy density is infinitaly large i.e. $\rho \to \infty$ and with the expansion of the universe it is decreases and at large expansion it is null i.e. $\rho \to 0$.Thus, our derived universe is free from big rip.

Pressure,

$$p = \alpha_6 - \frac{1}{2} e^{-6m_1 t}. \qquad (4.17)$$

It is observe that, in the exponential expansion of the universe at the initial epoch $t \to 0$ when universe start to expand the fluid pressure is infinitely large throughout the universe $p \to \infty$ and decreases with the expansion of the universe while at infinite expansion $t \to \infty$, $p \to 0$.

V. CONCLUSION

Bianchi type-III cosmological model with linear equation of state have investigated in $f(R)$ theory of gravitation. The models have studied in relation to volumetric power law and exponential law expansion. Our derived universe exhibited both accelerating as well as decelerating phase, in both phases universe is expanding and expansion of the universe is much faster and then slows down for later time.

In power law expansion, the universe starts with zero volume, the Hubble parameter and the scalar expansion are the functions of time and initially $t \to 0$ they attain infinitely large value and decreases with expansion and approaches to zero at large expansion. In an exponential expansion the universe starts with constant volume for $c_2 > 0$ and for $c_2 = 0$ it starts with zero volume and expand exponentially with infinite volume. The expansion scalar and the shear scalar are constant.

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