© 2017 IJSRSET | Volume 3 | Issue 6 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section: Engineering and Technology

Co axian – 2 General Service Queuing Model with Restricted Admissibility

P. Suthersan, S. Maragathasundari*
Department of Mathematics, Kalasalingam University, Tamilnadu, India

ABSTRACT

We explore the steady state conduct of a bulk arrival queuing model with compulsory vacation. Here the arrival follows a poisson distribution. Service following a general service distribution is rendered in two stages in which the second stage is optional. After the completion of service, the server has to undergo a compulsory vacation. An important phenomenon of restricted admissibility is carried out in this model which plays a wide role in all the categories of system which are following the queuing strategy. We obtain in closed form, the steady state probability generating functions for the number of customers in the queue for various states of the server, the average number of customers as well as their average waiting time in the queue and the system.

Keywords : Bulk arrival, optional second stage, compulsory vacation, restricted admissibility.

I. INTRODUCTION

A few creators have examined queueing models in sorts of services in differing charges that incorporate stages of service and standby server. Chae.K.C et.al[1] inquired about M/G/1-sort lines with summed up get-aways. Choi. B.D.et.al.[2] made an examination on a M/G/1 line with various sorts of information, and gated get-aways. Cox. D.R[4] made an examination on Non-Markovian Stochastic Processes by the joining of Supplementary Variables.. Madan and Anabosi [19] concentrated two sorts of administrations with single get-away and Bernoulli design outing. Maragathasundari[15] concentrated a mass arriving queueing model with three periods of administrations took after by benefit intrusion and postpone time. Choudhury[3] explored a bulk arrival queue with an excursion time under single vacatiom strategy. Madan and Abu-Dayyeh [12] concentrated a solitary server line with stage sort server departure and optional stage sort server vacation. Maraghi et.al [19] made an examination on the bunch landing queueing framework with second discretionary administration and irregular breakdown. Maragathasundari and Srinivasan [17] made an examination on a Non Markovian line with three phases of service and numerous vacations. Kavitha and Maragathasundari [10] researched the idea of limited tolerability and optional sorts of repair in a Non Markovian queue. A Non Markoviaan queue with discretionary services has been investigated by Srinivasan and Maragathasundari [22].A Non Markovian line with multi phases of service and reneging have been considered by Maragathasundari et.al[24]. Karthikeyan and Maragathasundari [9] made an examination on a bulk landing of two periods of administration with standby server in the midst of general get-away time and general repair time. Haridass and Arumuganathan[5] concentrated a retrial line in which modified outings under N course of action is solidified. Jain. M [6] investigated a working vacation queueing model with various sorts of server breakdowns. Time-subordinate properties of symmetric M/G/1 lines are considered by Kella.O et.al [7] Kumar.R and Sharma.S.K [8] reneging and Balking in a non markovian line. Madan. K.C. additionally, Chodhury. G[13] made an examination on M[x]/G/1 line with Bernoulli escape timetable under restricted admissibility...Maraghi. Et.al [20], concentrated a bunch Arrival fixing system with Random Breakdowns and Bernoulli Schedule server excursion taking after general conveyance. Ranjitham .A. furthermore, Maragathasundari .S[22] ,contemplated the two periods of administration in mass landing queueing model. In their audit, if an arriving gathering of customers find the server involved or in outing,, at that point the entire group joins the hover in order to search for the organization afresh.. Sowmiyah and Maragathasundari[23] investigated a mass queueing models with opened up journey and stages in repair. Kendall. D.G [11] concentrated the stochastic Processes occurring in the theory of lines and their examination by the system for embedded Markov chains. Maragathasundari et.al[18] studied the Queuing model in web hosting service.Vignesh et.al[25] investigated a Queuing process of restricted admissibility.

II. The Mathematical Description of the Model

The model is based on the following assumptions:

a) Customers' arrive one by one follows a compound Poisson process with a rate of arrival λ. Let λb_i dt(i = 1,2) be the first order probability that customers in batches of size i arrive at the system at a short interval of time (x. x+dt) , where $0\leq b_i \leq 1$. Not all the arriving customers are allowed to join the system .Let β be the probability of arriving customers to join the system during vacation and α be the probability of arriving customers to join the system during Non Vacation time.

b) The server provides service in two stages with optional second stage of service. As soon as the first stage of the server closes; the server has the choice of taking a optional second stage of service with the likelihood r. The service follows general distribution The service discipline is assumed to be on a first come first served basis (FCFS).Let us assume that the service time $\psi_i(i = 1,2)$ of the i[th] stage of service follows a general probability distribution with a distribution function $M(x)$, $S(x)$ and probability density function $m(x), s(x)$ respectively. Therefore we have

$$\psi_1(x) = \frac{m(x)}{1-M(x)} \quad , m(v) = \psi_1(v) e^{\left[-\int_0^v \psi_1(x)dx\right]} \; \& \quad \psi_2(x) = \frac{s(x)}{1-S(x)} \quad , \; s(v) = \psi_2(v) e^{\left[-\int_0^v \psi_2(x)dx\right]} \tag{a}$$

c) Once the service of a unit is finished, the server is accepted to take a compulsory vacation of general distribution. As soon as the first stage of the server closes and if no customer waits for optional second stage of service, then the server has the choice of taking a compulsory vacation with the likelihood φ. Otherwise, after the completion of customer's second stage of optional service, vacation will be followed. At the culmination of necessary vacation, it joins the system to proceed with a service to the holding up clients. Give us a chance to expect the Compulsory vacation time to be an irregular variable, after the general conditional probability law with distribution function Q(x) and density function q(x). Here, let us consider that $\gamma(x)$ is the conditional probability of a vacation period amid the interval (x,x+dx), given that the slipped by time is x, which can be given as

$$\gamma(x) = \frac{q(x)}{1-Q(x)} \; , \quad q(t) = \gamma(t) e^{\left[-\int_0^t \gamma(x)dx\right]} \tag{b}$$

2.1. Notations

$R_n^{(i)}(x,t)$: Probability that at time t,the server is active providing service and there are $n (n \geq 0)$ customers in the queue excluding the one being served in the i th stage of service and the elapsed service time for this customer is x.Consequently, $R_n^{(i)}(t) = \int_0^\infty R_n^{(i)}(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue excluding the one customer in the i th optional stage of service irrespective of the value of x. $S_n(x,t)$:Probability that at time t, the server is on compulsory vacation with elapsed vacation time x and there are $n (n > 0)$ customers waiting in the queue for service. Consequently,$K_n(t) = \int_0^\infty K_n(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue and the server is on compulsory vacation irrespective of the value of x. $A(t)$: Probability that at time t, there are no customers in the system and the server is idle but available in the system.

III. Steady State Equation Governing the System

$$\frac{d}{dx} R_n^{(1)}(x,t) + (\lambda + \psi_1(x)) R_n^{(1)}(x) = \lambda\alpha \sum_{j=1}^{n-1} b_i \, R_{n-i}^{(1)}(x) + \lambda(1-\alpha) R_n^{(1)}(x) \; , n \geq 1 \tag{3.1}$$

$$\frac{d}{dx} R_0^{(1)}(x,t) + (\lambda + \psi_1(x)) R_0^{(1)}(x) = \lambda(1-\alpha) R_0^{(1)}(x) \tag{3.2}$$

$$\frac{d}{dx}R_n^{(2)}(x,t) + (\lambda + \psi_2(x))R_n^{(2)}(x) = \lambda\alpha\sum_{j=1}^{n-1} b_i\, R_{n-i}^{(2)}(x) + \lambda(1-\alpha)R_n^{(2)}(x)\ \ , n \geq 1 \tag{3.3}$$

$$\frac{d}{dx}R_0^{(2)}(x,t) + (\lambda + \psi_2(x))R_0^{(2)}(x) = \ \ \lambda(1-\alpha)R_0^{(2)}(x) \tag{3.4}$$

$$\frac{d}{dx}K_n(x) + \big(\lambda + \gamma(x)\big)K_n(x) = \lambda\beta\sum_{j=1}^{n-1} b_i\, K_{n-i}(x)\ + \lambda(1-\beta)\ K_n(x) \tag{3.5}$$

$$\frac{d}{dx}K_0(x) + \big(\lambda + \gamma(x)\big)K_0(x) = \ \ \lambda(1-\beta)\ K_n(x) \tag{3.6}$$

$$\lambda A = \int_0^\infty K_0(x)\gamma(x)dx \tag{3.7}$$

Equations 3.1 – 3.7 are to be solved subject to the following boundary conditions:

$$R_n^{(1)}(0) = \int_0^\infty K_{n+1}(x)\,\gamma(x)dx + \quad \lambda\alpha b_{n+1}A\ \ , n \geq 0$$

$$R_n^{(2)}(0) = r\int_0^\infty R_n^{(1)}(x)\psi_1\,(x)dx \qquad n \geq 0$$

$$K_n(0) = \varphi\int_0^\infty R_n^{(1)}(x)\psi_1\,(x)dx\ + \int_0^\infty R_n^{(2)}(x)\psi_2\,(x)dx \tag{3.8}$$

3.1 Queue size Distribution at Random Epoch

We define the probability generation function as follows:

$$R_q^{(i)}(x,z) = \textstyle\sum_{n=0}^\infty z^n\,R(x), \qquad R_q^{(i)}(z,t) = \sum_{n=0}^\infty z^n\,R \qquad i = 1\,,2$$

$$K_q(x,z) = \textstyle\sum_{n=0}^\infty z^n K_q\,(x,t)$$

$$K_q(z,t) \quad = \textstyle\sum_{n=0}^\infty z^n\,K_q \tag{A}$$

$$B(z) = \textstyle\sum_{n=1}^\infty b_i z^i\ ; \quad |z| \leq 1$$

IV. Steady state queue size distribution at a random epoch

Cox (1955) has investigated non-Markovian models by changing them into Markovian ones, through the presentation of at least one supplementary factors. A stable recursive plan for the estimation of the restricting probabilities can be created, in view of an adaptable regenerative approach

Multiplying eq. 3.1 by z^n ,summing over n and adding the result to eq. (3.2) and again using (A), we get

$$\frac{d}{dx}R_q^{(1)}(x,z) + \big(\lambda\alpha - \lambda\alpha B(z) + \psi_1(x)\big)R_q^{(1)}(x,z)(x,z) =\ 0 \tag{4.1}$$

Similarly,

$$\frac{d}{dx}R_q^{(2)}(x,z) + \big(\lambda\alpha - \lambda\alpha B(z) + \psi_2(x)\big)R_q^{(2)}(x,z)(x,z) =\ 0 \tag{4.2}$$

$$\frac{d}{dx}K_q(x,z) + \big(\lambda\beta - \lambda\beta B(z) + \gamma(x)\big)K_q(x,z) =\ 0 \tag{4.3}$$

Next, similar operations are carried out on the boundary conditions 3.8, we getType equation here.

$$zR_q^{(1)}(0,z) = \int_0^\infty K_q(x,z)\,\gamma(x)dx + \quad \lambda\alpha B(z) - \lambda\alpha\lambda A \tag{4.4}$$

$$R_q^{(2)}(0,z) = r\int_0^\infty R_q^{(1)}(x,z)\psi_1\,(x)dx \tag{4.5}$$

$$S_q(0,z) = \varphi\int_0^\infty R_q^{(1)}(x,z)\psi_1\,(x)dx + \int_0^\infty R_q^{(2)}(x,z)\psi_2\,(x)dx \tag{4.6}$$

Now we integrate equations 4.1 – 4.3 between the limits 0 and x.

$$R_q^{(1)}(x,z)\ =\ Q_q^{(1)}(0,z)\,e^{(\alpha\lambda B(z)-\lambda\alpha)x-\int_0^x\psi_1(t)dt} \tag{4.7}$$

$$R_q^{(2)}(x,z) = R_q^{(2)}(0,z)\, e^{(\alpha\lambda B(z)-\lambda\alpha)x-\int_0^x \psi_2(t)dt} \quad (4.8)$$

$$K_q(x,z) = K_q(0,z)\, e^{(\alpha\lambda B(z)-\lambda\alpha)x-\int_0^x \gamma(t)dt} \quad (4.9)$$

$K_q^{(1)}(0,z)$, $Q_q^{(2)}(0,z)$, $S_q(0,z)$ are given in equations (4.4) - (4.6)

Next we integrate 4.7 – 4.9 with respect to x, by parts, we get

$$R_q^{(1)}(z) = R_q^{(1)}(0,z)\frac{1-\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)}{-\alpha\lambda B(z)+\lambda\alpha} \quad (4.10)$$

$$R_q^{(2)}(z) = R_q^{(2)}(0,z)\frac{1-\bar{S}(-\alpha\lambda B(z)+\lambda\alpha)}{-\alpha\lambda B(z)+\lambda\alpha} \quad (4.11)$$

$$K_q(z) = K_q(0,z)\frac{1-\bar{Q}((-\beta\lambda B(z)+\lambda\beta))}{-\beta\lambda B(z)+\lambda\beta} \quad (4.12)$$

Where $\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)$, $\bar{S}(-\alpha\lambda B(z)+\lambda\alpha)$, $\bar{Q}((-\beta\lambda B(z)+\lambda\beta)$ are the Laplace Stieltjes transform first stage of service time, second stage of service time and compulsory vacation respectively..

To find $\int_0^\infty R_q^{(1)}(x,z)\psi_1(x)dx$, $\int_0^\infty R_q^{(2)}(x,z)\psi_2(x)dx$ and $\int_0^\infty K_q(x,z)\,\gamma(x)dx$

For this purpose, we multiply the equations 4.7 – 4.9 by $\psi_1(x),\psi_2(x)$ and $\gamma(x)$ respectively and integrate each with respect to x. Hence we get

$$\int_0^\infty R_q^{(1)}(x,z)\psi_1(x)dx = R_q^{(1)}(0,z)\bar{M}(-\alpha\lambda B(z)+\lambda\alpha) \quad (4.13)$$

$$\int_0^\infty R_q^{(2)}(x,z)\psi_2(x)dx = R_q^{(2)}(0,z)\ \bar{S}(-\alpha\lambda B(z)+\lambda\alpha) \quad (4.14)$$

$$\int_0^\infty K_q(x,z)\,\gamma(x)dx = K_q(0,z)\bar{Q}((-\beta\lambda B(z)+\lambda\beta)) \quad (4.15)$$

Using equations 4.13 -4.15 into equations 4.4 – 4.6 and further applying the equations 4.10 – 4.12, we get

$$R_q^{(1)}(z) = \frac{(\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)-1)A}{z-\left[\varphi\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)+r\bar{S}\left(-\alpha\lambda B(z)+\lambda\alpha\right)\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)\right)\right]\bar{Q}\left((-\beta\lambda B(z)+\lambda\beta)\right)} \quad (4.16)$$

$$R_q^{(2)}(z) = \frac{rA(\bar{S}(-\alpha\lambda B(z)+\lambda\alpha)-1)\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)}{z-\left[\varphi\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)+r\bar{S}\left(-\alpha\lambda B(z)+\lambda\alpha\right)\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)\right)\right]\bar{Q}\left((-\beta\lambda B(z)+\lambda\beta)\right)} \quad (4.17)$$

$$K_q(z) = \frac{-\left[1-\bar{Q}\left((-\beta\lambda B(z)+\lambda\beta)\right)\right](-\alpha\lambda B(z)+\lambda\alpha)[\varphi+r\bar{S}(-\alpha\lambda B(z)+\lambda\alpha)-1)\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)\bar{Q}\left((-\beta\lambda B(z)+\lambda\beta)\right)}{\{z-\left[\varphi\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)+r\bar{S}\left(-\alpha\lambda B(z)+\lambda\alpha\right)\bar{M}(-\alpha\lambda B(z)+\lambda\alpha)\right)\right]\bar{Q}\left((-\beta\lambda B(z)+\lambda\beta)\right)\}(-\beta\lambda B(z)+\lambda\beta)} \quad (4.18)$$

Let $W_q(z) = R_q^{(1)}(z) + R_q^{(2)}(z) + K_q(z)$ be the probability generating function of the queue size.

To determine the idle time A

Using the normalizing condition A + $W_q(1)$ = 1, we get A. (4.19)

For this purpose we apply the following steps:

Eq.3.27 is indeterminate of the form $\frac{0}{0}$ at z = 1.Hence L Hopital's rule is applied. As a result ,

$$W_q(1) = \frac{\left(-2\lambda\beta\, E(I)\right)\left[\left(\lambda\alpha\, E(I)E(M)\right)+\left(r\alpha\lambda\, E(I)E(S)\right)+2\lambda^2\,\alpha\beta\left(E(I)\right)^2(\varphi+r)E(Q)\right.}{-\lambda\beta\, E(I(I-1)[1-(\beta+r)][1-(\lambda\beta\alpha\, E(I)E(M)+r\lambda\alpha\, E(I)(E(M)+E(S))-(\varphi+r)E(Q)\lambda\beta\, E(I)]\lambda\beta\, E(I)} \quad (4.20)$$

Where E(I) and E(I(I-1)) are the first and second moment of the bulk arrival. $E(M), E(S)$ are the mean service time of the first stage and second stage of service and E(Q) is the mean vacation time.

Applying (3.29) in (3.28), we get

$$A = \frac{\begin{array}{c}-\lambda\beta\, E(I(I-1)[1-(\beta+r)][1-(\lambda\beta\alpha\, E(I)E(M)+r\lambda\alpha\, E(I)(E(M)+E(S)) \\ -(\varphi+r)E(Q)\lambda\beta\, E(I)]\lambda\beta\, E(I)\end{array}}{\begin{array}{c}-\lambda\beta\, E(I(I-1)[1-(\beta+r)][1-(\lambda\beta\alpha\, E(I)E(M)+r\lambda\alpha\, E(I)(E(M)+E(S)) \\ -(\varphi+r)E(Q)\lambda\beta\, E(I)]\lambda\beta\, E(I)+ \\ (-2\lambda\beta\, E(I))[(\lambda\alpha\, E(I)E(M))+(r\alpha\lambda\, E(I)E(S))+ \\ 2\lambda^2\,\alpha\beta(E(I))^2(\varphi+r)E(Q)\end{array}} \tag{4.21}$$

Also we obtain the utilization factor ρ using the relation ρ = 1- A

i.e

$$\rho = \frac{\begin{array}{c}(-2\lambda\beta\, E(I))[(\lambda\alpha\, E(I)E(M))+(r\alpha\lambda\, E(I)E(S))+ \\ 2\lambda^2\,\alpha\beta(E(I))^2(\varphi+r)E(Q)\end{array}}{\begin{array}{c}-\lambda\beta\, E(I(I-1)[1-(\beta+r)][1-(\lambda\beta\alpha\, E(I)E(M)+r\lambda\alpha\, E(I)(E(M)+E(S)) \\ -(\varphi+r)E(Q)\lambda\beta\, E(I)]\lambda\beta\, E(I)+ \\ (-2\lambda\beta\, E(I))[(\lambda\alpha\, E(I)E(M))+(r\alpha\lambda\, E(I)E(S))+ \\ 2\lambda^2\,\alpha\beta(E(I))^2(\varphi+r)E(Q)\end{array}} \tag{4.22}$$

V. Steady State Mean Queue Size at a Random Epoch

Let L_q denote the steady state mean queue size at a random epoch.

Then using the Tauberian property,we have $L_q = \frac{d}{dq} W_q(z)_{z=1}$

Since $D_q(z)$ is indeterminate of the form $\frac{0}{0}$ at z = 1, we apply the following formula:

$$L_q = \lim_{z\to 1} \frac{D''N''' - N''D'''}{2(D'')^2} \tag{5.1}$$

Where double and triple primes denote the second and third order derivatives as follows:

$$D'' = \begin{array}{c}-\lambda\beta\, E(I(I-1)[1-(\beta+r)][1-(\lambda\beta\alpha\, E(I)E(M)+r\lambda\alpha\, E(I)(E(M)+E(S)) \\ -(\varphi+r)E(Q)\lambda\beta\, E(I)]\lambda\beta\, E(I)\end{array}$$

$N''' = -\lambda^2\,\alpha\beta\, \mathrm{E}(I(I-1)E(I)(E(M)+rE(S)) - \lambda\beta\, E(I(I-1)[\lambda\alpha\, E(I)E(M)+r\lambda\alpha\, E(I)E(S))] - 2(\lambda\beta\, E(I))[\alpha\lambda E(I(I-1)E(M)$
$+(\lambda\alpha\, E(I))^2\left((E(M)E(S)2+E(M^2))+r\lambda\alpha\, E(I)E(S^2)\right] + 2\lambda^2 E(I)E(Q)E(I(I-1))((r+\varphi)\lambda\beta + 2$
$(\lambda\alpha\, E(I))^2\, \lambda\beta\, E(I)rE(Q)E(S) + (r+\varphi)(\lambda\alpha\, E(I))^2 E(Q)E(M) - (\lambda\alpha\, E(I))^2\, \lambda\beta\, E(I)(E(Q))^2$ $+2$
$\lambda^2 E(I)E(I(I-1))E(Q)(\varphi+r)\alpha\beta$

$N'' = (-2\lambda\beta\, E(I))[(\lambda\alpha\, E(I)E(M)) + (r\alpha\lambda\,(E(I)+E(S)) + 2\lambda^2\,\alpha\beta(E(I))^2(\varphi+r)E(Q)$

$D''' = (-\lambda\beta\, E(I(I-1)\ )[1\text{-}\ \varphi\lambda\alpha\, E(I)E(M)$ $+\mathrm{r}\ \lambda\alpha\, E(I)(E(M)+E(S)) - (r+\varphi\,)\lambda\beta\, E(I)E(Q)] -$
$(2\lambda\beta\, E(I))[-\varphi E(M^2\ )\ (\lambda\alpha\, E(I))^2 + E(I(I-1)\varphi\alpha\, E(M) + (r\,\lambda\beta + r\lambda\alpha\, E(Q))\, E(I)(E(M)+E(S))$
$+r(\lambda\alpha\, E(I))^2[E(S^2)+E(M^2)] + (\lambda^2\alpha\,\beta E(I))^2(\varphi(E(M)+r((E(M)+E(S))E(Q) - (\lambda\beta\, E(I))^2(r+$
$\varphi)E(Q^2) + (r+\varphi)\lambda\beta\, E(I(I-1)]$ (5.2)

Substituting (5.2) in (5.1) , we obtain L_q in a closed form. Further using L_q in Little's formula we obtain the other required performance measures as follows:

Average number of customers in the system $L = L_q + \rho$ where ρ is given by eq.4.22

Average waiting time in the queue $W_q = \frac{L_q}{\lambda}$ Average waiting time in the system $W = \frac{L}{\lambda}$

VI. CONCLUSION

In this paper, we have studied a bulk arrival queuing model with two stages of service in which second stage is optional. The first service is essential and the second service is optional. Based on the interest of the customers, the choice of optional service can be selected. Moreover the important concept of restricted admissibility is introduced in this model. The compulsory vacation is followed after the completion of stages of service in this model by the server. It helps the server to continue with the process of maintenance activities during the time of working vacation. The probability generating functions of the number of customers in the system is obtained by using the supplementary variable technique. The performance measures of the model are derived. This model finds prospective application in manufacturing industries and communication networks. As a future work, other parameters like standby server, phases of vacation, reneging and balking can also be included in this model.

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