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Fuzzy Mapping and their Fixed Point Theorems in Complete Metric Spaces

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ABSTRACT

In this paper we establish some fixed point theorems of fuzzy mapping in complete metric spaces using new fuzzy contraction mappings.

Keywords: Fuzzy Mappings, Fixed Point, Membership Function, Metric Linear Space

I. INTRODUCTION

In 1981, Heilpern [2] introduced the concept of fuzzy mappings and proved some fixed point theorems for fuzzy contraction mappings. Since then, many authors have generalized Bose and Sahanis [4] (1987) results in different directions. Also Bose and mukherjee [1] (1977) considered common fixed points of a pair of multivalued mappings and a sequence of single valued mappings. We extent and solved some fixed point theorem of fuzzy mapping in complete metric space which are generalizations of the earlier results in the literature.

II. PRELIMINARIES

Definition 1.1 : Let (X, d) be a metric linear space and F(X) , the collection of all fuzzy sets in X. Let A∈ F(X) and α ∈ [0, 1]. The α-level set of A, denoted by A_α, and $A_\alpha = \{x : A(x) \geq \alpha\}$ if $\alpha \in (0,1]$
$A_0 =\overline{\{x : A(x) > 0\}}$ where $\bar{A}$ stands for the (non fuzzy) closure of A.

Definition 1.2 :A fuzzy subset A of X is an approximate quantity iff its $sup_{x\epsilon X}$ A(x) = 1. From the collection F(X), a sub collection of approximate quantities is denoted as W(X).

Definition 1.3 : The distance between two approximate quantities is defined by the following scheme.
Let A, B ∈ W(X), and $\alpha \epsilon$ [0,1];
D_α (A,B) = $inf_{x\epsilon A_\alpha, y\in B_\alpha,}$ d(x, y); and D_α is a non decreasing function of α.
H_α(A,B) = dist(A_α,B_α);
H(A,B) = $sup_a D_\alpha$(A,B);
Where in the dist is in the sense of Hausdorff distance.
The function D_α is called a α −distance (induced by d), H_α is called a a-distance (induced by dist), and H a distance between A and B.

Definition 1.4 : Let A, B ∈ W(X). An approximate quantity A is more accurate than B, denoted by A ⊂ B iff A(x) ≤ B(x), for each x ∈ X. It is clear that ⊂ is a partial order relation determined on the family W(X).

Definition 1.5 :Let Y be an arbitrary set and X any metric linear space. F is called fuzzy mapping iff F is a mapping from the set Y into W(X), that is F(y) ∈ W(X) for each y ∈ Y and the function value F(y, x) stands for the grade of membership of x in F(y).
Let A∈ F(Y), B ∈ F(X). Then the fuzzy set F(A) in F(X) is defined by F(A)(x) = $sup_{y\epsilon Y}$ (F(y, x) ^ A(y)), x ∈ X and the fuzzy set F^{-1} (B) in F(Y) is defined by F^{-1}(B)(y) = $sup_{x\epsilon X}$ (F(y,x) ^ B(x)), y ϵ Y

Lemma 2.1. [2]. Let x ∈ X , A ∈ W(x) and $\{x\}$ a fuzzy set with membership function equal to a characteristic function of $\{x\}$. If $\{x\}$⊂ A, then D_α(x,A) = 0 for each α ∈ [0, 1].

Lemma 2.2. [2]: D_α (x,A) ≤ $d(x,y)$ + D_α (y,A) for each x, y ∈ X.

Lemma2.3.[2]:If $\{x_0\}$ ⊂ A, then
$D_\alpha(x_0$,B) ≤ H_α(A,B) for each B ∈ W(x).

III. MAIN RESULTS

***Theorem 3.1*:** Let X be a complete metric linear space and T be fuzzy mapping from X to W(x) satisfying

$$H\big(T(x),T(y)\big) \le a_1 \frac{D_\alpha\big(x,T(x)\big)\, D_\alpha\big(y,T(y)\big) + D_\alpha\big(x,T(y)\big)\, D_\alpha\big(y,T(x)\big)}{D_\alpha(x,y)}$$
$$+a_2 \frac{D_\alpha\big(x,T(y)\big)\big[D_\alpha\big(x,T(x)\big) + D_\alpha\big(y,T(y)\big)\big]}{D_\alpha(x,y)\, D_\alpha\big(y,T(y)\big) + D_\alpha\big(y,T(x)\big)}$$
$$+a_3 \frac{D_\alpha\big(x,T(x)\big)\, D_\alpha\big(y,T(x)\big) + D_\alpha\big(y,T(y)\big)\, D_\alpha\big(x,T(y)\big)}{D_\alpha\big(x,T(x)\big)\, D_\alpha\big(y,T(x)\big) + D_\alpha\big(y,T(y)\big)\, D_\alpha\big(x,T(y)\big)} + a_4\big[D_\alpha\big(x,T(x)\big) + D_\alpha\big(y,T(y)\big)\big]$$
$$+a_5\big[D_\alpha\big(y,T(x)\big) + D_\alpha\big(x,T(y)\big)\big] + a_6\, D_\alpha(x,y)$$

for all x $x,y \in X, x \ne y$ and for $a_1,a_2,a_3,a_4,a_5,a_6 \in [0,1]$ and $2a_1 + a_3 + 4a_4 + 4a_5 + 2a_6 < 2$, and $a_3 + 2a_4 + 2a_5 < 2$. There exists a point $z \in X$, such that $\{z\} \subset T(z)$.

Proof.

Take $x_0 \in$ X. Let $\{x_1\} \subset T(x_0)$, choose x_2 such that $\{x_2\} \subset T(x_1)$, and
$d(x_1,x_2) \le H_1(T(x_0), T(x_1))$.
Continuing this process we construct a sequence $\{x_n\}$ such that $\{x_{n+1}\} \subset T(x_n)$ and
$d(x_{n+1},x_{n+2}) \le H_1(T(x_n),T(x_{n+1}))$ n = 0, 1, 2….

$$d(x_n,x_{n+1}) \le H_1(T(x_n),T(x_{n-1}))$$
$$\le H(T(x_n),T(x_{n-1}))$$
$$\le a_1 \frac{D_\alpha\big(x_n,T(x_n)\big)\, D_\alpha\big(x_{n-1},T(x_{n-1})\big) + D_\alpha\big(x_n,T(x_{n-1})\big)\, D_\alpha\big(x_{n-1},T(x_n)\big)}{D_\alpha(x_n,x_{n-1})}$$
$$+a_2 \frac{D_\alpha\big(x_n,T(x_{n-1})\big)\big[D_\alpha\big(x_n,T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_{n-1})\big)\big]}{D_\alpha(x_n,x_{n-1}) + D_\alpha\big(x_{n-1},T(x_{n-1})\big) + D_\alpha\big(x_{n-1},T(x_n)\big)}$$
$$+a_3 \frac{D_\alpha\big(x_n,T(x_n)\big)\, D_\alpha\big(x_{n-1},T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_{n-1})\big)\, D_\alpha\big(x_n,T(x_{n-1})\big)}{D_\alpha\big(x_n,T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_{n-1})\big) + D_\alpha\big(x_n,T(x_{n-1})\big)}$$
$$+a_4\big[D_\alpha\big(x_n,T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_{n-1})\big)\big] + a_5\big[D_\alpha\big(x_{n-1},T(x_n)\big) + D_\alpha\big(x_n,T(x_{n-1})\big)\big]$$
$$+a_6\, D_\alpha(x_n,x_{n-1})$$

$$\le a_1 \frac{d(x_n,x_{n+1})d(x_{n-1},x_n) + d(x_n,x_n)d(x_{n-1},x_{n+1})}{d(x_n,x_{n-1})} + a_2 \frac{d(x_n,x_n)[d(x_n,x_{n+1}) + d(x_{n-1},x_n)]}{d(x_n,x_{n-1}) + d(x_{n-1},x_n) + d(x_{n-1},x_{n+1})}$$
$$+ a_3 \frac{d(x_n,x_{n+1})d(x_{n-1},x_{n+1}) + d(x_{n-1},x_n)d(x_n,x_n)}{d(x_n,x_{n+1}) + d(x_{n-1},x_{n+1}) + d(x_{n-1},x_n) + d(x_n,x_n)}$$
$$+ a_4[d(x_n,x_{n+1}) + d(x_{n-1},x_n)] + a_5[d(x_{n-1},x_{n+1}) + d(x_n,x_n)] + a_6 d(x_n,x_{n-1})$$
$$\le a_1 d(x_n,x_{n+1}) + \tfrac{a_3}{2} d(x_n,x_{n+1}) + a_4\, d(x_n,x_{n+1}) + a_4 d(x_{n-1},x_n)$$
$$+a_5\, d(x_{n-1},x_{n+1}) + a_6 d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \le \left(a_1 + \tfrac{a_3}{2} + a_4 + a_5\right) d(x_n,x_{n+1}) + (a_4 + a_5 + a_6)\, d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \le \left(\frac{a_4 + a_5 + a_6}{1 - a_1 - \frac{a_3}{2} - a_4 - a_5}\right) d(x_n,x_{n-1})$$

$d(x_n,x_{n+1}) \le \lambda\, d(x_n,x_{n-1})$ for each $n = 0, 1, 2 \ldots$

Where $\lambda = \left(\frac{a_4+a_5+a_6}{1-a_1-\frac{a_3}{2}-a_4-a_5}\right) < 1$

Hence $\{x_n\}$ is a Cauchy sequence in X. It hasa limit in X. Call it z. In the view of Lemmas (2.2) - (2.3)

$D_0(z,T(z)) \leq d(z,x_{n+2}) + H_0(T(z),T(x_{n+1}))$

$$\leq d(z,x_{n+2}) + a_1 \frac{D_0(z,T(z))\,D_0(x_{n+1},T(x_{n+1})) + D_0(z,T(x_{n+1}))\,D_0(x_{n+1},T(z))}{D_0(z,x_{n-1})}$$

$$+a_2 \frac{D_0(z,T(x_{n+1}))[D_0(z,T(z)) + D_0(x_{n+1},T(x_{n+1}))]}{D_0(z,x_{n+1}) + D_0(x_{n+1},T(x_{n+1})) + D_0(x_{n+1},T(z))}$$

$$+a_3 \frac{D_0(z,T(z))\,D_0(x_{n+1},T(z)) + D_0(x_{n+1},T(x_{n+1}))\,D_0(z,T(x_{n+1}))}{D_0(z,T(z)) + D_0(x_{n+1},T(z)) + D_0(x_{n+1},T(x_{n+1})) + D_0(z,T(x_{n+1}))}$$

$$+a_4[D_0(z,T(z)) + D_0(x_{n+1},T(x_{n+1}))] + a_5[D_0(x_{n+1},T(z)) + D_0(z,T(x_{n+1}))]$$

$$+a_6\, D_0(z,x_{n+1})$$

$$D_0(z,T(z)) \leq d(z,x_{n+2}) + a_1 \frac{D_0(z,T(z))d(x_{n+1},x_{n+2}) + d(z,x_{n+2})\left(d(x_{n+1},z) + D_0(z,T(z))\right)}{d(x_{n+1}, z)}$$

$$+a_2 \frac{d(z,x_{n+2})[D_0(z,T(z)) + d(x_{n+1},x_{n+2})]}{d(z,x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+1},z) + D_0(z,T(z))}$$

$$+a_3 \frac{D_0(z,T(z))[d(x_{n+1},z) + D_0(z,T(z))] + d(x_{n+1},x_{n+2})d(z,x_{n+2})}{D_0(z,T(z)) + d(x_{n+1},T(z)) + D_0(x,T(z)) + d(x_{n+1},x_{n+2}) + d(z,x_{n+2})}$$

$$+a_4[D_0(z,T(z)) + d(x_{n+1},x_{n+2})] + a_5[d(x_{n+1},z) + D_0(z,T(z)) + d(z,x_{n+2})]$$

$$+a_6 d(z,x_{n+1})$$

Making n $\to \infty$ we get

$D_0(z,T(z)) \leq \frac{a_3}{2} D_0(z,T(z)) + a_4 D_0(z,T(z)) + a_5 D_0(z,T(z))$

$D_0(z,T(z)) \leq \left(\frac{a_3}{2} + a_4 + a_5\right) D_0(z,T(z))$

$D_0(z,T(z)) \leq K\, D_0(z,T(z))$, where $K = \left(\frac{a_3}{2} + a_4 + a_5\right)$ and $K < 1$

Which gives $D_0(z, T(z)) = 0$.

Hence $z \subset T(z)$.

***Theorem 3.2*:** Let X be a complete metric linear space and T be fuzzy mapping from X to W(x) satisfying

$H(T(x),T(y)) \leq$

$$a_1 \left[\frac{D_\alpha(x,T(x))\,D_\alpha(y,T(y)) + D_\alpha(x,T(y))\,D_\alpha(y,T(x))}{D_\alpha(x,y)} + \frac{D_\alpha(x,T(y))[D_\alpha(x,T(x)) + D_\alpha(y,T(y))]}{D_\alpha(x,y)\,D_\alpha(y,T(y)) + D_\alpha(y,T(x))}\right]$$

$$+a_2 \frac{D_\alpha(x,T(x))\,D_\alpha(y,T(x)) + D_\alpha(y,T(y))\,D_\alpha(x,T(y))}{D_\alpha(x,T(x))\,D_\alpha(y,T(x)) + D_\alpha(y,T(y))\,D_\alpha(x,T(y))} + a_3[D_\alpha(x,T(x)) + D_\alpha(y,T(y))]$$

$$+a_4[D_\alpha(y,T(x)) + D_\alpha(x,T(y))] + a_5\, D_\alpha(x,y)$$

for all x $x, y \in X, x \neq y$ and for $a_1, a_2, a_3, a_4, a_5 \in [0,1]$ and $2a_1 + a_2 + 4a_3 + 4a_4 + 2a_5 < 2$, and $a_2 + 2a_3 + 2a_4 < 2$. There exists a point $z \in X$, such that $\{z\} \subset T(z)$.

Proof.

Take $x_0 \in$ X. Let $\{x_1\} \subset \mathrm{T}(x_0)$, choose x_2 such that $\{x_2\} \subset \mathrm{T}(x_1)$, and $\mathrm{d}(x_1, x_2) \leq H_1(\mathrm{T}(x_0), T(x_1))$.

Continuing this process we construct a sequence $\{x_n\}$ such that $\{x_{n+1}\} \subset \mathrm{T}(x_n)$ and $\mathrm{d}(x_{n+1}, x_{n+2}) \leq H_1(\mathrm{T}(x_n), T(x_{n+1}))$ n = 0, 1, 2….

$$\mathrm{d}(x_n, x_{n+1}) \leq H_1(\mathrm{T}(x_n), T(x_{n-1}))$$

$$\leq H(T(x_n), T(x_{n-1}))$$

$$\leq a_1\left[\frac{D_\alpha(x_n,T(x_n))\,D_\alpha(x_{n-1},T(x_{n-1}))+D_\alpha(x_n,T(x_{n-1}))\,D_\alpha(x_{n-1},T(x_n))}{D_\alpha(x_n,x_{n-1})}+\frac{D_\alpha(x_n,T(x_{n-1}))[D_\alpha(x_n,T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))]}{D_\alpha(x_n,x_{n-1})+D_\alpha(x_{n-1},T(x_{n-1}))+D_\alpha(x_{n-1},T(x_n))}\right]$$

$$+a_2\frac{D_\alpha(x_n,T(x_n))\,D_\alpha(x_{n-1},T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))\,D_\alpha(x_n,T(x_{n-1}))}{D_\alpha(x_n,T(x_n))+D_\alpha(x_{n-1},T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))+D_\alpha(x_n,T(x_{n-1}))}$$

$$+a_3[D_\alpha(x_n,T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))]+a_4[D_\alpha(x_{n-1},T(x_n))+D_\alpha(x_n,T(x_{n-1}))]+a_5\,D_\alpha(x_n,x_{n-1})$$

$$\leq a_1\left[\frac{d(x_n,x_{n+1})d(x_{n-1},x_n)+d(x_n,x_n)d(x_{n-1},x_{n+1})}{d(x_n,x_{n-1})}+\frac{d(x_n,x_n)[d(x_n,x_{n+1})+d(x_{n-1},x_n)]}{d(x_n,x_{n-1})+d(x_{n-1},x_n)+d(x_{n-1},x_{n+1})}\right]$$

$$+a_2\frac{d(x_n,x_{n+1})d(x_{n-1},x_{n+1})+d(x_{n-1},x_n)d(x_n,x_n)}{d(x_n,x_{n+1})+d(x_{n-1},x_{n+1})+d(x_{n-1},x_n)+d(x_n,x_n)}+a_3[d(x_n,x_{n+1})+d(x_{n-1},x_n)]$$

$$+a_4[d(x_{n-1},x_{n+1})+d(x_n,x_n)]+a_5 d(x_n,x_{n-1})$$

$$\leq a_1 d(x_n,x_{n+1})+\frac{a_2}{2}d(x_n,x_{n+1})+a_3\,d(x_n,x_{n+1})+a_3 d(x_{n-1},x_n)$$

$$+a_4\,d(x_{n-1},x_{n+1})+a_5 d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \leq \left(a_1+\frac{a_2}{2}+a_3+a_4\right)d(x_n,x_{n+1})+(a_3+a_4+a_5)\,d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \leq \left(\frac{a_3+a_4+a_5}{1-a_1-\frac{a_2}{2}-a_3-a_4}\right)d(x_n,x_{n-1})$$

$d(x_n,x_{n+1}) \leq \delta\, d(x_n,x_{n-1})$for each $n = 0, 1, 2 \ldots$

Where $\delta = \left(\frac{a_3+a_4+a_5}{1-a_1-\frac{a_2}{2}-a_3-a_4}\right) < 1$

Hence $\{x_n\}$ is a Cauchy sequence in X. It hasa limit in X. Call it z. In the view of Lemmas (2.2) - (2.3)

$$D_0(z,T(z)) \leq d(z,x_{n+2}) + H_0(\mathrm{T}(z),T(x_{n+1}))$$

$$\leq \mathrm{d}(z,x_{n+2}) +$$

$$a_1\left[\frac{D_0(z,T(z))\,D_0(x_{n+1},T(x_{n+1}))+D_0(z,T(x_{n+1}))\,D_0(x_{n+1},T(z))}{D_0(z,x_{n-1})}+\frac{D_0(z,T(x_{n+1}))[D_0(z,T(z))+D_0(x_{n+1},T(x_{n+1}))]}{D_0(z,x_{n+1})+D_0(x_{n+1},T(x_{n+1}))+D_0(x_{n+1},T(z))}\right]$$

$$+a_2\frac{D_0(z,T(z))\,D_0(x_{n+1},T(z))+D_0(x_{n+1},T(x_{n+1}))\,D_0(z,T(x_{n+1}))}{D_0(z,T(z))+D_0(x_{n+1},T(z))+D_0(x_{n+1},T(x_{n+1}))+D_0(z,T(x_{n+1}))}$$

$$+a_3[D_0(z,T(z))+D_0(x_{n+1},T(x_{n+1}))]+a_4[D_0(x_{n+1},T(z))+D_0(z,T(x_{n+1}))]$$

$$+a_5\,D_0(z,x_{n+1})$$

$$D_0(z,T(z)) \le d(z,x_{n+2})$$
$$+a_1\left[\frac{D_0(z,T(z))d(x_{n+1},x_{n+2})+d(z,x_{n+2})\left(d(x_{n+1},z)+D_0(z,T(z))\right)}{d(x_{n+1},z)}+\frac{d(z,x_{n+2})[D_0(z,T(z))+d(x_{n+1},x_{n+2})]}{d(z,x_{n+1})+d(x_{n+1},x_{n+2})+d(x_{n+1},z)+D_0(z,T(z))}\right]$$
$$+a_2\frac{D_0(z,T(z))[d(x_{n+1},z)+D_0(z,T(z))]+d(x_{n+1},x_{n+2})d(z,x_{n+2})}{D_0(z,T(z))+d(x_{n+1},T(z))+D_0(x,T(z))+d(x_{n+1},x_{n+2})+d(z,x_{n+2})}$$
$$+a_3[D_0(z,T(z))+d(x_{n+1},x_{n+2})]+a_4[d(x_{n+1},z)+D_0(z,T(z))+d(z,x_{n+2})]$$
$$+a_5 d(z,x_{n+1})$$

Making n → ∞ we get

$$D_0(z,T(z)) \le \frac{a_2}{2}D_0(z,T(z))+a_3D_0(z,T(z))+a_4D_0(z,T(z))$$
$$D_0(z,T(z)) \le \left(\frac{a_2}{2}+a_3+a_4\right)D_0(z,T(z))$$

D_0(z, T(z))≤ η D_0(z, T(z)) , where $\eta=\left(\frac{a_2}{2}+a_3+a_4\right)$ and $\eta<1$

Which gives D_0(z, T (z)) = 0.

Hence z ⊂T(z).

***Theorem 3.3*:** Let X be a complete metric linear space and T be fuzzy mapping from X to W(x) satisfying

$$H(T(x),T(y)) \le a_1\frac{D_\alpha(x,T(x))\,D_\alpha(y,T(y))+D_\alpha(x,T(y))\,D_\alpha(y,T(x))}{D_\alpha(x,y)}$$
$$+a_2\frac{D_\alpha(x,T(y))[D_\alpha(x,T(x))+D_\alpha(y,T(y))]}{D_\alpha(x,y)\,D_\alpha(y,T(y))+D_\alpha(y,T(x))}$$
$$+a_3\frac{D_\alpha(x,T(x))\,D_\alpha(y,T(x))+D_\alpha(y,T(y))\,D_\alpha(x,T(y))}{D_\alpha(x,T(x))\,D_\alpha(y,T(x))+D_\alpha(y,T(y))\,D_\alpha(x,T(y))}$$
$$+a_4[D_\alpha(x,T(x))+D_\alpha(y,T(y))+D_\alpha(y,T(x))+D_\alpha(x,T(y))]+a_5D_\alpha(x,y)$$

for all x $x,y\in X, x\ne y$ and for $a_1,a_2,a_3,a_4,a_5\in[0,1]$ and $2a_1+a_3+8a_4+2a_5<2$, and $a_3+4a_4<2$.

There exists a point $z\in X$, such that $\{z\}\subset T(z)$.

Proof.

Take x_0 ∈ X. Let $\{x_1\}$⊂T(x_0), choose x_2 such that $\{x_2\}\subset$ T(x_1), and
d(x_1,x_2) ≤ H_1(T(x_0), $T(x_1)$).
Continuing this process we construct a sequence$\{x_n\}$ such that $\{x_{n+1}\}\subset$ T(x_n) and
d(x_{n+1},x_{n+2}) ≤ H_1(T(x_n), $T(x_{n+1})$) n = 0, 1, 2….
d(x_n,x_{n+1}) ≤ H_1(T(x_n), $T(x_{n-1})$)

$$\le H(T(x_n),T(x_{n-1}))$$
$$\le a_1\frac{D_\alpha(x_n,T(x_n))\,D_\alpha(x_{n-1},T(x_{n-1}))+D_\alpha(x_n,T(x_{n-1}))\,D_\alpha(x_{n-1},T(x_n))}{D_\alpha(x_n,x_{n-1})}$$
$$+a_2\frac{D_\alpha(x_n,T(x_{n-1}))[D_\alpha(x_n,T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))]}{D_\alpha(x_n,x_{n-1})+D_\alpha(x_{n-1},T(x_{n-1}))+D_\alpha(x_{n-1},T(x_n))}$$
$$+a_3\frac{D_\alpha(x_n,T(x_n))\,D_\alpha(x_{n-1},T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))\,D_\alpha(x_n,T(x_{n-1}))}{D_\alpha(x_n,T(x_n))+D_\alpha(x_{n-1},T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))+D_\alpha(x_n,T(x_{n-1}))}$$
$$+a_4[D_\alpha(x_n,T(x_n))+D_\alpha(x_{n-1},T(x_{n-1}))\,D_\alpha(x_{n-1},T(x_n))+D_\alpha(x_n,T(x_{n-1}))]$$
$$+a_5D_\alpha(x_n,x_{n-1})$$

$$\leq a_1 \frac{d(x_n,x_{n+1})d(x_{n-1},x_n)+d(x_n,x_n)d(x_{n-1},x_{n+1})}{d(x_n,x_{n-1})} + a_2 \frac{d(x_n,x_n)[d(x_n,x_{n+1})+d(x_{n-1},x_n)]}{d(x_n,x_{n-1})+d(x_{n-1},x_n)+d(x_{n-1},x_{n+1})}$$
$$+ a_3 \frac{d(x_n,x_{n+1})d(x_{n-1},x_{n+1})+d(x_{n-1},x_n)d(x_n,x_n)}{d(x_n,x_{n+1})+d(x_{n-1},x_{n+1})+d(x_{n-1},x_n)+d(x_n,x_n)}$$
$$+ a_4[d(x_n,x_{n+1})+d(x_{n-1},x_n)d(x_{n-1},x_{n+1})+d(x_n,x_n)] + a_5 d(x_n,x_{n-1})$$

$$\leq a_1 d(x_n,x_{n+1}) + \frac{a_3}{2} d(x_n,x_{n+1}) + a_4\, d(x_n,x_{n+1}) + a_4 d(x_{n-1},x_n)$$
$$+ a_4\, d(x_{n-1},x_{n+1}) + a_5 d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \leq \left(a_1 + \frac{a_3}{2} + 2a_4\right) d(x_n,x_{n+1}) + (2a_4 + a_5)\, d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \leq \left(\frac{2a_4 + a_5}{1 - a_1 - \frac{a_3}{2} - 2a_4}\right) d(x_n,x_{n-1})$$

$d(x_n,x_{n+1}) \leq \beta\, d(x_n,x_{n-1})$for each $n = 0, 1,2 ...$

Where $\beta = \left(\frac{2a_4+a_5}{1-a_1-\frac{a_3}{2}-2a_4}\right) < 1$

Hence $\{x_n\}$ is a Cauchy sequence in X. It hasa limit in X. Call it z. In the view of Lemmas (2.2) - (2.3)

$D_0(z,T(z)) \leq d(z,x_{n+2}) + H_0(\mathrm{T}(z),T(x_{n+1}))$

$$\leq d(z,x_{n+2}) + a_1 \frac{D_0(z,T(z))\, D_0(x_{n+1},T(x_{n+1})) + D_0(z,T(x_{n+1}))\, D_0(x_{n+1},T(z))}{D_0(z,x_{n-1})}$$
$$+a_2 \frac{D_0(z,T(x_{n+1}))[\, D_0(z,T(z)) + D_0(x_{n+1},T(x_{n+1}))]}{D_0(z,x_{n+1}) + D_0(x_{n+1},T(x_{n+1})) + D_0(x_{n+1},T(z))}$$
$$+a_3 \frac{D_0(z,T(z))\, D_0(x_{n+1},T(z)) + D_0(x_{n+1},T(x_{n+1}))\, D_0(z,T(x_{n+1}))}{D_0(z,T(z)) + D_0(x_{n+1},T(z)) + D_0(x_{n+1},T(x_{n+1})) + D_0(z,T(x_{n+1}))}$$
$$+a_4[\, D_0(z,T(z)) + D_0(x_{n+1},T(x_{n+1})) + D_0(x_{n+1},T(z)) + D_0(z,T(x_{n+1}))]$$
$$+a_5\, D_0(z,x_{n+1})$$

$$D_0(z,T(z)) \leq \mathrm{d}(z,x_{n+2}) + a_1 \frac{D_0(z,T(z))d(x_{n+1},x_{n+2}) + d(z,x_{n+2})\left(d(x_{n+1},z) + D_0(z,T(z))\right)}{d(x_{n+1},\ z)}$$
$$+a_2 \frac{d(z,x_{n+2})[\, D_0(z,T(z)) + d(x_{n+1},x_{n+2})]}{d(z,x_{n+1}) + d(x_{n+1},\ x_{n+2}) + d(x_{n+1},z) + D_0(z,T(z))}$$
$$+a_3 \frac{D_0(z,T(z))[d(x_{n+1},z) + D_0(z,T(z))] + d(x_{n+1},x_{n+2})d(z,x_{n+2})}{D_0(z,T(z)) + d(x_{n+1},T(z)) + D_0(x,T(z)) + d(x_{n+1},x_{n+2}) + d(z,x_{n+2})}$$
$$+a_4[\, D_0(z,T(z)) + d(x_{n+1},x_{n+2}) + d(x_{n+1},z) + D_0(z,T(z)) + d(z,x_{n+2})]$$
$$+a_5 d(z,x_{n+1})$$

Making n $\rightarrow \infty$ we get

$$D_0(z,T(z)) \leq \frac{a_3}{2} D_0(z,T(z)) + 2a_4 D_0(z,T(z))$$
$$D_0(z,T(z)) \leq \left(\frac{a_3}{2} + 2a_4\right) D_0(z,T(z))$$

$D_0(\mathrm{z},\mathrm{T(z)}) \leq L\, D_0(\mathrm{z},\ \mathrm{T(z)})$, where $L = \left(\frac{a_3}{2} + 2a_4\right)$ and $L < 1$

Which gives $D_0(\mathrm{z},\ \mathrm{T\ (z)}) = 0$.

Hence z $\subset$T(z).

Theorem 3.4: Let X be a complete metric linear space and T be fuzzy mapping from X to W(x) satisfying

$$H\big(T(x),T(y)\big) \le a_1 \frac{D_\alpha\big(x,T(x)\big)\,D_\alpha\big(y,T(y)\big) + D_\alpha\big(x,T(y)\big)\,D_\alpha\big(y,T(x)\big)}{D_\alpha(x,y)}$$

$$+a_2 \frac{D_\alpha\big(x,T(y)\big)\big[D_\alpha\big(x,T(x)\big) + D_\alpha\big(y,T(y)\big)\big]}{D_\alpha(x,y)\,D_\alpha\big(y,T(y)\big) + D_\alpha\big(y,T(x)\big)}$$

$$+a_3 \frac{D_\alpha\big(x,T(x)\big)\,D_\alpha\big(y,T(x)\big) + D_\alpha\big(y,T(y)\big)\,D_\alpha\big(x,T(y)\big)}{D_\alpha\big(x,T(x)\big)\,D_\alpha\big(y,T(x)\big) + D_\alpha\big(y,T(y)\big)\,D_\alpha\big(x,T(y)\big)}$$

$$+a_4\big[D_\alpha\big(x,T(x)\big)+D_\alpha\big(y,T(y)\big)\big] + a_5\big[D_\alpha\big(y,T(x)\big)+D_\alpha\big(x,T(y)\big)\big]$$

for all x $x,y \in X, x \neq y$ and for $a_1,a_2,a_3,a_4,a_5 \in [0,1]$ and $2a_1 + a_3 + 4a_4 + 4a_5 < 2$, and $a_3 + 2a_4 + 2a_5 < 2$. There exists a point $z \in X$, such that $\{z\} \subset T(z)$.

Proof.

Take $x_0 \in$ X. Let $\{x_1\} \subset$T(x_0), Choose x_2 such that $\{x_2\} \subset$ T(x_1), and

d$(x_1,x_2) \le$ H_1(T(x_0), $T(x_1)$).

Continuing this process we construct a sequence$\{x_n\}$ such that $\{x_{n+1}\} \subset$ T(x_n) and

d$(x_{n+1},x_{n+2}) \le$ H_1(T$(x_n),T(x_{n+1})$) n = 0, 1, 2….

$$d(x_n,x_{n+1}) \le H_1(T(x_n),T(x_{n-1}))$$

$$\le H(T(x_n),T(x_{n-1}))$$

$$\le a_1 \frac{D_\alpha\big(x_n,T(x_n)\big)\,D_\alpha\big(x_{n-1},T(x_{n-1})\big) + D_\alpha\big(x_n,T(x_{n-1})\big)\,D_\alpha\big(x_{n-1},T(x_n)\big)}{D_\alpha(x_n,x_{n-1})}$$

$$+ a_2 \frac{D_\alpha\big(x_n,T(x_{n-1})\big)\big[D_\alpha(x_n,T(x_n)) + D_\alpha\big(x_{n-1},T(x_{n-1})\big)\big]}{D_\alpha(x_n,x_{n-1}) + D_\alpha\big(x_{n-1},T(x_{n-1})\big) + D_\alpha\big(x_{n-1},T(x_n)\big)}$$

$$+ a_3 \frac{D_\alpha\big(x_n,T(x_n)\big)\,D_\alpha\big(x_{n-1},T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_{n-1})\big)\,D_\alpha\big(x_n,T(x_{n-1})\big)}{D_\alpha\big(x_n,T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_n)\big) + D_\alpha\big(x_{n-1},T(x_{n-1})\big)+D_\alpha\big(x_n,T(x_{n-1})\big)}$$

$$+ a_4\big[D_\alpha\big(x_n,T(x_n)\big)+D_\alpha\big(x_{n-1},T(x_{n-1})\big)\big]$$

$$+a_5\big[D_\alpha\big(x_{n-1},T(x_n)\big)+D_\alpha\big(x_n,T(x_{n-1})\big)\big]$$

$$\le a_1 \frac{d(x_n,x_{n+1})d(x_{n-1},x_n) + d(x_n,x_n)d(x_{n-1},x_{n+1})}{d(x_n,x_{n-1})} + a_2 \frac{d(x_n,x_n)[d(x_n,x_{n+1}) + d(x_{n-1},x_n)]}{d(x_n,x_{n-1}) + d(x_{n-1},x_n) + d(x_{n-1},x_{n+1})}$$

$$+ a_3 \frac{d(x_n,x_{n+1})d(x_{n-1},x_{n+1}) + d(x_{n-1},x_n)d(x_n,x_n)}{d(x_n,x_{n+1}) + d(x_{n-1},x_{n+1}) + d(x_{n-1},x_n) + d(x_n,x_n)}$$

$$+ a_4[d(x_n,x_{n+1}) + d(x_{n-1},x_n)] + a_5[d(x_{n-1},x_{n+1}) + d(x_n,x_n)]$$

$$\le a_1 d(x_n,x_{n+1}) + \frac{a_3}{2} d(x_n,x_{n+1}) + a_4\, d(x_n,x_{n+1}) + a_4 d(x_{n-1},x_n) + a_5\, d(x_{n-1},x_{n+1})$$

$$d(x_n,x_{n+1}) \le \left(a_1 + \frac{a_3}{2} + a_4 + a_5\right) d(x_n,x_{n+1}) + (a_4 + a_5)\, d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \le \left(\frac{a_4 + a_5}{1 - a_1 - \frac{a_3}{2} - a_4 - a_5}\right) d(x_n,x_{n-1})$$

$d(x_n,x_{n+1}) \le S\, d(x_n,x_{n-1})$for each $n = 0,1,2$ ...

Where $S = \left(\frac{a_4+a_5}{1-a_1-\frac{a_3}{2}-a_4-a_5}\right) < 1$

Hence $\{x_n\}$ is a Cauchy sequence in X. It hasa limit in X. Call it z. In the view of Lemmas (2.2) - (2.3)

$D_0(z,T(z)) \le$ d(z,x_{n+2}) + H_0(T$(z),T(x_{n+1})$)

$$\leq \ d(z, x_{n+2}) + a_1 \frac{D_0(z, T(z))\, D_0(x_{n+1}, T(x_{n+1})) + \ D_0(z, T(x_{n+1}))\, D_0(x_{n+1}, T(z))}{D_0(z, x_{n-1})}$$
$$+a_2 \frac{D_0(z, T(x_{n+1}))[\, D_0(z, T(z)) + \ D_0(x_{n+1}, T(x_{n+1}))]}{D_0(z, x_{n+1}) + D_0(x_{n+1}, T(x_{n+1})) + \ D_0(x_{n+1}, T(z))}$$
$$+a_3 \frac{D_0(z, T(z))\, D_0(x_{n+1}, T(z)) + \ D_0(x_{n+1}, T(x_{n+1}))\, D_0(z, T(x_{n+1}))}{D_0(z, T(z)) + D_0(x_{n+1}, T(z)) + \ D_0(x_{n+1}, T(x_{n+1})) + D_0(z, T(x_{n+1}))}$$
$$+a_4[\, D_0(z, T(z)) + D_0(x_{n+1}, T(x_{n+1}))] + a_5[\, D_0(x_{n+1}, T(z)) + D_0(z, T(x_{n+1}))]$$

$$D_0(z, T(z)) \ \leq \ d(z, x_{n+2}) + a_1 \frac{D_0(z, T(z)) d(x_{n+1}, x_{n+2}) + d(z, x_{n+2}) \left(d(x_{n+1}, z) + \ D_0(z, T(z))\right)}{d(x_{n+1}, \ z)}$$
$$+a_2 \frac{d(z, x_{n+2})[\, D_0(z, T(z)) + d(x_{n+1}, x_{n+2})]}{d(z, x_{n+1}) + d(x_{n+1}, \ x_{n+2}) + d(x_{n+1}, z) + D_0(z, T(z))}$$
$$+a_3 \frac{D_0(z, T(z))[d(x_{n+1}, z) + \ D_0(z, T(z))] + d(x_{n+1}, x_{n+2}) d(z, x_{n+2})}{D_0(z, T(z)) + d(x_{n+1}, T(z)) + \ D_0(x, T(z)) + d(x_{n+1}, x_{n+2}) + d(z, x_{n+2})}$$
$$+a_4[\, D_0(z, T(z)) + d(x_{n+1}, x_{n+2})] + a_5[d(x_{n+1}, z) + D_0(z, T(z)) + d(z, x_{n+2})]$$

Making n → ∞ we get

$$D_0(z, T(z)) \ \leq \frac{a_3}{2} D_0(z, T(z)) + a_4 D_0(z, T(z)) + a_5 D_0(z, T(z))$$
$$D_0(z, T(z)) \ \leq \left(\frac{a_3}{2} + a_4 + a_5\right) D_0(z, T(z))$$

D_0(z, T(z))≤ R D_0(z, T(z)) , where $R = \left(\frac{a_3}{2} + a_4 + a_5\right)$ and $R < 1$

Which gives D_0(z, T (z)) = 0.

Hence z ⊂T(z).

IV. CONCLUSION

We have proved some fixed point theorems of fuzzy function in complete metric spaces. The presented results generalize the results proved in various spaces and extend some results from the literature.

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