© 2017 IJSRSET | Volume 3 | Issue 8| Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section: Engineering and Technology

Properties of Lower Level Subsets of Intuitionistic Anti L-Fuzzy M-Subgroups

P. Pandiammal[*1], Nivetha Martin[2]
[*1]Department of Mathematics, G T N Arts College (Autonomous), Dindigul, Tamil Nadu, India
[2]Department of Mathematics, Arul Anandar College (Autonomous), Karumathur, Madurai, Tamil Nadu, India

ABSTRACT

In this paper, we introduce the concept of lower level subsets of Intuitionistic Anti L-fuzzy M- subgroups and investigate some related properties.

Keywords: Intuitionistic Fuzzy Subsets; Intuitionistic Anti Fuzzy Subgroups; Intuitionistic Anti L-Fuzzy M-Subgroups; Intuitionistic Anti Fuzzy Characteristic.

I. INTRODUCTION

A fuzzy set theory has developed in many directions and finding application in a wide variety of fields. Zadeh's classical paper [21] of 1965 introduced the concepts of fuzzy sets and fuzzy set operations. The study of fuzzy groups was started by Rosenfeld [17] and it was extended by Roventa [18] who have introduced the concept of fuzzy groups operating on fuzzy sets and many researchers [1,7,9,10] are engaged in extending the concepts. The concept of intuitionistic fuzzy set was introduced by Atanassov. K.T [2,3], as a generalization of the notion of fuzzy sets. Choudhury. F.P et al [6] defined a fuzzy subgroup and fuzzy homomorphism. **Palaniappan. N** and **Muthuraj,** [11] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. **Pandiammal. P, Natarajan. R** and **Palaniappan. N,** [13] defined the homomorphism, anti-homomorphism of an anti L-fuzzy M-subgroups, **Pandiammal. P,** [14] defined Intuitionistic Anti L-fuzzy M- subgroups of M-groups, **Pandiammal. P,** [15] defined Intuitionistic Anti L-fuzzy Normal M-subgroups of M-groups. In this paper we introduce and discuss the algebraic properties of lower level subsets of Intuitionistic Anti L-fuzzy M-subgroups of M-group with operator and obtain some related results.

II. PRELIMINARIES

2.1 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be **anti L-fuzzy M-subgroup** (ALFMSG) of G if its satisfies the following axioms:

(i) $\mu_A(mxy) \le \mu_A(x) \vee \mu_A(y)$,
(ii) $\mu_A(x^{-1}) \le \mu_A(x)$,

for all x and y in G.

2.2 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an **intuitionistic L-fuzzy M-subgroup (ILFMSG)** of G if the following conditions are satisfied:

(i) $\mu_A(mxy) \ge \mu_A(x) \wedge \mu_A(y)$,
(ii) $\mu_A(x^{-1}) \ge \mu_A(x)$,
(iii) $\nu_A(mxy) \le \nu_A(x) \vee \nu_A(y)$,
(iv) $\nu_A(x^{-1}) \le \nu_A(x)$,

for all x and y in G.

2.3 Definition: Let (G,·) and (G',·) be any two M-groups. Let f : G → G' be any function and A be an intuitionistic L-fuzzy M-subgroup in G, V be an intuitionistic L-fuzzy M-subgroup in f (G) = G', defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in G and y in G^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2.4 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB = { 〈 (x, y), μ_{AxB}(x, y) , ν_{AxB}(x, y) 〉 / for all x in G and y in H }, where $\mu_{AxB}(x, y) = \mu_A(x) \wedge \mu_B(y)$ and $\nu_{AxB}(x, y) = \nu_A(x) \vee \nu_B(y)$.

2.5 Definition: Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **conjugate intuitionistic L-fuzzy M-subgroups** of G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$ and $\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

2.6 Definition: Let A be an intuitionistic L-fuzzy subset in a set S, the **strongest intuitionistic L-fuzzy relation** on S, that is an intuitionistic L-fuzzy relation on A is V given by $\mu_V(x, y) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_V(x, y) = \nu_A(x) \vee \nu_A(y)$, for all x and y in S.

2.7 Definition: Let A be a L-fuzzy subset of X. For t in L, the lower level subset of A is the set, $A_t = \{ x \in X : \mu_A(x) \leq t \}$. This is called an **anti L-fuzzy lower level subset** of A.

2.8 Definition: Let A be an intuitionistic L-fuzzy subset of X. For α and β in L, the (α , β)-level subset of A is the set $A_{(\alpha, \beta)} = \{x \in X : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}$. This is called an **intuitionistic L-fuzzy level subset** of A.

III. LOWER LEVEL SUBSETS OF INTUITIONISTIC ANTI L-FUZZY M-SUBGROUPS

3.1 Definition: An intuitionistic fuzzy subset μ in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following axioms are satisfied.

(i) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$,(ii) $\mu_A(x^{-1}) \leq \mu_A(x)$, (iii)$\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$, (iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x and y in G.

3.2 Proposition: Let G be a group. An intuitionistic fuzzy subset μ in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following conditions are satisfied.

(i) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, (ii) $\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$, for all x ,y in G.

3.3 Definition: Let G be an M-group and μ be an intuitionistic anti fuzzy group of G. If $\mu_A(mx) \leq \mu_A(x)$ and $\nu_A(mx) \geq \nu_A(x)$ for all x in G and m in M then μ is said to be an intuitionistic anti fuzzy subgroup with operator of G. We use the phrase μ is an **intuitionistic anti L-fuzzy M-subgroup** of G.

3.4 Example: Let H be M-subgroup of an M-group G and let A = (μ_A,ν_A) be an intuitionistic fuzzy set in G defined by

$$\mu_A(x) = \begin{cases} 0.3 ; & x \in H \\ 0.5; & \text{otherwise} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.6; & x \in H \\ 0.3; & \text{otherwise} \end{cases}$$

for all x in G. Then it is easy to verify that A = (μ_A,ν_A) is an anti fuzzy M- subgroup of G.

3.5 Definition: Let A and B be any two intuitionistic anti L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **conjugate intuitionistic anti L-fuzzy M-subgroups** of G if for some g in G, $\mu_A(x)= \mu_B(g^{-1}xg)$ & $\nu_A(x)=\nu_B(g^{-1}xg)$, for every x in G.

3.6 Proposition: If μ = (δμ , λμ) is an intuitionistic anti fuzzy M-subgroup of an M- group G, then for any x, y∈ G and m ∈ M.

(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,
(ii) $\mu_A(mx^{-1}) \leq \mu_A(x)$ and
(iii) $\nu_A(mxy) \geq \nu_A(x) \wedge \nu_A(y)$,
(iv) $\nu_A(mx^{-1}) \leq \nu_A(x)$, for all x and y in G.

3.7 Theorem: A is an intuitionistic anti L-fuzzy M-subgroup of a M-group (G, ·) if and only if $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x & y in G.

3.8 Definition: Let A be an Intuitionistic Anti L-fuzzy subset of X. For α and β in L, the (α , β)-level subset of A is the set $A_{(\alpha, \beta)} = \{x \in X : \mu_A(x) \leq \alpha \text{ and } \nu_A(x) \geq \beta\}$. This is called an **Intuitionistic Anti L-fuzzy level subset** of A.

IV. PROPERTIES OF INTUITIONISTIC ANTI L-fuzzy LEVEL SUBSETS

4.1 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Then for α and β in L such that $\alpha \leq \mu_A(e)$ and $\beta \geq \nu_A(e)$, $A_{(\alpha, \beta)}$ is a M-subgroup of G, where e is the identity element of G.

Proof: For all x and y in $A_{(\alpha, \beta)}$, we have, $\mu_A(x) \leq \alpha$ and $\nu_A(x) \geq \beta$ and $\mu_A(y) \leq \alpha$ and $\nu_A(y) \geq \beta$.

Now, $\mu_A(mxy^{-1}) \leq \mu_A(x) \wedge \mu_A(y)$, (as A is an IALFMSG of a M-group G) $\leq \alpha \wedge \alpha = \alpha$,

which implies that, $\mu_A(mxy^{-1}) \leq \alpha$.
And also, $\nu_A(mxy^{-1}) \geq \nu_A(x) \vee \nu_A(y)$, (as A is an IALFMSG of a M-group G) $\geq \beta \vee \beta = \beta$,
which implies that, $\nu_A(mxy^{-1}) \geq \beta$.

Therefore, $\mu_A(mxy^{-1}) \leq \alpha$ and $\nu_A(mxy^{-1}) \geq \beta$, we get mxy^{-1} in $A_{(\alpha, \beta)}$.
Hence $A_{(\alpha, \beta)}$ is a M-subgroup of a M-group G.

4.2 Definition: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G. The level M-subgroup $A_{(\alpha, \beta)}$, for α and β in L such that $\alpha \geq \mu_A(e)$ and $\beta \leq \nu_A(e)$ is called an **intuitionistic anti L-fuzzy level M-subgroup** of A.

4.3 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Then two intuitionistic anti L-fuzzy level M-subgroups $A_{(\alpha_1,\beta_1)}$, $A_{(\alpha_2,\beta_2)}$ and α_1 and α_2 in L, β_1 and β_2 in L and $\alpha_1 \geq \mu_A(e)$, $\alpha_2 \geq \mu_A(e)$, $\beta_1 \leq \nu_A(e)$ and $\beta_2 \leq \nu_A(e)$ with $\alpha_1 < \alpha_2$ and $\beta_2 < \beta_1$ of A are equal iff there is no x in G such that $\alpha_1 < \mu_A(x)$, $\mu_A(x) < \alpha_2$, $\beta_1 > \nu_A(x)$ and $\nu_A(x) > \beta_2$, where e is the identity element of G.

Proof: Assume that $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.
Suppose there exists x in G such that $\alpha_1 < \mu_A(x)$, $\mu_A(x) < \alpha_2$, $\beta_1 > \nu_A(x)$ and $\nu_A(x) > \beta_2$.
Then $A_{(\alpha_1,\beta_1)} \subseteq A_{(\alpha_2,\beta_2)}$, which implies that x belongs to $A_{(\alpha 2, \beta 2)}$, but not in $A_{(\alpha_1,\beta_1)}$.
This is contradiction to $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.
Therefore there is no x in G such that $\alpha_1 < \mu_A(x)$, $\mu_A(x) < \alpha_2$, $\beta_1 > \nu_A(x)$ and $\nu_A(x) > \beta_2$.

Conversely,
if there is no x in G such that $\alpha_1 < \mu_A(x)$, $\mu_A(x) < \alpha_2$, $\beta_1 > \nu_A(x)$ and $\nu_A(x) > \beta_2$.
Then $A_{(\alpha_2,\beta_1)} = A_{(\alpha_2,\beta_2)}$.

4.4 Theorem: Let G be a M-group and A be an intuitionistic anti L-fuzzy subset of G such that $A_{(\alpha, \beta)}$ be a M-subgroup of G. If α and β in L satisfying $\alpha \geq \mu_A(e)$ and $\beta \leq \nu_A(e)$, then A is an intuitionistic anti L-fuzzy M-subgroup of G, where e is the identity element in G.

Proof: Let G be a M-group. For x and y in G and m in M.
Let $\mu_A(x) = \alpha_1$ and $\mu_A(y) = \alpha_2$, $\nu_A(x) = \beta_1$ and $\nu_A(y) = \beta_2$.

Case (i):
If $\alpha_1 < \alpha_2$ and $\beta_1 > \beta_2$, then x and y in $A_{(\alpha 1, \beta 1)}$.
As $A_{(\alpha_1,\beta_1)}$ is a level M-subgroup of G, xy^{-1} in $A_{(\alpha_1,\beta_1)}$.
Now, $\mu_A(mxy^{-1}) \leq \alpha_1 = \alpha_1 \vee \alpha_2 = \mu_A(x) \vee \mu_A(y)$,
which implies that $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.
And , $\nu_A(mxy^{-1}) \geq \beta_1 = \beta_1 \vee \beta_2 = \nu_A(x) \wedge \nu_A(y)$,
which implies that $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Case (ii):
If $\alpha_1 < \alpha_2$ and $\beta_1 < \beta_2$, then x and y in $A_{(\alpha_1,\beta_2)}$.
As $A_{(\alpha_1,\beta_2)}$ is a level M-subgroup of G, xy^{-1} in $A_{(\alpha_1,\beta_2)}$.
Now, $\mu_A(mxy^{-1}) \leq \alpha_1 = \alpha_1 \vee \alpha_2 = \mu_A(x) \vee \mu_A(y)$,
which implies that $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G. And,
$\nu_A(mxy^{-1}) \geq \beta_2 = \beta_2 \wedge \beta_1 = \nu_A(y) \wedge \nu_A(x)$,
which implies that $\nu_A(mxy^{-1}) \geq \nu_A(y) \wedge \nu_A(x)$, for all x and y in G.

Case (iii):
If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$, then x and y in $A_{(\alpha_2,\beta_1)}$.
As $A_{(\alpha_2,\beta_1)}$ is a level M-subgroup of G, xy^{-1} in $A_{(\alpha_2,\beta_1)}$.
Now, $\mu_A(mxy^{-1}) \leq \alpha_2 = \alpha_2 \vee \alpha_1 = \mu_A(y) \vee \mu_A(x)$,
which implies that $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.
And, $\nu_A(mxy^{-1}) \geq \beta_1 = \beta_1 \wedge \beta_2 = \nu_A(x) \wedge \nu_A(y)$,
which implies that $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Case (iv):
If $\alpha_1 > \alpha_2$ and $\beta_1 < \beta_2$, then x and y in $A_{(\alpha_2,\beta_2)}$.
As $A_{(\alpha_2,\beta_2)}$ is a level M-subgroup of G, xy^{-1}in $A_{(\alpha_2,\beta_2)}$.
Now,$\mu_A(mxy^{-1}) \leq \alpha_2 = \alpha_2 \vee \alpha_1 = \mu_A(y) \vee \mu_A(x)$,
which implies that $\mu_A(mxy^{-1}) \leq \mu_A(y) \vee \mu_A(x)$, for all x and y in G.
And, $\nu_A(mxy^{-1}) \geq \beta_2 = \beta_2 \wedge \beta_1 = \nu_A(y) \wedge \nu_A(x)$,
which implies that $\nu_A(mxy^{-1}) \geq \nu_A(y) \wedge \nu_A(x)$, for all x and y in G.

Case (v):
If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.
It is trivial.
In all the cases, A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.
Hence A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

4.5 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. If any two level M-subgroups of A belongs to G, then their intersection is also level M-subgroup of A in G.
Proof: For $\alpha_1$and α_2 in L, $\beta_1$and β_2 in L, α_1 and $\alpha_2 \geq \mu_A(e)$, β_1 and $\beta_2 \leq \nu_A(e)$, where e is the identity element in G.

Case (i):
If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha_2,\beta_2)} \subseteq A_{(\alpha_1,\beta_1)}$.
Therefore, $A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_2,\beta_2)}$, but $A_{(\alpha_2,\beta_2)}$ is a level M-subgroup of A.
Case(ii):
If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha_1,\beta_1)} \subseteq A_{(\alpha_2,\beta_2)}$.
Therefore, $A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_1,\beta_1)}$, but $A_{(\alpha1, \beta1)}$ is a level M-subgroup of A.
Case (iii):
If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A_{(\alpha1, \beta2)}$.
Therefore, $A_{(\alpha_2,\beta_1)} \cap A_{(\alpha1, \beta2)} = A_{(\alpha_2,\beta_1)}$, but $A_{(\alpha_2,\beta_1)}$ is a level M-subgroup of A.
Case (iv):
If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha_1,\beta_2)} \subseteq A_{(\alpha_2,\beta_1)}$.
Therefore, $A_{(\alpha_1,\beta_2)} \cap A_{(\alpha_2,\beta_1)} = A_{(\alpha_1,\beta_2)}$, but $A_{(\alpha_1,\beta_2)}$ is a level M-subgroup of A.

Case (v):
If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.
In all cases, intersection of any two levels M-subgroup is a level M-subgroup of A.

4.6 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. If α_i and β_j in L, $\alpha_i \geq \mu_A(e)$, $\beta_j \leq \nu_A(e)$ and $A_{(\alpha i, \beta j)}$, i and j in I, is a collection of level M-subgroups of A, then their intersection is also a level M-subgroup of A.
Proof: It is trivial.

4.7 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. If any two level M-subgroups of A belongs to G, then their union is also a level M-subgroup of A in G.
Proof: Let $\alpha_1$and α_2 in L, β_1 and β_2 in L, α_1 and $\alpha_2 \leq \mu_A(e)$, β_1 and $\beta_2 \geq \nu_A(e)$.
Case (i):
If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha_2,\beta_2)} \subseteq A_{(\alpha_1,\beta_1)}$.
Therefore, $A_{(\alpha_1,\beta_1)} \cup A_{(\alpha_2,\beta_2)} = A_{(\alpha_1,\beta_1)}$, but $A_{(\alpha_1,\beta_1)}$ is a level M-subgroup of A.

Case (ii):
If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha_1,\beta_1)} \subseteq A_{(\alpha_2,\beta_2)}$.
Therefore, $A_{(\alpha_1,\beta_1)} \cup A_{(\alpha_2,\beta_2)} = A_{(\alpha_2,\beta_2)}$, but $A_{(\alpha_2,\beta_2)}$ is a level M-subgroup of A.

Case (iii):
If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A_{(\alpha_1,\beta_2)}$.
Therefore, $A_{(\alpha2, \beta1)} \cup A_{(\alpha_1,\beta_2)} = A_{(\alpha_1,\beta_2)}$, but $A_{(\alpha_1,\beta_2)}$ is a level M-subgroup of A.

Case (iv):

If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha_1,\beta_2)} \subseteq A_{(\alpha_2,\beta_1)}$.

Therefore, $A_{(\alpha_1,\beta_2)} \cup A_{(\alpha_2,\beta_1)} = A_{(\alpha_2,\beta_1)}$, but $A_{(\alpha_2,\beta_1)}$ is a level M-subgroup of A.

Case (v):

If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.

In all cases, union of any two level subgroups is also a level M-subgroup of A.

4.8 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. If α_i and β_j in L, $\alpha_i \geq \mu_A(e)$ and $\beta_j \leq \nu_A(e)$ and $A_{(\alpha i, \beta j)}$, i and j in I, is a collection of level M-subgroups of A, then their union is also a level M-subgroup of A.

Proof: It is trivial.

4.9 Theorem: Any M-subgroup H of a M-group G can be realized as a level M-subgroup of some intuitionistic anti L-fuzzy M-subgroup of G.

Proof: Let A be the intuitionistic anti L-fuzzy subset of G defined by

$$\mu_A(x) = \begin{cases} \alpha & \text{if } x \in H, 0 < \alpha < 1 \\ 0 & \text{if } x \notin H \end{cases}$$ and

$$\nu_A(x) = \begin{cases} \beta & \text{if } x \in H, 0 < \beta < 1 \\ 0 & \text{if } x \notin H \end{cases}$$

and $\alpha + \beta \leq 1$, where H is M-subgroup of a M-group G.

We claim that A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

Let x and y in G.

Case (i):

If x and y in H and m in M, then mxy^{-1} in H,

Since H is a M-subgroup of G,

Therefore, $\mu_A(mxy^{-1}) = \alpha$, $\mu_A(x) = \alpha$, $\mu_A(y) = \alpha$.

So, $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

Also, $\nu_A(mxy^{-1}) = \beta$, $\nu_A(x) = \beta$, $\nu_A(y) = \beta$.

So, $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Case (ii):

If x in H, y not in H, then mxy^{-1} not in H.

Then, $\mu_A(mxy^{-1}) = 0$, $\mu_A(x) = \alpha$, $\mu_A(y) = 0$.

Therefore, $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

And $\nu_A(mxy^{-1}) = 0$, $\nu_A(x) = \beta$, $\nu_A(y) = 0$.

Therefore, $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Case (iii):

If x and y not in H, then mxy^{-1} may or may not belong to H.

Clearly $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

Also, $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

In any case, $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G .

Thus in all the cases, A is an intuitionistic anti L-fuzzy M-subgroup of G.

4.10 Theorem: Let I be the subset of L and let G be a M-group with M-subgroups $\{H_i\}$, i in I such that $\cup H_i = G$ and $i < j$ implies that $H_i \subset H_j$. Then an intuitionistic anti L-fuzzy subset A of G defined by $\mu_A(x) = \wedge\{i / x \in H_i\}$ and $\nu_A(x) = \vee\{ i / x \in H_i \}$ is an intuitionistic anti L-fuzzy M-subgroup of G.

Proof: Let A be an intuitionistic anti L-fuzzy subset of G defined by

$\mu_A(x) = \wedge\{ i / x \in H_i \}$ and $\nu_A(x) = \vee\{ i / x \in H_i \}$, where i in $I \subseteq L$.

Let x and y in G and $\mu_A(x) = m_1$ and $\mu_A(y) = n_1$.

If $\mu_A(mxy) = \wedge\{i / mxy \in H_i\} < m_1 \vee n_1$, then there exists j such that x and y are elements of H_j, but xy is not an element of H_j, since H_j is a M-subgroup of G.

This is a contradiction.

Therefore, $\mu_A(mxy) \leq m_1 \vee n_1$,

which implies that $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$.

Clearly $\mu_A(x^{-1}) = \mu_A(x)$. Also, $\nu_A(x) = m_2$ and $\nu_A(y) = n_2$.

If $\nu_A(mxy) = \wedge\{ i / mxy \in H_i \} > m_2 \wedge n_2$, then there exists j such that x and y are elements of H_j, but xy is not an element of H_j, since H_j is a subgroup of G.

This is a contradiction.

Therefore, $\nu_A(mxy) \geq m_2 \wedge n_2$,

which implies that $\nu_A(mxy) \geq \nu_A(x) \wedge \nu_A(y)$.

Clearly $\nu_A(x^{-1}) = \nu_A(x)$.

Hence A is an intuitionistic anti L-fuzzy M-subgroup of G.

4.11 Theorem: If A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G, then for each level M-subgroup $A_{(\alpha,\beta)}$, α and β in L, $\alpha \geq \mu_A(e)$ and $\beta \leq \nu_A(e)$ is a normal M-subgroup of G.

Proof: Let A be an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

Let $A_{(\alpha,\beta)}$ be any level M-subgroup of A.

To prove that $A_{(\alpha,\beta)}$ is a normal M-subgroup in G.

Let x in $A_{(\alpha,\beta)}$ and g in G and m in M.

Then, $\mu_A(mx) \leq \alpha$ and $\nu_A(mx) \geq \beta$.
Now, $\mu_A(mg^{-1}xg) = \mu_A(mxgg^{-1})$, (A is a IALFNMSG of G) $= \mu_A(mx) \leq \alpha$.
And, $\nu_A(mg^{-1}xg) = \nu_A(mxgg^{-1})$, (A is a IALFNMSG of G) $= \nu_A(mx) \geq \beta$.
Hence $\mu_A(mg^{-1}xg) \leq \alpha$ and $\nu_A(mg^{-1}xg \geq \beta$.
Therefore, $mg^{-1}xg$ in $A_{(\alpha,\beta)}$ and hence $A_{(\alpha,\beta)}$ is a normal M-subgroup of G.

4.12 Theorem: Let A and B be intuitionistic anti L-fuzzy subsets of the sets G and H, respectively, and let α and β in L. Then $(A\times B)_{(\alpha,\beta)} = A_{(\alpha,\beta)}$ x $B_{(\alpha,\beta)}$.

Proof: Let α and β be in L and (x,y) be in $(A\times B)_{(\alpha,\beta)}$

$\Leftrightarrow \mu_{AxB}(x, y) \leq \alpha$ and $\nu_{AxB}(x, y) \geq \beta$

$\Leftrightarrow \mu_A(x) \vee \mu_B(y) \leq \alpha$ and $\nu_A(x) \wedge \nu_B(y) \geq \beta$

$\Leftrightarrow \mu_A(x) \leq \alpha$ and $\mu_B(y) \leq \alpha$ and $\nu_A(x) \geq \beta$ and $\nu_B(y) \geq \beta$

$\Leftrightarrow \mu_A(x) \leq \alpha$ and $\nu_A(x) \geq \beta$ and $\mu_B(y) \leq \alpha$ and $\nu_B(y) \geq \beta$

$\Leftrightarrow$ x in $A_{(\alpha,\beta)}$ and y in $B_{(\alpha,\beta)}$

$\Leftrightarrow$ (x,y) in $A_{(\alpha,\beta)}$ x $B_{(\alpha,\beta)}$.

Therefore, $(A\times B)_{(\alpha,\beta)} = A_{(\alpha,\beta)}$ x $B_{(\alpha,\beta)}$.

4.13 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Then $aA_{(\alpha,\beta)} = (aA)_{(\alpha,\beta)}$, for every a in G, α and β in L.
Proof : Let A be intuitionistic anti L-fuzzy M-subgroup of a M-group G and let x in G.

Now , $x \in (aA)_{(\alpha,\beta)}$

$\Leftrightarrow(a\mu_A)(x) \leq \alpha$ and $(a\nu_A)(x) \geq \beta$
$\Leftrightarrow \mu_A(a^{-1}x) \leq \alpha$ and $\nu_A(a^{-1}x) \geq \beta$
$\Leftrightarrow a^{-1}x \in A_{(\alpha,\beta)}$
$\Leftrightarrow x \in a A_{(\alpha,\beta)}$.
Therefore, $a A_{(\alpha,\beta)} = (aA)_{(\alpha,\beta)}$ for every x in G.

V. CONCLUSION

Further work is in progress in order to develop the homomorphism and anti-homomorphism of intuitionistic anti L- fuzzy normal M -subgroups, homomorphism and anti-homomorphism of lower level subsets of intuitionistic anti L- fuzzy M-subgroup and intuitionistic anti L-fuzzy normal M-N-subgroups.

VI. REFERENCES

[1]. Aktas, H and Cagman, N., "Generalized product of fuzzy subgroup and t -level subgroups", Math. Commun, Vol. 11, pp.121 -128, 2006.
[2]. Atanassov. K. T.," Intuitionistic fuzzy sets", Fuzzy sets and systems, Vol.20, pp. 87 -96,1986
[3]. Atanassov. K. T., "New operations defined over the intuitionistic fuzzy sets", Fuzzy sets and systems, Vol. 61, pp.137 –142, 1994
[4]. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications 35, (1971) 512-517.
[5]. R. Biswas, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy Sets and Systems 35 (1990), 121-124.
[6]. Choudhury. F. P and Chakraborty. A. B and Khare. S. S., "A note on fuzzy subgroups and fuzzy homomorphism", Journal of mathematical analysis and applications, Vol.131,537 - 553,1988.
[7]. J.A. Goguen, L-fuzzy sets, J. Math. Anal. Appl.18(1967),145-179.
[8]. Jacobson. N., Lectures in abstract algebras. East - west press, 1951.
[9]. Mohamed Asaad, Groups and Fuzzy Subgroups, Fuzzy Sets and Systems, 39(1991)323-328.
[10]. Massa'deh. M. O and For a. A.A., "Centralizer of upper fuzzy groups", Pioneer journal of mathematics and mathematical sciences, Vol.1, pp. 81-88, 2011
[11]. N.Palaniappan, R.Muthuraj, The homomorphism, Anti-homomorphism of a fuzzy and an anti-fuzzy group, Varahmihir Journal of mathematical Sciences, 4 (2)(2004) 387-399.
[12]. N.Palaniappan , S. Naganathan, & K. Arjunan, A Study on Intuitionistic L-Fuzzy Subgroups, Applied mathematical Sciences, 3 (53) (2009) 2619-2624.

[13]. Pandiammal. P, Natarajan. R, and Palaniappan. N , Anti L-fuzzy M-subgroups, Antarctica J. Math., Vol. 7, number 6, 683-691, (2010).
[14]. Pandiammal. P, Intuionistic Anti L-fuzzy M-subgroups, International Journal of computer and organization Trends, Volume 5, Feb. 2014.
[15]. Pandiammal. P, A Study on Intuionistic Anti L-fuzzy Normal M-subgroups, International Journal of computer and organization Trends, Volume 13, number 1, Oct. 2014.
[16]. Prabir Bhattacharya, Fuzzy subgroups: Some characterizations, J. Math. Anal. Appl. 128 (1981) 241-252.
[17]. Rosenfeld. A.," Fuzzy groups", J. Math. Anal. Appl, Vol. 35, pp. 512 -517, 1971.
[18]. Roventa. A and Spircu. T.," Groups operating on fuzzy sets", Fuzzy sets and systems, Vol. 120 , pp. 543 -548,2001.
[19]. Sulaiman. R and Ahmad. A. G.," The number of fuzzy subgroups of finite ncyclic groups", Int. Math. Forum, Vol.6 , pp. 987 -994,2011.
[20]. Tarnauceanu. M and Bentea.L.," On the number of fuzzy subgroup of finite abelian groups", Fuzzy sets and systems, Vol. 159, pp. 1084 -1096,2008.
[21]. L.A. Zadeh, Fuzzy sets, Information and control, 8, (1965) 338-353.