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The Role of Mathematical Definitions in Mathematics

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ABSTRACT

The central focus of teaching mathematics is solving problems via definitions and concern results. Problem solving is not a distinct topic, but a process that should be a focus for learning Mathematical definitions and hence in proof of results. Definitions play a key role in mathematics, but their creation and use differs from those of "everyday language" definitions. Mathematical definitions are of fundamental importance in the axiomatic formation that characterizes mathematics. Mathematics students into the field of mathematics includes their getting and of the role of mathematical definitions. Mathematical definitions have many features, some critical to their nature and others, while not necessary to the categorization of mathematical definitions, preferred by the mathematics community. Undergraduate mathematics students' have difficulties in writing formal mathematical proofs, weak understanding of logic and/or mathematical concepts due to Mathematical definitions. In this study we first discuss a structure for thinking about mathematical definitions derived from literature in the fields of mathematics. Next, we discuss research on student understanding and use of definition and on the role of definitions in the teaching of mathematics. Finally, we discuss the implications of this research and important educational decisions that should rule the use of mathematical in the teaching of mathematics.

Keywords : Mathematics, Mathematical Definition

I. INTRODUCTION

Mathematics is ultimately about formalizing systems and understanding space, shape and structure. It is the "language of nature" and is utilized heavily in all of the quantitative sciences. It is also fascinating in its own right. If you are heavily interested in learning more about deeper areas of mathematics, but lack the ability to carry it out in a formal setting, this article series will help you gain the necessary mathematical maturity, if you are willing to put in the effort.

The discipline of mathematics now covers - in addition to the more or less standard fields of (1)Real analysis: Real analysis is a staple course in first year undergraduate mathematics. The subject is primarily about real numbers and functions between sets of real numbers. The main topics discussed include sequences, series, convergence, limits, calculus and continuity.(2)Linear algebra :Linear Algebra is one of the most important, if not the most important. In an abstract sense Linear Algebra is about the study of linear maps between vector spaces.(3) Geometry – Euclidean: Geometry is one of the most fundamental areas of mathematics. It is absolutely essential for many areas of deeper mathematics. Geometry is one of the most fundamental areas of mathematics. It is absolutely essential for many areas of deeper mathematics.

Mathematics is mental activity which consists in carrying out, one after the other and those mental constructions which are inductive and effective. Mathematics is the manipulation of the meaningless symbols of a first-order language according to explicit, syntactical rules. It seems to be common knowledge in mathematics departments that many students do not "know" the definitions they need to know in order to perform mathematical tasks such as proving theorems. Often, in an attempt to solve this problem students are asked to memorize the pertinent definitions in the course and sometimes they are given credit in examinations for repeating those definitions. A definition explains the meaning of a piece of terminology. There are logical problems with even this simple idea, for consider the first definition that we are going to formulate. The definitions give us then a language for doing mathematics. We formulate our results, or theorems, by using the words that have been established in the definitions. In any subject area of mathematics, one begins with a brief list of definitions and a brief list of axioms in form of definitions. Once these are in place, and are accepted and understood, then one can begin proving theorems. Many students seem to have trouble with the idea of a mathematical definitions and hence in mathematical proof. Students that come to a course like - Calculus, Trigonometry, Vector geometry, differential equations and Abstract algebra, all of the unexpected come to meet a new kind of mathematics, an abstract mathematics that requires proofs.

II. The Concept of Definition

Euclid had definitions and axioms and then theorems—in that order. There is no gainsaying the assertion that Euclid set the paradigm by which we have been practicing mathematics for 2300 years. This was mathematics done right. Now, following Euclid, in order to address the issue of the infinitely regressing chain of reasoning, we begin our studies by putting into place a set of Definitions. What is a definition? A definition explains the meaning of a piece of terminology. There are logical problems with even this simple idea, for consider the first definition that we are going to formulate. Suppose that we wish to define metric space. This will be the first piece of terminology in our mathematical system. What words and notations can we use to define it? Suppose that we define metric space in terms of distance function and together it on given set. That begs the questions: What is a set? What is a distance? What is a function? How do we define "function"? What is a distance function? Thus we see that our first definition(s) must be formulated in terms of commonly accepted words and notations that require no further explanation. It was Aristotle (384 B.C.E.–322 B.C.E.) who insisted that a definition must describe the concept being defined in terms of other concepts already known. This is often quite difficult. As an example, Georg cantor, one of the founders of the set theory, gave the definition of set.A set is a gathering together into a whole of definite, distinct objects of our thought. which has no part. Thus he is using words outside of mathematics, that are commonly accepted part of everyday argot, to explain the precise mathematical notion of "set". Once "set" is defined, then one can use that term in later definitions—for example, to define "function". And one will also use everyday language that does not require further explication. That is how we build up our system of definitions.

III. The Study

Definition activities in mathematics courses can have several pedagogical objectives, some of which could be promoting deeper conceptual understanding of the mathematics involved, promoting an understanding of the nature or the characteristics of mathematical definitions, and/or promoting an understanding of the role of definitions in mathematics. Definitions are frequently and most obviously used to promote the first objective, deeper conceptual understanding of mathematics. Indeed the traditional method of communicating mathematics between professional mathematicians begins with a statement of the pertinent definition or definitions. Activities that involve studying a mathematical definition carefully and deciding from a collection of items which are and which are not examples of the defined concept are plentiful. Some of the tasks of the interviews for both the algebra study and the analysis study are examples of such activities. These activities also indirectly address the role of mathematical definitions. For a related discussion on this topic see Wilson (1990). Activities that address the second objective are common also, although we feel that these alone might also not be sufficient for encouraging proper use of definitions in formal mathematics. An activity used by the author is to ask students working in groups to define a given concept, for example, set. Each group agrees upon a definition for set and all definitions are then displayed for the whole class to compare and then choose the "best" definition. Of course, what it means to be "best" is also discussed. Students often mention criteria from Van Dormolen and Zaslavsky (2003) and when necessary we guide the discussion toward considering the entire list. We also encourage discussions about the consequences of various conditions included in the definition for set. It seems that the key issue for many of the students from the analysis and algebra studies, however, understood that mathematical definitions are stipulated and thus different from everyday definitions. To develop this understanding requires treating mathematical definition as a concept in its own right by promoting an understanding of the role of definitions in mathematics, our third objective. If students are asked to create definitions in such a way that their task is actually one of discovering the "correct' definition for a concept, it may give them the impression that definitions can be right or wrong and that they are extracted rather than stipulated. Later students can compare their definitions to the definitions that were created before them, probably with a much greater understanding of the defining process. This is possibly a more difficult and longer process, but it is more authentic. It seems to be common knowledge in mathematics departments that many students do not "know" the definitions they need to know in order to perform mathematical tasks such as proving theorems. Often, in an attempt to solve this problem students are asked to memorize the pertinent definitions in the course and sometimes they are given credit in examinations for repeating those definitions. However, just knowing the definition may not be enough. If students had mathematically correct definitions in front of them at all times during interviews and written tasks, it would be possible to see evidence of their understanding of how mathematical definitions should be used, unencumbered by worry about the actual wording of a particular definition. An introductory real analysis course that had as one of its goals helping students learn to write proofs. Specifically, the course was described as an introduction to rigorous analytic proofs in the context of the properties of real numbers, continuity, differentiation, integration and infinite sequences and series. Even with the definitions in front of them, many of the undergraduate mathematics students had some difficulty using mathematical definitions in a mathematically appropriate way. The question arose, however, whether this result could have been influenced by the fact that many of the definitions in concepts that students had encountered in elementary calculus or were in some way related to those concepts.

IV. Design of the Study

The purpose of study was to look beneath students' understandings of the content of mathematical definitions to discern their understandings of the role played by formal definitions in mathematics. This is somewhat tricky since it is possible that a student might apply a definition in a mathematically incorrect way for at least two reasons. A student could have an incomplete or faulty understanding of the content of a particular definition; or A student could have a mathematically incorrect understanding of the role or nature of mathematical definitions in general. For example, a student may decide that f(x) = 2011 is not a function because the symbolic form "has no x in it" (a faulty understanding of the function definition itself); or he may decide that the particular example is not a function even after reading the definition because the requirement of having an x is something we just know and it does not need to be mentioned in the definition (a faulty understanding of the role and character of mathematical definitions in general). A further difficulty influencing the design of these studies arose from the possibility that, if asked directly, students might profess a seemingly adequate understanding of the role of formal definitions in mathematics without really understanding this role. It is not uncommon for students to repeat something they have heard without full understanding. If the definition had been encountered before but had not been discussed in the course, the students were asked first to explain in their own words their understanding of the associated concept and then to provide a definition for it if they could do so. The students were then given a copy of the stipulated definition and were asked to explain its meaning and to discuss how their previous explanation agreed or did not agree with their understanding of the given formal definition.

The nature of mathematical definition and the aims of teaching it are universal. Different students can give different ways in form notations and uses its own logical arguments, but basically they are the same. The following are some importance of mathematical definition:

- ✓ Developing the basic skills in dealing with mathematical terms in definition,
- ✓ Developing the ability to think critically for notations in definition,
- ✓ Developing the ability to communicate definition precisely in symbolic form,

V. RESULT

The result of study is many undergraduate mathematics students do not categorize mathematical definitions as stipulated, and that they may, under some circumstances, defer to their image of a concept rather than the definition if the two do not agree. one must consider the impact of these pedagogical decisions and perhaps do something to mitigate against later misunderstandings by the student.

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