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Some methods of Mathematical Proof

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ABSTRACT

The word "proof" comes from the Latin probare meaning "to test". In mathematics, a proof is an inferential argument for a mathematical statement . Proofs remain important in mathematics because they are our bellwether for what we can believe in, and what we can depend on. They are timeless and rigid and dependable. They are what hold the subject together, and what make it one of the glories of human thought. In the argument, other previously established statements, such as theorems, can be used. In principle, a proof can be traced back to self-evident or assumed statements, known as axioms, along with accepted rules of inference. Axioms may be treated as conditions that must be met before the statement applies. Proofs make use of logic but usually include some amount of natural language which usually admits some ambiguity. In fact, the vast majority of proofs in written mathematics can be considered as applications of rigorous informal logic. Purely formal proofs, written in symbolic language instead of natural language, are considered in proof theory. The distinction between formal and informal proofs has led to much examination of current and historical mathematical practice in mathematics. The concept of a proof is formalized in the field of mathematical logic. In this study we first discuss, The role of some methods of Mathematical Proof of theorems of mathematics. For this two main methods are formal proof and other one is direct proof. A formal proof is written in a formal language instead of a natural language. In direct proof, the conclusion is established by logically combining the axioms, definitions, and earlier theorems. Also aim of this study is to see impact of methods**,** namely: Proof by mathematical induction, Proof by contraposition, Proof by contradiction, Proof by construction, Proof by exhaustion, Probabilistic proof, combinatorial proof, Nonconstructive proof, Undecidable statements, Elementary proof of some mathematical results.
Keywords : Proof, Formal Proof, Direct Proof.

I. INTRODUCTION

The unique feature that sets mathematics apart from other sciences is the use of exact proof. It is the proof concept that makes the subject comes together, that gives it its timelessness, and that enables it to travel well. The purpose of this discussion is to describe proof, to put it in context, and to explain its significance. Proofs may be viewed as aesthetic objects, admired for their mathematical beauty. The mathematician Paul Erdős was known for describing proofs he found particularly elegant as coming from "The Book", a hypothetical tome containing the most beautiful method(s) of proving each theorem. The book Proofs from the book, published in 2003, is devoted to presenting 32 proofs its editors find particularly pleasing. A classic question in philosophy asks whether mathematical proofs are analytic or synthetic. Kant, who introduced the analytic-synthetic distinction, believed mathematical proofs are synthetic. The field of proof theory studies formal proofs and their properties, for example, the property that a statement has a formal proof. An application of proof theory is to show that certain undecidable statements are not provable. Proofs may be viewed as aesthetic objects, admired for their mathematical beauty. In direct proof, the conclusion is established by logically combining the axioms, definitions, and earlier theorems. Generally speaking, in any subject area of mathematics, one begins with a brief list of definitions and a brief list of axioms. Once these are in place, and are accepted and understood, then one can begin proving theorems. And what is a proof? A proof is a symbolic device for convincing another mathematician that a given statement (the theorem) is true. Thus a proof can take many different forms. The most traditional form of mathematical proof is that it is a tightly knit sequence of statements linked together by strict rules of logic. But the purpose of the study is to discuss and consider the various forms that a proof might take. While early mathematicians such as Eudoxus of Cnidus did not use proofs, from Euclid to the foundational mathematics developments of the late 19th and 20th centuries, proofs were an essential part of mathematics. With the increase in computing power in the 1960s, significant work began to be done investigating mathematical objects outside of the proof-theorem framework. Most of the steps of a mathematical proof are applications of the elementary rules of logic. This is a slight oversimplification, as there are a great many proof techniques that have been developed over the past two centuries. These include proof by mathematical induction, proof by contradiction, proof by exhaustion, proof by enumeration, and many others. But they are all built on one simple rule: modus ponendo ponens. This rule of logic says that if we know that "p implies q", and if we know "p", then we may conclude q. Thus a proof is a sequence of steps linked together by modus ponendo ponens. The theorems that Euclid and Pythagoras proved 2500 years ago are still valid today; and we use them with confidence because we know that they are just as true today as they were when those great masters first discovered them. What is marvelous is that, in spite of the appearance of some artificiality in the mathematical process, mathematics provides beautiful models for nature (see the lovely essay [WIG], which discusses this point). Over and over again, and more with each passing year, mathematics has helped to explain how the world around us works. For example illustrates the point: Isaac Newton derived Kepler's three laws of planetary motion from just his universal law of gravitation and calculus.

II. The study

A formal proof : A formal proof is defined as sequence of formulas in a formal language, in which each formula is a logical consequence of preceding formulas. Having a definition of formal proof makes the concept of proof amenable to study. Indeed, the field of proof theory studies formal proofs and their properties, for example, the property that a statement has a formal proof. An application of proof theory is to show that certain undecidable statements are not provable. The definition of a formal proof is intended to capture the concept of proofs as written in the practice of mathematics. The soundness of this definition amounts to the belief that a published proof can, in principle, be converted into a formal proof. However, outside the field of automated proof assistants, this is rarely done in practice.

Direct proof : In direct proof, the conclusion is established by logically combining the axioms, definitions, and earlier theorems. For example, direct proof can be used to establish that the sum of two even integers is always even: Consider two even integers m and n. Since they are odd, they can be written as m = 2x and n = 2y, respectively, for integers x and y. Then the sum m + n = 2x + 2y = 2(x + y). Therefore m + n has 2 as a factor and, by definition, is even. Hence the sum of any two even integers is even. This proof uses the definition of even integers, the integer properties of closure under addition and multiplication, and distributivity.

Proof by Mathematical Induction:

Mathematical induction is a method of deduction, not a form of inductive reasoning. In proof by mathematical induction, a single "base case" is proved, and an "induction rule" is proved that establishes that any arbitrary case implies the next case. This avoids having to prove each case individually. A common application of proof by mathematical induction is to prove that a property known to hold for one number holds for all natural numbers: Let N = {1,2,3,4,...} be the set of natural numbers, and P(n) be a mathematical statement involving the natural number n belonging to N such that(i) P(1) is true. (ii) P (K) is true ⇒P(K+1) is true. Then P (n) is true for all natural numbers n.

Proof by contradiction:

In proof by contradiction, it is shown that if some statement were true, a logical contradiction occurs, hence the statement must be false. For example, prove that ∅ is subset of every set S.

Proof: suppose that $\emptyset \not\subset S$ ∴ By definition of subset ther exit atleast one element in ∅ which is not in S. But it is not true as ∅ have no elements. Hence our supposition is wrong.

Hence $\emptyset \subset S$

Proof by construction:

Proof by construction, or proof by example, is the construction of a concrete example with a property to show that something having that property exists. Joseph Liouville, for instance, proved the existence of transcendental numbers by constructing an explicit example. It can also be used to construct a counterexample to disprove a proposition that all elements have a certain property.

Combinatorial proof:

A combinatorial proof establishes the equivalence of different expressions by showing that they count the same object in different ways. Often a bisection between two sets is used to show that the expressions for their two sizes are equal. Alternatively, a double counting argument provides two different expressions for the size of a single set, again showing that the two expressions are equal.

Undecidable statements:

A statement that is neither provable nor disprovable from a set of axioms is called undecidable (from those axioms). One example is the parallel postulate, which is neither provable nor refutable from the remaining axioms of Euclidean geometry.

Elementary proof: An elementary proof is a proof which only uses basic techniques. More specifically, the term is used in number theory to refer to proofs that make no use of complex analysis. For some time it was thought that certain theorems, like the prime number theorem, could only be proved using "higher" mathematics. However, over time, many of these results have been reproved using only elementary techniques.

Contra positive: Some propositions that take the form of if P then Q can be hard to prove. It is sometimes useful to consider the contra positive of the statement. Before I explain what contra positive is let us see an example. If n^2 is odd then n is also odd and it is harder to prove than if n is even then n^2 is also even although they mean the same thing. So instead of proving the first proposition directly, we prove the second proposition instead. This technique is called proof by contra positive.

III. Result

This study shows that for proof of mathematical theorems in abstract algebra and in mathematical analysis given methods gives right direction of mathematical proof of such results/theorems.

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