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Boundary Layer Flow and Heat Transfer over A Continuous Porous Surface with Porous Medium Moving in an Oscillating Free Stream

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ABSTRACT

In this work we investigate the boundary layer flow of an incompressible, unsteady, viscous fluid, with porous medium and free stream oscillation, over a continuous moving flat surface. Let us make an assumption that the oscillations created by the roll are neglected, we observed that velocity fieldu_o, Temperature θ_0 and Nusselt number increase as the permeability of the medium increase, skin fraction has antiproportional relation with β, but proportional relation with permeability is constant.

Keyword: Permeability, Nusselt Number, Skin Fraction, Temperature.

I. INTRODUCTION

The problem of boundary layar flow of a fluid layer trough a porous medium subjected to a temperature gradient is of importance in geo-Physics, ground water, hydrology, soil sciences and so on. The analysis of such flows finds application in different areas , such as the aerodynamic extension of plastic sheets, the boundary layer along material handling conveyers , the cooling of a metallie plate in acool bath, and the boundary along a liquid film in condensation processes.

Sakiadis[1] has studied the orically the boundary layer on a continuous semi- infinite sheet moving steadily through an otherwise quiescent fluid environment. The boundary layer solution of sakiadis resulted in a skin – fraction of about 30% higher than that of Blasius[2] for the flow past a stationary flat plate. Later,for different values of the Prandt1 number experimental and theoretical stdies were made by Tsou et al[3]. Recently, Abdelhafez[4] has made a very interesting analysis by including the effects of accompanying parallel free stream.

The analysis temperature field, as modified by generation or absorption of heat in moving fluids, is important in view of several physical problems such as: (a) problems dealing with chemical reactions[5] , (b) problems concerned with dissociating fluids[6].In fact , literature is replete with examples dealing with heat transfer in laminar flow of viscous fluids. Heat generation has been assumed to be constant or a function of space variable by some authors. Others have considered directly the frictional heating and the expansion effects. Sparrow and CEN[7] have obtained solutions of the steady flow and heat transfer of the stagnation point flow, taking into account the temperature dependent heat generation. Foraboschi and Federico [8] have amed a volumetric rate of heat generation as

$$Q = Q_0(T - T_0) \text{ when } T \geq T_0$$
$$Q = 0 \text{ when } T < T_0 \qquad \ldots\ldots \quad \text{(A)}$$

In their study of the steady state temperature profiles for linear parabolic and piston flow in circular tubes. The relation (A) described by Foraboschi and Federico, is valid as an approximation of the state of some exothermic process having to as the onset temperature.
This analysis is ,therefore, an attempt to investigate the effects of unsteadiness ,suction , and internal heat generation/absorpition (with and without viscous dissipation) on the continuous flat surface problem. The volumetric rate of heat generation (or absorption) is taken as

$$\theta \sim (T' - T'_\infty)$$

Where T'_∞ is the free stream temperature .The flow and heat transfer characteristics are found to depend on the new dimensional numbers $= u'_w/u'_\infty$, $w = w'4\nu/V_0^2$,$E = C_p(T'_w - T'_\infty)$ and $\alpha = Q\nu^2/k\nu_0^2$. Lahurikar and Pohanerkar[9] has been studied on unsteady forced and free convective flow past an infinite vertical plate through a porous medium.Vajravelu [10] has been studied on boundary layer flow and heat transfer over a continuous porous, surface moving in an oscillating free stream.

This work is the extension of the paper of Vajaraveluk. With the application of permeability of the medium.

II. FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider a long continuous sheet of finite length on which the boundary layer would grow in the direction opposite to the direction of motion of the plate. Let us investigate the boundary layer flow of

Incompressible, unsteady viscous fluid, with free stream oscillations and porous medium, over a continuous moving flat surface. Let us make a assumption that the disturbances created by the roll are neglected. The boundary layer equations for flow and heat transfer with uniform suction and internal heat generation are in the usual notation.

$$\frac{\partial u'}{\partial t'} - \upsilon_0 \frac{\partial u'}{\partial y'} = \frac{\partial u'}{\partial t'} + \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{K'_0} u' \quad \ldots\ldots(1)$$

$$\rho C_P \left(\frac{\partial T'}{\partial t'} - \nu_0 \frac{\partial T'}{\partial y'}\right) = k \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 + Q(T' - T'_\infty) \quad \ldots..(2)$$

The boundary condition for the velocity and temperature and temperature fields are-

$u' = u'_w$, $T' = T'_W$ at $y' = 0$

$u' \Rightarrow U'(t')$ $T' \rightarrow T'_\infty$ as $y' \rightarrow 0$ ….. (3)

Defining non-dimensional variables

$$y = \frac{y'\upsilon_0}{\upsilon}, \quad t = \frac{t'\upsilon_0^2}{4\upsilon}, \quad w = \frac{w'4\upsilon}{4\upsilon}, \quad u = \frac{u'}{u'_\infty}, \quad U = \frac{U'}{U'_\infty}, \quad Q = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \quad K = \frac{\upsilon_0 K'}{\upsilon^2}.$$

Equation (1) and (2) and conditions (3) can be written as

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial u}{\partial t} + \frac{\partial y}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{U}{K_0} \quad \ldots\ldots (4)$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - P\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 + \alpha\theta. \quad \ldots.(5)$$

and boundary condition are-

$u = \beta, \theta = 1$ at $y = 0$

$u_\infty = U(t), \theta \rightarrow 0$ as $\theta \rightarrow \infty$ …..(6) where

$\beta = \frac{u'_w}{u'_\infty}$ the velocity ratio parameter.

$P = \mu \frac{C_p}{K}$ the number Prandt1 number.

$E = \frac{{u'_\infty}^2}{C_p}(T'_W - T'_\infty)$ the Eckert number

$\alpha = = Q\upsilon^2/K\, v_0^2$ the heat source /sink parameter. And K_0 is the permeability constant. We now assume

$u(y,t) = u_0(y) + \epsilon e^{iwt} u_1(y)$

$\theta(y,t) = \theta_0(y) + \epsilon e^{iwt} \theta_1(y)$

$U(t) = 1 + \epsilon e^{iwt}$ …… (7)

Where ϵ is a small constant <<1

Substituting (7) in (4) and (5) and equating harmonic terms neglecting coefficients of ϵ^2 and higher, we get

$$u''_0 + u'_0 - \frac{U_0}{K} = 0 \quad \ldots.. (8)$$

$$\theta''_0 + P\theta'_0 + \alpha\theta_0 = -PE(u'_0)^2 \quad \ldots. (9)$$

$$u''_1 + u'_1 - \left(\frac{1}{4}iw + \frac{1}{K_0}\right)u_1 = -\frac{1}{4}iw \quad \ldots. (10)$$

$$\theta''_1 + P\,\theta'_1 - \frac{1}{4}Piw\theta_1 + \alpha\theta_1 = -2PEu'_0u'_1 \quad \ldots(11)$$

Here the prices denote differentiation w.r.t.y. The corresponding boundary conditions are

$u_0 = \beta = \theta_0 = 1$ at $y = 0$

$u_0 \rightarrow 1, \theta_0 \rightarrow 0$ as $y \rightarrow \infty$ ….. (12)

$u_1 = 0, \theta_1 = 0$ at $y = 0$

$u_1 \rightarrow 1, \theta_1 \rightarrow 0$ as $y \rightarrow \infty$ …..(13)

Solving Equation (8) and (9) with the boundary conditions (12) , we get ,

$$u_0(y) = \beta\, e^{-\left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{K_0+4}{K_0}}\right)y} \quad \ldots..(14)$$ and

$$\theta_0(y) = \left[\frac{K_0[(2-P+\alpha) + PE\beta^2] + (2-P)(k_0^2 + 4k_0 + 4)^{1/2}}{K_0(2-P+\alpha) + (2-P)(K_0^2 + 4K_0 + 4)^{1/2}}\right] .\, e^{-\frac{(P)+\sqrt{P^2-4\alpha}}{2}y} - \frac{PEK_0\beta^2 e^{-\left(1+\frac{\sqrt{K_0+4}}{K_0}\right)y}}{K_0(2-P+\alpha) + (2-P)(K_0^2 + 4K_0 + 4)^{1/2}} \quad \ldots..(15)$$

The mean sharing stress and the mean heat transfer coefficient (or the Nusselt number) at the surface are defined respectively, in nondimensional form as

$$\tau_0 = \frac{\tau'_0}{\rho U'_\infty V_0} = \frac{du_0}{dy} / y = 0$$

$$\tau_0 = -\beta\left[(1/2) + (1/2)\left(\frac{K_0+4}{K_0}\right)^{1/2}\right] \quad \ldots(16)$$ and

$$Nu_0 = -\frac{h\upsilon}{K v_0 (T'_w - T'_\infty)} = \frac{d\theta_0}{dy} / y = 0$$

$$Nu_0 = \left\{\frac{K_0(2-P+\alpha)+PE\beta^2+(2-P)\left(K_0^2+4K_0+4\right)^{1/2}}{K_0(2-P+\alpha)+(2-P)\left(K_0^2+4K_0+4\right)^{1/2}}\right\}\left(-\left(\frac{P+\sqrt{P^2-4\alpha}}{2}\right)\right) + \frac{PEK_0\beta^2\left(1+\frac{\sqrt{K_0+4}}{4}\right)}{K_0(2-P+\alpha)+(2-P)\left(K_0^2+4K_0+4\right)^{1/2}} \quad \ldots(17)$$

The velocity field u_0 , temperature field θ_0, skin-fraction τ_0 , and heat transfer coefficient Nu_0 are calculated.

III. RESULT AND DISCUSSION

The graph (i) plotted between mean velocity field u_0 and permeability constant K_0 in which we observe that the velocity field increases considerably as permeability of the medium increases as constant

$\beta = 1$, it is also observed that mean velocity field has proportional relation with β at constant permeability of mediumK_0.

In graph (ii) sketch temperature θ_0 versus y.We find that temperature increases as permeability constant increase at constant $\beta = 1$, $\alpha = .1$,P=.71 and E=.02.

From graph (iii) we see that Nusselt number increases as the permeability of medium K increases at constant $\beta = 1$, P=.71, $\alpha = .1$ and E=.02.

From equation (16) skin friction has anti proportional relation with β but proportional relation with permeability constant K.

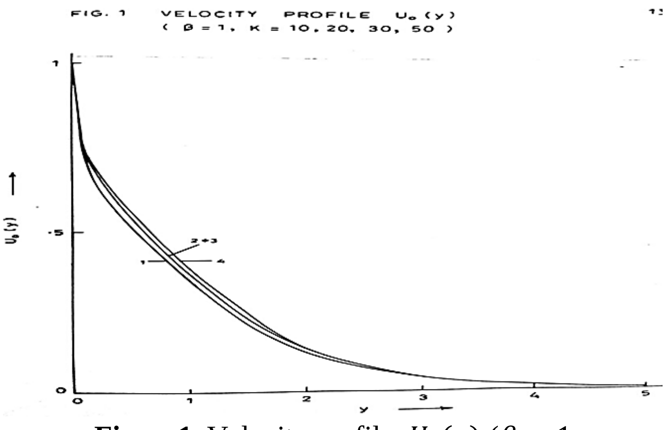

Figure1. Velocity profile $U_0(y)$ ($\beta = 1$, K=10,20,30,50.)

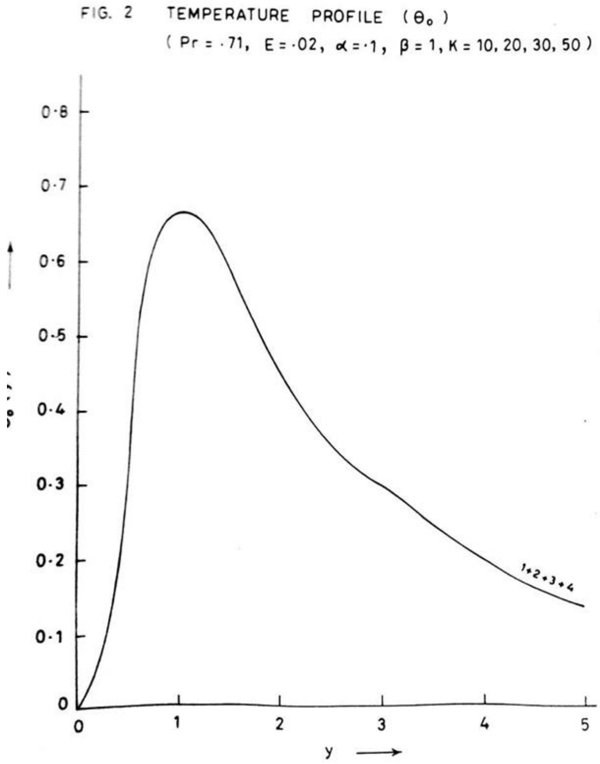

Figure 2. Temperature profile (θ_0) P=.71, E=.02, $\alpha = .1$, $\beta = 1$ K=10,20,30.50

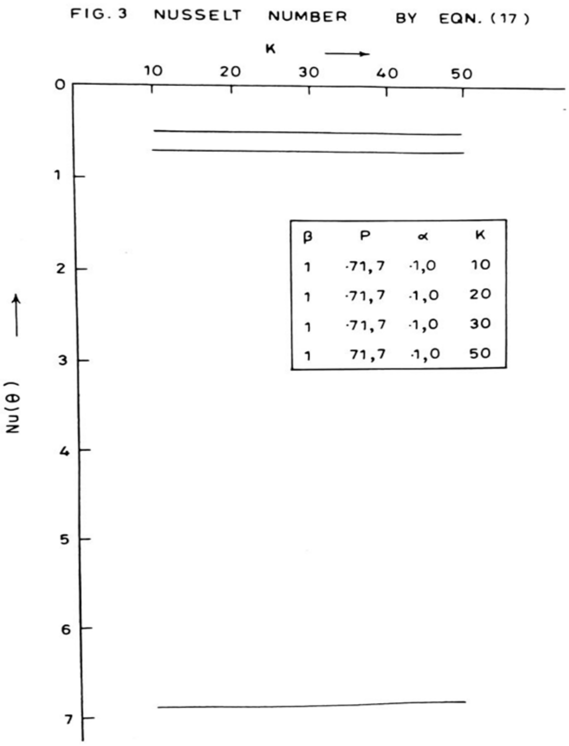

Figure 3. Nusselt number by eqn. (17)

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