

Wave Propagation at an Interface of Heat Conducting Micropolar Solid (Viscoelastic) and Fluid Media

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ABSTRACT

The present investigation is concerned with wave propagation at an interface of micropolar generalized viscothermoelastic solid half space and heat conducting micropolar fluid half space. Reflection and transmission phenomenon of plane waves impinging obliquely at a plane interface between a micropolar generalized viscothermoelastic solid half space and heat conducting micropolar fluid half space are investigated. The incident wave is assumed to be striking at the plane interface after propagating through the micropolar generalized viscothermoelastic solid with two temperatures. Amplitude ratios of the various reflected and transmitted waves are obtained in closed form and it is observed that Amplitude ratios are function of angle of incidence, frequency and are affected by the micropolar viscoelastic properties of the media. Viscosity, Micropolarity and thermal relaxation effects are shown on these amplitude ratios for a specific model. Results of some earlier workers have also been deduced from the present investigation.

Keywords: Micropolar Viscothermoelastic Solid, Micropolar Fluid, Viscothermoelastic, Reflection Coefficient, Transmission Coefficient half Space.

I. INTRODUCTION

The theory of microfluids was introduced by Eringen[1]. A microfluid in addition to its classical translatory degrees of freedom represented by velocity field, possesses three gyration vector fields. As a subclass of these fluids, Eringen introduced the micropolar fluids[2] in which the local fluid elements were allowed to undergo only rigid rotations without stretch. Micropolar fluids can support couple stress, the body couples, asymmetric stress tensor and possesses a rotational field, which is independent of the velocity of fluid. A large class of fluids such as anisotropic fluids, liquid crystals with rigid molecules, magnetic fluids, cloud with dust, muddy fluids, biological tropic fluids, dirty fluids (dusty air, snow) over airfoil can be modeled more realistically as micropolar fluids. Various authors notably Ariman et.al[3,4], Riha[5], Eringen and Kafadar[6], Brulin[7], Aggarwal and Dhanapal[8], Payne and Straughan[9], Gorla[10], Eringen[11], Aydemir and Venart[12], Yerofeyev and Soldatov[13], Yeremeyev and Zubov[14], Hsia and Cheng[15], Hsia, Chiu, Su and

Chen[16] investigated different types of problem in micropolar fluid and heat conducting micropolar fluid.

The inelastic behaviour of the earth materials play an important role in changing the characteristics of seismic waves in defining seismic source functions. Viscoelastic materials are those for which the relationship between stress and strain depends on time. All materials exhibit some viscoelastic response. In common metals such as steel, aluminum, copper etc. The general theory of viscoelasticity describes the linear behaviour of both elastic and inelastic materials and provides the basis for describing the attenuation of seismic waves due to inelasticity.

The general linear theory of micropolar elasticity was given by Eringen [17]. Under this theory, solids can undergo macro-deformations and micro-rotations and support couple stresses in addition to force stresses. The problem of micropolar viscoelastic waves has been discussed by Eringen[18]. The theory of thermoelasticity deals with the effect of mechanical and thermal

disturbances on an elastic body. The theory of uncoupled thermoelasticity consists of the heat equation, which is independent of mechanical effects and the equation of motion contains the temperature as a known function. Biot[19] gave a satisfactory theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories are of parabolic type predicting infinite speed of propagation for heat waves contrary to physical observations. To overcome this drawback, two generalizations of coupled theory were introduced. The first generalization is due to Lord and Shulman[20], who modified heat conduction law which involves heat flux rate. This contains the heat flux vector. This thermoelastic theory is including the finite velocity of thermal wave by correcting the Fourier thermal conduction law by introducing one relaxation time of thermoelastic process. Since the governing equation of this theory is of the wave-type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and uncoupled theories of thermoelasticity.

The second generalization to the coupled theory of elasticity is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate dependent thermoelasticity. Mullar[21], in a review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws[22]. Green and

Lindsay[23](G-L theory) obtained another version of the constitutive equations. These were also obtained independently and more explicitly by Suhubi[24]. This theory contains two constants that act as relaxation times and modifies all the equations of the coupled theory. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry.

The problems of reflection and transmission of plane waves at an interface of micropolar/micropolar elastic/micropolar viscoelastic half spaces have been investigated by various authors e.g. Tomar and Gogna[25,26,27]. Kumar, Nidhi and Ram[28,29]. Kumar[30]. Singh and Tomar[31]. have discussed the longitudinal waves at an interface of micropolar fluid/micropolar solid half spaces.

In this paper, we studied the problem of reflection of plane waves at an interface of micropolar generalized viscothermoelastic solid half space and heat conducting micropolar fluid half space. Viscosity, Micropolarity and thermal relaxation effects are depicted graphically on the amplitude ratios for incidence of various plane waves that is Longitudinal displacement wave (LD wave), Thermal wave (T wave), Coupled transverse wave (CD-I wave and CD-II wave).

II. BASIC EQUATIONS

Following Eringen[17]. Lord and Shulman[20], and Green and Lindsay[23] the field equations for an isotropic and homogeneous micropolar viscoelastic medium in the context of generalized theory of thermoelasticity, without body forces, body couples and heat sources, are given by

$$(\lambda_1 + 2\mu_1 + \kappa_1)\nabla(\nabla \cdot \vec{u}) - (\mu_1 + \kappa_1)\nabla \times (\nabla \times \vec{u}) + \kappa_1(\nabla \times \vec{\phi}) - \nu \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla T = \rho \frac{\partial^2 \vec{u}}{\partial t^2}, \quad (1)$$

$$(\alpha_1 + \beta_1 + \gamma_1)\nabla(\nabla \cdot \vec{\phi}) - \gamma_1 \nabla \times (\nabla \times \vec{\phi}) + \kappa_1 \nabla \times \vec{u} - 2\kappa_1 \vec{\phi} = \rho \hat{j} \frac{\partial^2 \vec{\phi}}{\partial t^2}, \quad (2)$$

$$K^* \nabla^2 T = \rho c^* \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \nu T_0 \left(\frac{\partial}{\partial t} + \eta_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) (\nabla \cdot \vec{u}), \quad (3)$$

and the constitutive relations are

$$t_{ij} = \lambda_1 u_{r,r} \delta_{ij} + \mu_1 (u_{i,j} + u_{j,i}) + \kappa_1 (u_{j,i} - \varepsilon_{ijr} \phi_r) - \nu \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \delta_{ij}, \quad (4)$$

$$m_{ij} = \alpha_1 \phi_{r,r} \delta_{ij} + \beta_1 \phi_{i,j} + \gamma_1 \phi_{j,i}, \quad i, j, r = 1, 2, 3$$

$$\text{Where } \lambda_1 = \lambda + \lambda_v \frac{\partial}{\partial t}, \mu_1 = \mu + \mu_v \frac{\partial}{\partial t}, k_1 = k + k_v \frac{\partial}{\partial t}, \alpha_1 = \alpha + \alpha_v \frac{\partial}{\partial t}, \beta_1 = \beta + \beta_v \frac{\partial}{\partial t}, \gamma_1 = \lambda + \gamma_v \frac{\partial}{\partial t} \quad (5)$$

where λ and μ are Lamé's constants. $\lambda_v, \mu_v, \kappa_v, \alpha_v, \beta_v$ and γ_v are micropolar constants. t_{ij} are the components of the stress tensor and m_{ij} are the components of couple stress tensor. \vec{u} and $\vec{\phi}$ are the displacement and microrotation vectors, ρ is the density, \hat{j} is the microinertia, K^* is the thermal conductivity, c^* is the specific heat at constant strain, T_0 is the uniform temperature, T is the temperature change, $\nu = (3\lambda_1 + 2\mu_1 + \kappa_1)\alpha_T$, where α_T is the coefficient of linear thermal expansion, δ_{ij} is the Kronecker delta, ε_{ijr} is the alternating symbol. For Lord Shulman (L-S) theory $\eta_0 = 1, \tau_1 = 0$ and for Green-Lindsay (G-L) theory $\eta_0 = 0, \tau_1 > 0$. The thermal relaxation time τ_0 and τ_1 satisfy the inequalities $\tau_0 \geq \tau_1 \geq 0$ for G-L theory only.

Following Ciarletta[32], the field equations and the constitutive relations for heat conducting micropolar fluids without body forces, body couples and heat sources are given by

$$D_1 \vec{v} + (\lambda^f + \mu^f) \nabla(\nabla \cdot \vec{v}) + \kappa^f (\nabla \times \vec{\Psi}) - b \nabla T^f - c_0 \nabla \phi^{*f} = 0, \quad (6)$$

$$D_2 \vec{\Psi} + (\alpha^f + \beta^f) \nabla(\nabla \cdot \vec{\Psi}) + \kappa^f (\nabla \times \vec{v}) = 0, \quad (7)$$

$$K_1^* \nabla^2 T^f - b T_0^f (\nabla \cdot \vec{v}) = \rho^f a T_0^f \frac{\partial T^f}{\partial t}, \quad (8)$$

$$\rho^f \frac{\partial \phi^{*f}}{\partial t} = \nabla \cdot \vec{v}, \quad (9)$$

where

$$D_1 = (\mu^f + \kappa^f) \Delta - \rho^f \frac{\partial}{\partial t}, \quad D_2 = \gamma^f \Delta - I \frac{\partial}{\partial t} - 2\kappa^f, \\ D_3 = \kappa^f \Delta - \rho^f a T_0^f \frac{\partial}{\partial t}, \quad \Delta g = g_{,ii} \quad (10)$$

and superscript f denotes physical quantities and material constants related to fluid

The constitutive relations are

$$t_{ij}^f = -p \delta_{ij} + \sigma_{ij}^f, \quad p = b T^f + c_0 \phi^{*f}, \\ \sigma_{ij}^f = \lambda^f \gamma_{rr} \delta_{ij} + (\mu^f + \kappa^f) \gamma_{ij} + \mu^f \gamma_{ji}, \\ m_{ij}^f = \alpha^f \nu_{rr} \delta_{ij} + \beta^f \nu_{ij} + \gamma^f \nu_{ji}, \quad (11)$$

where $\gamma_{ij} = \nu_{j,i} + \varepsilon_{jir} \Psi_r$, $\nu_{ij} = \Psi_{j,i}$ and $\lambda^f, \mu^f, \kappa^f, \alpha^f, \beta^f, \gamma^f$ and c_0 are material constants of the fluid. σ_{ij}^f are the components of stress tensor in the fluid and m_{ij}^f are the components of couple stress tensor in the fluid. \vec{v} and $\vec{\Psi}$ are the velocity vector and microrotation velocity vector, ρ^f is the density, I is a scalar constant with the dimension of moment of inertia of unit mass, p is the pressure, K_1^* is the thermal conductivity, $a T_0^f$ is the specific heat at constant strain, T_0^f is the absolute temperature, T^f is the temperature change, ϕ^{*f} is the variation in specific volume, $b = (3\lambda^f + 2\mu^f + \kappa^f) \alpha_{T^f}$, where α_{T^f} is the coefficient of linear thermal expansion.

III. FORMULATION OF THE PROBLEM AND BOUNDARY CONDITIONS

We consider a homogeneous, isotropic micropolar generalized thermoelastic solid half space (medium M_1) in contact with heat conducting micropolar fluid half space (medium M_2). The rectangular Cartesian co-ordinate system $Ox_1x_2x_3$ having origin on the surface $x_3=0$ separating the two media is taken. Let us take the x_1 -axis along the interface between two half spaces namely M_1 ($0 < x_3 < \infty$) and M_2 ($-\infty < x_3 < 0$) in such a way that x_3 -axis is pointing vertically downward into the medium M_1 . The geometry of the problem is shown in Figure 1.

We consider two dimensional problem in x_1x_3 -plane, so that the displacement vector \vec{u} and microrotation vector $\vec{\phi}$ for the solid medium M_1 and velocity vector \vec{v} and microrotation velocity vector $\vec{\Psi}$ for fluid medium M_2 are taken as

$$\begin{aligned}\vec{u} &= (u_1(x_1, x_3), 0, u_3(x_1, x_3)), & \vec{\phi} &= (0, \phi_2(x_1, x_3), 0), \\ \vec{v} &= (v_1(x_1, x_3), 0, v_3(x_1, x_3)), & \vec{\Psi} &= (0, \Psi_2(x_1, x_3), 0),\end{aligned}\quad (12)$$

For convenience, the following non dimensional quantities are introduced

$$\begin{aligned}x'_1 &= \frac{\omega^* x_1}{c_1}, & z'_1 &= \frac{\omega^* x_3}{c_1}, & u'_1 &= \frac{\rho \omega^* c_1}{\nu T_0} u_1, & u'_3 &= \frac{\rho \omega^* c_1}{\nu T_0} u_3, & v'_1 &= \frac{\rho c_1}{\nu T_0} v_1, \\ v'_3 &= \frac{\rho c_1}{\nu T_0} v_3, & \phi'_2 &= \frac{\rho c_1^2}{\nu T_0} \phi_2, & \psi'_2 &= \frac{\rho c_1^2}{\omega^* \nu T_0} \psi_2, & t' &= \omega^* t, & \tau'_1 &= \omega^* \tau_1, & T' &= \frac{T}{T_0}, \\ t'_{ij} &= \frac{1}{\nu T_0} t_{ij}, & m'_{ij} &= \frac{\omega^*}{c_1 \nu T_0} m_{ij}, & t'^f_{ij} &= \frac{1}{\nu T_0} t^f_{ij}, & m'^f_{ij} &= \frac{\omega^*}{c_1 \nu T_0} m^f_{ij}, \\ \phi'^{*f'} &= \rho \phi'^{*f}, & T'^f &= \frac{T^f}{T_0}\end{aligned}\quad (13)$$

where

$$\omega^* = \frac{\rho c^* c_1^2}{K^*}, \quad c_1^2 = \frac{\lambda_1 + 2\mu_1 + \kappa_1}{\rho}$$

The expressions relating the displacement components u_1, u_3 and velocity component v_1, v_3 to the potential functions ϕ^s, ϕ^f and ψ^s, ψ^f in dimensionless form are taken as

$$\begin{aligned}u_1 &= \frac{\partial \phi^s}{\partial x_1} - \frac{\partial \psi^s}{\partial x_3}, & u_3 &= \frac{\partial \phi^s}{\partial x_3} + \frac{\partial \psi^s}{\partial x_1}, \\ v_1 &= \frac{\partial \phi^f}{\partial x_1} - \frac{\partial \psi^f}{\partial x_3}, & v_3 &= \frac{\partial \phi^f}{\partial x_3} + \frac{\partial \psi^f}{\partial x_1},\end{aligned}\quad (14)$$

Making use of equation (13) in equations (1)-(3) and (6)-(9) and with the aid of equations (12) and (14), (after suppressing the primes), we obtain

$$\nabla^2 \phi^s - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T - \frac{\partial^2 \phi^s}{\partial t^2} = 0, \quad (15)$$

$$\nabla^2 \psi^s + a_1 \phi_2 - a_2 \frac{\partial^2 \psi^s}{\partial t^2} = 0, \quad (16)$$

$$\nabla^2 \phi_2 - a_3 \nabla^2 \psi^s - a_4 \phi_2 - a_5 \frac{\partial^2 \phi_2}{\partial t^2} = 0, \quad (17)$$

$$\nabla^2 T = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} + \varepsilon_1 \left(\frac{\partial}{\partial t} + \eta_0 \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla^2 \phi^s, \quad (18)$$

$$\nabla^2 \phi^f - b_1 T^f - b_2 \phi^{*f} - b_3 \frac{\partial \phi^f}{\partial t} = 0, \quad (19)$$

$$\nabla^2 \psi^f + b_4 \Psi_2 - b_5 \frac{\partial \psi^f}{\partial t} = 0, \quad (20)$$

$$\nabla^2 \Psi_2 - b_6 \nabla^2 \psi^f - b_7 \Psi_2 - b_8 \frac{\partial \Psi_2}{\partial t} = 0, \quad (21)$$

$$\nabla^2 T^f - b_9 \nabla^2 \phi^f - b_{10} \frac{\partial T^f}{\partial t} = 0, \quad (22)$$

$$b_{11} \nabla^2 \phi^f - \frac{\partial}{\partial t} \phi^{*f} = 0, \quad (23)$$

where

$$a_1 = \frac{\kappa_1}{\mu_1 + \kappa_1}, \quad a_2 = \frac{\rho c_1^2}{\mu_1 + \kappa_1}, \quad a_3 = \frac{\kappa c_1^2}{\gamma_1 \omega^{*2}}, \quad a_4 = 2 a_3, \quad a_5 = \frac{\rho j c_1^2}{\gamma_1},$$

$$\varepsilon_1 = \frac{v^2 T_0}{K^* \omega^* \rho}, \quad b_1 = \frac{b \rho c_1^2}{(\lambda^f + 2\mu^f + \kappa^f) \omega^* v}, \quad b_2 = \frac{c_0 c_1^2}{(\lambda^f + 2\mu^f + \kappa^f) \omega^* v T_0},$$

$$b_3 = \frac{\rho^f c_1^2}{(\lambda^f + 2\mu^f + \kappa^f) \omega^*}, \quad b_4 = \frac{\kappa^f}{\mu^f + \kappa^f}, \quad b_5 = \frac{\rho^f c_1^2}{(\mu^f + \kappa^f) \omega^*}, \quad b_6 = \frac{\kappa^f c_1^2}{\gamma^f \omega^{*2}},$$

$$b_7 = 2b_6, \quad b_8 = \frac{I c_1^2}{\gamma^f \omega^*}, \quad b_9 = \frac{b v T_0^f}{K_1^* \rho \omega^*}, \quad b_{10} = \frac{\rho^f a T_0^f c_1^2}{K_1^* \omega^*}, \quad b_{11} = \frac{v T_0}{\rho^f c_1^2}$$

and $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator

The boundary conditions at the interface $x_3 = 0$ are the continuity of components of normal stress, tangential stress, tangential couple stress, normal velocity, tangential velocity, microrotation vector, temperature and temperature gradient. Mathematically these can be written as

$$T_{33} = T_{33}^f, \quad T_{31} = T_{31}^f, \quad m_{32} = m_{32}^f, \quad \frac{\partial u_3}{\partial t} = v_3, \quad \frac{\partial u_1}{\partial t} = v_1, \quad \frac{\partial \phi_2}{\partial t} = \Psi_2,$$

$$T = T^f, \quad K^* \frac{\partial T}{\partial x_3} = K_1^* \frac{\partial T^f}{\partial x_3} \quad (24)$$

IV. REFLECTION AND TRANSMISSION

We consider Longitudinal displacement wave (LD-wave), Thermal wave (T-wave), Coupled transverse and microrotational waves (CD-I wave and CD-II wave) propagating through medium M_1 and incident at the plane $x_3 = 0$ with its direction of propagation with angle θ_0 normal to the surface. Corresponding to each incident wave, we get reflected LD-wave, T-wave, CD-I and CD-II waves in medium M_1 and two coupled longitudinal waves and two coupled transverse waves in medium M_2 as shown in Fig.1.

In order to solve the equations (15)-(20), we assume the solutions of the form

$$\{\phi^s, T, \psi^s, \phi_2, \phi^f, \phi^{*f}, T^f, \psi^f, \Psi_2\} = \{\overline{\phi^s}, \overline{T}, \overline{\psi^s}, \overline{\phi_2}, \overline{\phi^f}, \overline{\phi^{*f}}, \overline{T^f}, \overline{\psi^f}, \overline{\Psi_2}\} e^{i\{k(x_1 \sin\theta - x_3 \cos\theta) - \omega t\}} \quad (25)$$

where k is the wave number and ω is the angular frequency and $\overline{\phi^s}, \overline{T}, \overline{\psi^s}, \overline{\phi_2}, \overline{\phi^f}, \overline{\phi^{*f}}, \overline{T^f}, \overline{\psi^f}, \overline{\Psi_2}$ are arbitrary constants.

Making use of equation (25) in equations (15)-(23), yield

$$V^4 + DV^2 + E = 0, \quad (26)$$

$$V^4 + D_1V^2 + E_1 = 0, \quad (27)$$

$$V^4 + D_2V^2 + E_2 = 0, \quad (28)$$

$$V^4 + D_3V^2 + E_3 = 0, \quad (29)$$

where

$$D = -\frac{\left[1 + (1 - i\omega\tau_1)\varepsilon_1\left(\frac{l}{\omega} + \eta_0\tau_0\right)\right]}{\left(\frac{l}{\omega} + \tau_0\right)} - 1, \quad E = \frac{1}{\left(\frac{l}{\omega} + \tau_0\right)},$$

$$D_1 = \left(-\frac{a_1a_3}{\omega^2a_2} - 1\right) \frac{1}{\left(a_5 - \frac{a_4}{\omega^2}\right)} - \frac{1}{a_2}, \quad E_1 = \frac{1}{\left(a_5 - \frac{a_4}{\omega^2}\right)a_2},$$

$$D_2 = \frac{i\omega}{b_3} \left(1 - \frac{ib_2b_{11}}{\omega}\right) + \frac{i\omega}{b_{10}} \left(1 + \frac{ib_1b_9}{\omega b_3}\right), \quad E_2 = \frac{\omega^2}{b_3b_{10}} \left(-1 + \frac{ib_2b_{11}}{\omega}\right),$$

$$D_3 = \frac{i\omega}{b_5} + i\omega \frac{\left(1 - \frac{ib_4b_6}{\omega b_5}\right)}{\left(b_8 + \frac{l}{\omega}b_7\right)}, \quad E_3 = -\frac{\omega^2}{\left(b_8 + \frac{l}{\omega}b_7\right)b_5},$$

and $V^2 = \frac{\omega^2}{k^2}$

Equation (26) and (27) are quadratic in V^2 , therefore the roots of these equations give four values of V^2 . Corresponding to each value of V^2 , there exist two types of waves in solid medium in decreasing order of their velocities, namely a LD-wave, T-wave and CD-I wave, CD-II wave. Let V_1, V_2 are the velocities of reflected LD-wave, T-wave and V_3, V_4 are the velocities of reflected CD-I wave, CD-II wave in medium M_1 . Similarly equation (28) and (29) gives four velocities \bar{V}_1, \bar{V}_2 and \bar{V}_3, \bar{V}_4 of two coupled longitudinal waves and two coupled transverse waves in medium M_2 .

In view of equation (25), the appropriate solutions of equations (15)-(23) for medium M_1 and medium M_2 take the form

Medium M_1 :

$$\{\phi^s, T\} = \sum_{i=1}^2 \{1, f_i\} [S_{0i} e^{i\{k_i(x_1 \sin \theta_{0i} - x_3 \cos \theta_{0i}) - \omega_i t\}} + P_i], \quad (30)$$

$$\{\psi^s, \phi_2\} = \sum_{j=3}^4 \{1, f_j\} [T_{0j} e^{i\{k_j(x_1 \sin \theta_{0j} - x_3 \cos \theta_{0j}) - \omega_j t\}} + P_j], \quad (31)$$

where

$$f_i = \frac{\varepsilon_i \omega_i^2 (\frac{l}{\omega_i} + \eta_0 \tau_0)}{-\frac{1}{V_i^2} + (\frac{l}{\omega_i} + \tau_0) - l \varepsilon_i \omega_i (\frac{l}{\omega_i} + \eta_0 \tau_0) (\frac{l}{\omega_i} + \tau_1)}, \quad f_j = \frac{a_4 (-a_2 + \frac{1}{V_j^2}) - a_1 a_3 \frac{1}{V_j^2}}{a_1 (a_5 - \frac{1}{V_j^2})}$$

$$\text{and } P_i = S_i e^{i\{k_i(x_1 \sin \theta_{0i} + x_3 \cos \theta_{0i}) - \omega_i t\}}, \quad P_j = T_j e^{i\{k_j(x_1 \sin \theta_{0j} + x_3 \cos \theta_{0j}) - \omega_j t\}}$$

Medium M_2 :

$$\{\phi^f, T^f, \phi^{*f}\} = \sum_{i=1}^2 \{1, \bar{f}_i, \bar{g}_i\} \bar{S}_i e^{i\{\bar{k}_i(x_1 \sin \bar{\theta}_i - x_3 \cos \bar{\theta}_i) - \bar{\omega}_i t\}}, \quad (32)$$

$$\{\psi^f, \Psi_2\} = \sum_{j=3}^4 \{1, \bar{f}_j\} \bar{T}_j e^{i\{\bar{k}_j(x_1 \sin \bar{\theta}_j - x_3 \cos \bar{\theta}_j) - \bar{\omega}_j t\}}, \quad (33)$$

where

$$\bar{f}_i = \frac{-b_3 b_9}{b_1 b_9 \frac{l}{\omega_i} + l \omega_i (1 - \frac{b_2 b_{11}}{\omega_i}) (\frac{1}{V_i^2} - b_{10} \frac{l}{\omega_i})}, \quad \bar{f}_j = \frac{-b_6 b_5 \frac{l}{\omega_j}}{(-\frac{1}{V_j^2} - \frac{b_7}{\omega_j^2} + b_8 \frac{l}{\omega_j} + \frac{b_4 b_6}{\omega_j^2})}, \quad \bar{g}_i = \frac{-b_{11} \omega_i}{V_i^2}$$

S_{0i}, T_{0j} are the amplitudes of incident (LD-wave, T-wave) and (CD-I, CD-II) waves respectively. S_i and T_j are the amplitudes of reflected (LD-wave, T-wave) and (CD-I, CD-II) waves and \bar{S}_i, \bar{T}_j are the amplitudes of transmitted two coupled longitudinal waves and transmitted (CD-I, CD-II) waves respectively.

Following Singh and Tomar[31], the extension of the Snell's law is given by

$$\frac{\sin \theta_0}{V_0} = \frac{\sin \theta_1}{V_1} = \frac{\sin \theta_2}{V_2} = \frac{\sin \theta_3}{V_3} = \frac{\sin \theta_4}{V_4} = \frac{\sin \bar{\theta}_1}{\bar{V}_1} = \frac{\sin \bar{\theta}_2}{\bar{V}_2} = \frac{\sin \bar{\theta}_3}{\bar{V}_3} = \frac{\sin \bar{\theta}_4}{\bar{V}_4} \quad (34)$$

$$\text{where } V_j = \frac{\omega}{k_j}, \bar{V}_j = \frac{\omega}{k_j} \quad (j=1, 2, 3, 4) \text{ at } x_3=0 \quad (35)$$

Making use of ϕ^s , ψ^s , ϕ^f and ψ^f in boundary conditions (24) and with the help of equations (34) and (35), we obtain a system of eight non-homogeneous equations which can be written as

$$\sum_{j=1}^8 a_{ij} Z_j = Y_i; \quad (i=1, 2, 3, 4, 5, 6, 7, 8) \quad (36)$$

where

$$\begin{aligned} a_{11} &= \left(d_1 + d_2 \left(1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0 \right) \right) + \left(\frac{V_1^2}{\omega^2} - \tau_1 \iota \frac{V_1^2}{\omega} \right) f_1, & a_{12} &= \left(d_1 + d_2 \left(1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0 \right) \right) \frac{V_2^2}{V_1^2} + \left(\frac{V_1^2}{\omega^2} - \tau_1 \iota \frac{V_1^2}{\omega} \right) f_2, \\ a_{17} &= d_3^f \frac{V_1^2}{V_3 V_0} \sin \theta_0 \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0}, & a_{18} &= d_3^f \frac{V_1^2}{V_4 V_0} \sin \theta_0 \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0}, \\ a_{21} &= -(2d_4 + d_5) \frac{V_1}{V_0} \sin \theta_0 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \\ a_{22} &= -(2d_4 + d_5) \frac{V_1^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\ a_{2j} &= d_4 \frac{V_1^2}{V_j^2} \left(1 - 2 \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{V_1^2}{V_j^2} \left(1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) - d_5 \frac{V_1^2}{\omega^2} f_j, \\ a_{25} &= -(2d_4^f + d_5^f) \frac{V_1^2}{V_1 V_0} \sin \theta_0 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \\ a_{26} &= -(2d_4^f + d_5^f) \frac{V_1^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\ a_{27} &= \left[d_4^f \frac{V_1^2}{V_3^2} \left(1 - 2 \frac{V_3^2}{V_0^2} \sin^2 \theta_0 \right) + d_5^f \left(\frac{V_1^2}{V_3^2} \left(\sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} \right) - \frac{V_1^2}{\omega^2} \bar{f}_3 \right) \right], \\ a_{28} &= \left[d_4^f \frac{V_1^2}{V_4^2} \left(1 - 2 \frac{V_4^2}{V_0^2} \sin^2 \theta_0 \right) + d_5^f \left(\frac{V_1^2}{V_4^2} \left(\sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} \right) - \frac{V_1^2}{\omega^2} \bar{f}_4 \right) \right], \\ a_{3i} &= 0, \quad a_{33} = \iota \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} f_3, \quad a_{34} = \iota \frac{V_3}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} f_4, \quad a_{35} = 0, \end{aligned}$$

$$\begin{aligned}
a_{36} &= 0, & a_{37} &= \omega p_1 \frac{V_3}{V_3} \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} \bar{f}_3, & a_{38} &= \omega p_1 \frac{V_3}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} \bar{f}_4, \\
a_{41} &= \omega \frac{V_1}{V_0} \sin \theta_0, & a_{42} &= \omega \frac{V_1}{V_0} \sin \theta_0, & a_{4j} &= -\omega \frac{V_1}{V_j} \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0}, \\
a_{45} &= a_{46} = -t \frac{V_1}{V_0} \sin \theta_0, & a_{47} &= -t \frac{V_1}{V_3} \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0}, \\
a_{48} &= -t \frac{V_1}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0}, & a_{51} &= \omega \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, & a_{52} &= \omega \frac{V_2}{V_1} \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\
a_{5j} &= \omega \frac{V_1}{V_0} \sin \theta_0, & a_{55} &= t \frac{V_1}{V_1} \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, & a_{56} &= t \frac{V_1}{V_2} \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\
a_{57} &= a_{58} = -t \frac{V_1}{V_0} \sin \theta_0, & a_{6i} &= f_i, & a_{6j} &= 0, & a_{65} &= -\bar{f}_1, \\
a_{66} &= -\bar{f}_2, & a_{67} &= a_{68} = 0, & a_{7i} &= 0, & a_{73} &= t \omega f_3, & a_{74} &= t \frac{V_3}{V_4} \omega f_4, \\
a_{75} &= 0, & a_{76} &= 0, & a_{77} &= \frac{V_3}{\omega} \bar{f}_3, & a_{78} &= \frac{V_3}{\omega} \bar{f}_4, & a_{81} &= t f_1 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \\
a_{82} &= t f_2 \frac{V_1}{V_2} \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, & a_{8j} &= 0, & a_{85} &= \omega p_2 \bar{f}_1 \frac{V_1}{\omega} \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \\
a_{86} &= \omega p_2 \bar{f}_2 \frac{V_1}{\omega} \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, & a_{87} &= 0, & a_{88} &= 0 \quad (i = 1, 2 \text{ and } j = 3, 4)
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
d_1 &= \frac{\lambda_1}{\rho c_1^2}, & d_2 &= \frac{(2\mu_1 + \kappa_1)}{\rho c_1^2}, & d_4 &= \frac{\mu_1}{\rho c_1^2}, & d_5 &= \frac{\kappa_1}{\rho c_1^2}, & d_1^{ff} &= \frac{b}{v}, & d_1^f &= \frac{c_0}{\rho v T_0}, \\
d_2^f &= \frac{\lambda^f \omega^*}{\rho c_1^2}, & d_3^f &= \frac{(2\mu^f + \kappa^f) \omega^*}{\rho c_1^2}, & d_4^f &= \frac{\mu^f \omega^*}{\rho c_1^2}, & d_5^f &= \frac{\kappa^f \omega^*}{\rho c_1^2}, & p_1 &= \frac{\gamma^f \omega^*}{\gamma_1}, \\
p_2 &= \frac{K_1^*}{K^*}
\end{aligned}$$

Considering the phase of the reflected waves for the incident LD-wave, T-wave and CD-I waves can be written by using equation (34) and (35) as

$$\frac{\cos \theta_i}{V_i} = \frac{1}{V_0} \left[\left(\frac{V_0}{V_i} \right)^2 - \sin^2 \theta_0 \right]^{1/2},$$

$$\frac{\cos \theta_j}{V_j} = \frac{1}{V_0} \left[\left(\frac{V_0}{V_j} \right)^2 - \sin^2 \theta_0 \right]^{1/2},$$

Following Schoenberg[36]. if we write

$$\frac{\cos \theta_i}{V_i} = \frac{\cos \theta'_i}{V'_i} + \frac{c_i}{V_0 2\pi}; \quad (i=1, 2, 3, 4, 5) \text{ then}$$

$$\frac{\cos \theta'_i}{V'_i} = \frac{1}{V_0} \operatorname{Re} \left\{ \left[\left(\frac{V_0}{V_i} \right)^2 - \sin^2 \theta_0 \right]^{1/2} \right\},$$

$$c_i = 2\pi \operatorname{Im} \left\{ \left[\left(\frac{V_0}{V_i} \right)^2 - \sin^2 \theta_0 \right]^{1/2} \right\},$$

where V'_i , the real phase speed and θ'_i , the angle of reflection are given by

$$\frac{V'_i}{V_0} = \frac{\sin \theta'_i}{\sin \theta_0} \left[\sin^2 \theta_0 + \left[\operatorname{Re} \left[\left(\frac{V_0}{V_i} \right)^2 - \sin^2 \theta_0 \right] \right]^2 \right]^{-1/2},$$

and c_i , the attenuation in a depth is equal to the wavelength of incident wave i.e. $2\pi \frac{V_0}{\omega}$

(1) For incident LD-wave:

$$A^* = S_{01}, \quad S_{02} = T_{03} = T_{04} = 0, \quad Y_1 = -a_{11}, \quad Y_2 = a_{21}, \quad Y_3 = a_{31} = 0, \quad Y_4 = -a_{41}, \\ Y_5 = a_{51}, \quad Y_6 = -a_{61}, \quad Y_7 = a_{71} = 0, \quad Y_8 = a_{81}$$

(2) For incident T-wave:

$$A^* = S_{02}, \quad S_{01} = T_{03} = T_{04} = 0, \quad Y_1 = -a_{12}, \quad Y_2 = a_{22}, \quad Y_3 = a_{32} = 0, \quad Y_4 = -a_{42}, \\ Y_5 = a_{52}, \quad Y_6 = -a_{62}, \quad Y_7 = a_{72} = 0, \quad Y_8 = a_{82}$$

(3) For incident CD-I wave:

$$A^* = T_{03}, \quad S_{01} = S_{02} = T_{04} = 0, \quad Y_1 = a_{13}, \quad Y_2 = -a_{23}, \quad Y_3 = a_{33}, \quad Y_4 = a_{43}, \\ Y_5 = -a_{53}, \quad Y_6 = a_{63} = 0, \quad Y_7 = -a_{73}, \quad Y_8 = a_{83} = 0$$

(4) For incident CD-II wave:

$$A^* = T_{04}, \quad S_{01} = S_{02} = T_{03} = 0, \quad Y_1 = a_{14}, \quad Y_2 = -a_{24}, \quad Y_3 = a_{34}, \quad Y_4 = a_{44}, \\ Y_5 = -a_{54}, \quad Y_6 = a_{64} = 0, \quad Y_7 = -a_{74}, \quad Y_8 = a_{84} = 0$$

and

$$Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{S_2}{A^*}, \quad Z_3 = \frac{T_3}{A^*}, \quad Z_4 = \frac{T_4}{A^*}, \quad Z_5 = \frac{\overline{S_1}}{A^*}, \quad Z_6 = \frac{\overline{S_2}}{A^*}, \quad Z_7 = \frac{\overline{T_3}}{A^*}, \quad Z_8 = \frac{\overline{T_4}}{A^*} \quad (38)$$

where Z_1, Z_2, Z_3, Z_4 are the complex amplitude ratios of reflected LD-wave, T-wave and coupled CD-I, CD-II waves in medium M_1 and Z_5, Z_6, Z_7, Z_8 are the complex amplitude ratios of transmitted LD-wave, T-wave and coupled CD-I, CD-II waves in medium M_2 .

V. PARTICULAR CASES

1. If $\eta_0 = 1, \tau_1 = 0$ in equation (36), then we obtain the corresponding amplitude ratios at an interface of micropolar thermoelastic solid with one relaxation time and heat conducting micropolar fluid half space.
2. If $\eta_0 = 0, \tau_1 > 0$ in equation (36), then we obtain the corresponding amplitude ratios at an interface of micropolar thermoelastic solid with two relaxation time and heat conducting micropolar fluid half space.
3. Neglecting the micropolarity effect in medium M_2 i.e. let $\kappa^f \rightarrow 0$, we obtain the amplitude ratios at the interface of micropolar generalized thermoelastic solid half space and thermal conducting fluid half space as

$$\sum_{j=1}^7 a_{ij} Z_j = Y_i ; (i = 1, 2, 3, 4, 5, 6, 7)$$

(1) For incident LD-wave:

$$A^* = S_{01}, S_{02} = T_{03} = T_{04} = 0, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = a_{31} = 0, Y_4 = -a_{41},$$

$$Y_5 = a_{51}, Y_6 = -a_{61}, Y_7 = a_{71} = 0$$

(2) For incident T-wave:

$$A^* = S_{02}, S_{01} = T_{03} = T_{04} = 0, Y_1 = -a_{12}, Y_2 = a_{22}, Y_3 = a_{32} = 0, Y_4 = -a_{42},$$

$$Y_5 = a_{52}, Y_6 = -a_{62}, Y_7 = a_{72} = 0$$

(3) For incident CD-I wave:

$$A^* = T_{03}, S_{01} = S_{02} = T_{04} = 0, Y_1 = a_{13}, Y_2 = -a_{23}, Y_3 = a_{33}, Y_4 = a_{43},$$

$$Y_5 = -a_{53}, Y_6 = a_{63} = 0, Y_7 = a_{73} = 0$$

(4) For incident CD-II wave:

$$A^* = T_{04}, S_{01} = S_{02} = T_{03} = 0, Y_1 = a_{14}, Y_2 = -a_{24}, Y_3 = a_{34}, Y_4 = a_{44},$$

$$Y_5 = -a_{54}, Y_6 = a_{64} = 0, Y_7 = a_{74} = 0$$

where the values of a_{ij} are given by equation (37) with changed values as

$$a_{25} = -2d_4^f \frac{V_1^2}{V_1 V_0} \sin \theta_0 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \quad a_{26} = -2d_4^f \frac{V_1^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{27} = -\left[d_4^f \frac{V_1^2}{V_3^2} \left(1 - 2 \frac{V_3^2}{V_0^2} \sin^2 \theta_0 \right) \right], \quad a_{37} = 0, \quad a_{71} = \iota \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0} f_1,$$

$$a_{72} = \iota \frac{V_1}{V_2} \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0} f_2, \quad a_{7j} = 0,$$

$$a_{75} = \varpi_2 \cos \bar{\theta}_1 f_1 \frac{V_1}{\omega}, \quad a_{76} = \varpi_2 \cos \bar{\theta}_2 f_2 \frac{V_1}{\omega}, \quad a_{77} = 0 \quad (i = 1, 2 \text{ and } j = 3, 4) \quad (39)$$

4. By neglecting the thermal effect in medium M_1 and medium M_2 , we obtain the amplitude ratios at the interface of micropolar elastic solid half space and fluid half space as

$$\sum_{j=1}^6 a_{ij} Z_j = Y_i ; (i = 1, 2, 3, 4, 5, 6)$$

where the values of a_{ij} are given by

$$\begin{aligned}
 a_{11} &= \left(d_1 + d_2 \left(1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0 \right) \right), & a_{12} &= d_2 \frac{V_1^2}{V_3 V_0} \sin \theta_0 \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{13} &= d_2 \frac{V_1^2}{V_4 V_0} \sin \theta_0 \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0}, & a_{14} &= - \left[\left(d_2^f + d_3^f \left(1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0 \right) \right) \frac{V_1^2}{V_2^2} \right], \\
 a_{15} &= d_3^f \frac{V_1^2}{V_3 V_0} \sin \theta_0 \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0}, & a_{16} &= d_3^f \frac{V_1^2}{V_4 V_0} \sin \theta_0 \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{21} &= -(2d_4 + d_5) \frac{V_1}{V_0} \sin \theta_0 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{22} &= d_4 \frac{V_1^2}{V_3^2} \left(1 - 2 \frac{V_3^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{V_1^2}{V_3^2} \left(1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0 \right) - d_5 \frac{V_1^2}{\omega^2} f_3, \\
 a_{23} &= d_4 \frac{V_1^2}{V_4^2} \left(1 - 2 \frac{V_4^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{V_1^2}{V_4^2} \left(1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0 \right) - d_5 \frac{V_1^2}{\omega^2} f_4, \\
 a_{24} &= -(2d_4^f + d_5^f) \frac{V_1^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{25} &= - \left[d_4^f \frac{V_1^2}{V_3^2} \left(1 - 2 \frac{V_3^2}{V_0^2} \sin^2 \theta_0 \right) + d_5^f \left(\frac{V_1^2}{V_3^2} \left(\sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} \right) - \frac{V_1^2}{\omega^2} f_3 \right) \right], \\
 a_{26} &= - \left[d_4^f \frac{V_1^2}{V_4^2} \left(1 - 2 \frac{V_4^2}{V_0^2} \sin^2 \theta_0 \right) + d_5^f \left(\frac{V_1^2}{V_4^2} \left(\sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} \right) - \frac{V_1^2}{\omega^2} f_4 \right) \right], \\
 a_{31} &= 0, & a_{32} &= \iota \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} f_3, & a_{33} &= \iota \frac{V_3}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} f_4, & a_{34} &= 0, \\
 a_{35} &= \varpi_1 \frac{V_3}{V_3} \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} \bar{f}_3, & a_{36} &= \varpi_1 \frac{V_3}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} \bar{f}_4, \\
 a_{41} &= \omega_1 \frac{V_1}{V_0} \sin \theta_0, & a_{42} &= -\omega_3 \frac{V_1}{V_3} \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0}, & a_{43} &= -\omega_4 \frac{V_1}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{44} &= -\iota \frac{V_1}{V_0} \sin \theta_0, & a_{45} &= -\iota \frac{V_1}{V_3} \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0}, & a_{46} &= -\iota \frac{V_1}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{51} &= \omega_1 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, & a_{52} &= \omega \frac{V_1}{V_0} \sin \theta_0, & a_{53} &= \omega \frac{V_1}{V_0} \sin \theta_0, \\
 a_{54} &= \iota \frac{V_1}{V_1} \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, & a_{55} &= a_{56} = -\iota \frac{V_1}{V_0} \sin \theta_0, & a_{61} &= 0, & a_{62} &= \iota \frac{V_1}{V_3} \omega_3 f_3,
 \end{aligned}$$

$$a_{63} = \iota \frac{V_1}{V_4} \omega f_4, \quad a_{64} = 0, \quad a_{65} = \frac{V_1}{\omega} \overline{f_3}, \quad a_{66} = \frac{V_1}{\omega} \overline{f_4} \quad (40)$$

and

$$Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{T_3}{A^*}, \quad Z_3 = \frac{T_4}{A^*}, \quad Z_4 = \frac{\overline{S_1}}{A^*}, \quad Z_5 = \frac{\overline{T_3}}{A^*}, \quad Z_6 = \frac{\overline{T_4}}{A^*}$$

where Z_1, Z_2, Z_3 are the complex amplitude ratios of reflected LD-wave and coupled CD-I, CD-II waves in medium M_1 and Z_4, Z_5, Z_6 are the complex amplitude ratios of transmitted LD-wave and coupled CD-I, CD-II waves in medium M_2 .

The above results are similar as those obtained by Singh and Tomar[31], by changing the dimensionless quantities into physical quantities.

5. If the upper medium i.e. M_2 is neglected then we obtain the amplitude ratios at the free surface of micropolar generalized thermoelastic solid half space as

$$\sum_{j=1}^4 a_{ij} Z_j = Y_i; (i = 1, 2, 3, 4)$$

where the values of a_{ij} are given by

$$a_{11} = \left(d_1 + d_2 \left(1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0 \right) \right) + \left(\frac{V_1^2}{\omega^2} - \tau_1 \iota \frac{V_1^2}{\omega} \right) f_1,$$

$$a_{12} = \left(d_1 + d_2 \left(1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0 \right) \right) \frac{V_2^2}{V_1^2} + \left(\frac{V_1^2}{\omega^2} - \tau_1 \iota \frac{V_1^2}{\omega} \right) f_2,$$

$$a_{1j} = d_2 \frac{V_j^2}{V_0^2} \sin \theta_j \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{21} = -(2d_4 + d_5) \frac{V_1}{V_0} \sin \theta_0 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{22} = -(2d_4 + d_5) \frac{V_1^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{2j} = d_4 \frac{V_1^2}{V_j^2} \left(1 - 2 \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{V_1^2}{V_j^2} \left(1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) - d_5 \frac{V_1^2}{\omega^2} f_j,$$

$$a_{3i} = 0, \quad a_{33} = \iota \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} f_3, \quad a_{34} = \iota \frac{V_3}{V_4} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} f_4,$$

$$a_{41} = \iota f_1 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \quad a_{42} = \iota f_2 \frac{V_1}{V_2} \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \quad a_{4j} = 0 \quad (j = 3, 4)$$

and

$$Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{S_2}{A^*}, \quad Z_3 = \frac{T_3}{A^*}, \quad Z_4 = \frac{T_4}{A^*}$$

where Z_1, Z_2, Z_3, Z_4 are the complex amplitude ratios of reflected LD-wave, T-wave and coupled CD-I, CD-II waves in medium M_1 .

The above results are similar as those obtained by Kumar and Singh[35] after changing the dimensionless quantities into the physical quantities.

(a) **Subcase:** If we neglect thermal effect in case 5 then we obtain the amplitude ratios at the free surface of micropolar elastic solid half space as

$$\begin{aligned}
 a_{11} &= \left(d_1 + d_2 \left(1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0 \right) \right) + \left(\frac{V_1^2}{\omega^2} - \tau_1 \iota \frac{V_1^2}{\omega} \right) f_1, \\
 a_{12} &= \left(d_1 + d_2 \left(1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0 \right) \right) \frac{V_2^2}{V_1^2}, \quad a_{1j} = d_2 \frac{V_j^2}{V_0^2} \sin \theta_j \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{21} &= -(2d_4 + d_5) \frac{V_1}{V_0} \sin \theta_0 \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{22} &= -(2d_4 + d_5) \frac{V_2^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\
 a_{2j} &= d_4 \frac{V_1^2}{V_j^2} \left(1 - 2 \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{V_1^2}{V_j^2} \left(1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) - d_5 \frac{V_1^2}{\omega^2} f_j, \\
 a_{31} &= a_{32} = 0, \\
 a_{33} &= \iota \sqrt{1 - \frac{V_3^2}{V_0^2} \sin^2 \theta_0} f_3, \quad a_{34} = \iota \frac{V_1}{V_3} \sqrt{1 - \frac{V_4^2}{V_0^2} \sin^2 \theta_0} f_4, \quad (j = 3, 4)
 \end{aligned} \tag{41}$$

and

$$Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{T_3}{A^*}, \quad Z_3 = \frac{T_4}{A^*}$$

where Z_1, Z_2, Z_3 are the complex amplitude ratios of reflected LD-wave and coupled CD-I, CD-II waves in medium M_1 .

The above results are similar as that obtained by Parfit and Eringen[33] by changing the dimensionless quantities into the physical quantities.

VI. NUMERICAL RESULTS AND DISCUSSION

The following values of relevant parameters for both the half spaces for numerical computations are taken.

Following Singh and Tomar[31], the values of micropolar constants for medium M_1 are taken as:

$$\begin{aligned}
 \lambda &= 0.209730 \times 10^{10} Nm^{-2}, & \mu &= 0.91822 \times 10^9 Nm^{-2}, \\
 \kappa &= 0.22956 \times 10^9 Nm^{-2}, & \gamma &= 0.0000423 \times 10^5 N, \\
 \hat{j} &= 0.037 \times 10^{-2} m^2, & \rho &= 0.0034 \times 10^3 Kgm^{-3},
 \end{aligned}$$

and thermal parameters are taken from Dhaliwal and Singh[34] :

$$\begin{aligned} \nu &= 0.268 \times 10^7 \text{ Nm}^{-2} \text{ K}^{-1}, & c^* &= 1.04 \times 10^3 \text{ NmKg}^{-1} \text{ K}^{-1}, \\ T_0 &= 0.298 \text{ K}, & K^* &= 1.7 \times 10^2 \text{ Nsec}^{-1} \text{ K}^{-1}, \\ \tau_0 &= 0.613 \times 10^{-12} \text{ sec}, & \tau_1 &= 0.813 \times 10^{-12} \text{ sec}, & \omega &= 1 \end{aligned}$$

Following Singh and Tomar[31], the values of micropolar constants for medium M_2 are taken as:

$$\begin{aligned} \lambda^f &= 0.15 \times 10^8 \text{ Nsecm}^{-2}, & \mu^f &= 0.03 \times 10^8 \text{ Nsecm}^{-2}, \\ \kappa^f &= 0.000223 \times 10^8 \text{ Nsecm}^{-2}, & \gamma^f &= 0.0000222 \times 10^8 \text{ Nsec}, \\ \rho^f &= 0.8 \times 10^3 \text{ Kgm}^{-3}, & I &= 0.00400 \times 10^{-16} \text{ Nsec}^2 \text{ m}^{-2} \end{aligned}$$

Thermal Parameters for the medium M_2 are taken as of comparable magnitude:

$$\begin{aligned} T_0^f &= 0.196 \text{ K}, & K_1^* &= 0.89 \times 10^2 \text{ Nsec}^{-1} \text{ K}^{-1}, \\ c_0 &= 0.005 \times 10^{11} \text{ Nsec}^2 \text{ m}^{-6}, & a &= 1.5 \times 10^5 \text{ m}^2 \text{ sec}^{-2} \text{ K}^{-2} \\ Q_1 &= 5, Q_2 = 10, Q_3 = 15, Q_4 = 20, \bar{Q}_1 = 3, \bar{Q}_2 = 8, \bar{Q}_3 = 10, \bar{Q}_4 = 12 \end{aligned}$$

The values of amplitude ratios have been computed at different angles of incidence.

In Figure 2((a)-(h))-4((a)-(h)), we represent the solid line for incident wave for L-S theory with viscoelastic effect (VLS), small dashes line for incident wave for G-L theory with viscoelastic effect(VGL), dash dot dash line for incident wave for L-S theory in the absence of viscoelastic effect (LS) and large dashes line for incident wave for G-L theory in the absence of viscoelastic effect (GL).

VII. LD-WAVE INCIDENT

Variations of amplitude ratios $|Z_i|$; $1 \leq i \leq 8$ with the angle of incidence θ_0 , for incident LD- wave are shown in Figure 2(a) through 2(h).

Figure 2(a) depicts that the values of amplitude ratio $|Z_1|$ for LS, VLS, GL and VGL decrease in the range $0^\circ < \theta_0 < 65^\circ$ and after that value of amplitude ratio increases in the further range. It is noticed that the values of $|Z_1|$ for VLS remain less than the values for LS which shows the effect of viscosity..

Figure 2(b) depicts that the values of $|Z_2|$ for LS, GL, VLS and VGL first increase in small range and after that the values of amplitude ratio decrease as angle of incidence increases. The values of amplitude ratio for VLS remain more than the values for VGL in the whole range which shows the effect of thermal relaxation time.

Figure 2(c) shows that the values of $|Z_3|$ for LS, GL, VLS and VGL increase in the interval $0^\circ < \theta_0 < 50^\circ$ and after that the values of amplitude ratio decrease as θ_0 increases. It is observed that the values of amplitude ratio for LS are greater than the values for VLS in the range $15^\circ < \theta_0 < 75^\circ$ which shows the effect of viscosity.

Figure 2(d) depicts that variation in the values of $|Z_4|$ for GL, LS, VGL and VLS is same as in $|Z_3|$ but different in magnitude. The values of $|Z_4|$ for VLS are greater in comparison than the values for LS and values for VGL are greater than the values for GL in the whole range.

Figure 2(e) depicts that the values of $|Z_5|$ for LS,VLS,GL and VGL decrease in the range .the values of amplitude ratio for VLS are greater than the values of amplitude ratio for VGL in the whole domain. Which shows the thermal relaxation effect.

Figure 2(f) shows that the values of $|Z_6|$ for LS.VLS,GL and VGL first remain constant after that the values of amplitude ratio decrease in the remaining domain. And values of amplitude ratio for LS remain more than the values for VLS in the whole range.

From figure 2(g) It is noticed that the values of $|Z_7|$ for LS,GL,VLS and VGL increase in the range $0^0 < \theta_0 < 45^0$ and decrease in the further range and maximum value of $|Z_7|$ for LS,VLS,GL and VGL is attained in the range $50 < \theta_0 < 55^0$.

From figure 2(h) It is noticed that the variation of the vales of $|Z_8|$ for LS,GL,VLS and VGL are same as variation in $|Z_7|$ with different in magnitude.

T-Wave Incident

Variations of amplitude ratios $|Z_i|; 1 \leq i \leq 8$ with the angle of incidence θ_0 , for incident T-wave are shown in Figure 3(a)-3(h)

Figure 3(a) depicts that in the beginning the value of $|Z_1|$ for LS, GL,VLS and VGL oscillates after that the values of amplitude ratio decrease with the increase in the angle of incidence.The values of amplitude ratio for VLS are greater than the values for LS in the range $35^0 < \theta_0 < 75^0$. Which shows the effect of viscosity.

It is depicted from Figure3(b) that the values of $|Z_2|$ for LS, GL, VLS and VGL first oscillate slightly as angle of incidence increases in some part of range and after that the value of amplitude ratio increase with increase in θ_0 and the values of amplitude ratio forVLS remain more than the values of amplitude ratio forVGL in $40^0 < \theta_0 < 85^0$.which shows the effect of thermal relaxation time

Figure 3(c) it is noticed that the values of $|Z_3|$ for VLS and VGL first increase sharply in the range $0^0 < \theta_0 < 55^0$ and after that the values of amplitude ratiodecrease sharply as the angle of incidence increases .The values of amplitude ratio for VLS are greater than the values for VGL in the range $40^0 < \theta_0 < 80^0$.Which shows the effect of thermal relaxation time.the values of amplitude ratio for LS and GL first increase slowly andthen decrease slowly.

The values of amplitude ratio for VLS remains more than the values of amplitude ratio for LS in the whole range which shows the effect of viscosity.

It is noticed fromFigure3 (d) . that the values of $|Z_4|$ for LS,GL,VLS and VGL first increase sharply in the range $0^0 < \theta_0 < 55^0$ and after that the values of amplitude ratiodecrease sharply as the angle of incidence increases the value of amplitude ratio for LS are greater than the value of amplitude ratio for VLS in the whole range .

Figure 3(e) depicts that the values of $|Z_5|$ for LS, GL,VLS and VGLfirst remain constant in some part of range after that the values of amplitude ratio decrease in the whole range. The values of amplitude ratio for VLS are greater than the values for VGLexcept near the grazing incidence where the values coincide.

Figure 3(f) depicts that the behaviour of variation of values of $|Z_6|$ for LS, GL, VLS and VGL decrease in the whole range. The values of amplitude ratio for VLS are greater than the values for VGL except near the grazing incidence where the values coincide.

From Figure 3(g) It is noticed that the variation in the values of $|Z_7|$ for LS, GL, VLS and VGL is similar to the behavior of variation of values $|Z_4|$ with difference in magnitude for the whole range. Due to thermal relaxation effect the values of amplitude ratio for VLS remain more than the values for VGL in the whole domain.

From figure 3(h) It is noticed that the values of $|Z_8|$ for similar to the behavior of variation of values $|Z_7|$ with difference in magnitude for the whole range.

CD-Wave Incident

Variations of amplitude ratios $|Z_i|; 1 \leq i \leq 8$, with the angle of incidence θ_0 , for incident SV-wave are shown in Figure 4(a)-4(h).

Figure 4(a) depicts that the value of $|Z_1|$ for LS, VLS, GL and VGL increase fastly as angle of incidence increases in the range $0^\circ < \theta_0 < 40^\circ$ after that the values of amplitude ratio decrease fastly as angle of incidence increases. The maximum values for LS, VLS, GL and VGL is attained at $\theta_0 = 40^\circ$. The values of amplitude ratio for LS, VLS, GL and VGL approximately coincide in the whole range.

Figure 4(b) shows that the values of $|Z_2|$ for LS, VLS and VGL increase in $0^\circ < \theta_0 < 40^\circ$ as the angle of incidence increases and then decrease in remaining range as the angle of incidence increases. The maximum value of amplitude ratio is attained between $35^\circ < \theta_0 < 45^\circ$. The value of amplitude ratio for LS are greater than the values of amplitude ratio for VLS in the range $35^\circ < \theta_0 < 65^\circ$. Which shows the effect of viscosity.

Figure 4(c) depicts that the values of $|Z_3|$ for LS, GL, VLS and VGL decreases in $0^\circ \leq \theta_0 \leq 30^\circ$ and the value of amplitude ratio increases in the remaining range. The value of amplitude ratio attained its minimum value for LS, VLS, GL and VGL between $30^\circ \leq \theta_0 \leq 35^\circ$. The values of amplitude ratio for LS, VLS, GL and VGL approximately coincide in the whole range.

Figure 4(d) shows that the values of $|Z_4|$ for LS, GL, VLS and VGL decrease with slight oscillation in the whole range. Due to viscosity the values of amplitude ratio for LS remain more than the values for VLS in the range $40^\circ \leq \theta_0 \leq 70^\circ$.

Figure 4(e) depicts that the variation of values of $|Z_5|$ is similar to the variation of $|Z_1|$ with slight difference in magnitude. Due to viscosity the value of amplitude ratio for LS remains more than the value for VLS in the domain $0^\circ \leq \theta_0 \leq 42^\circ$ and after that the behavior is reversed.

It is noticed from Figure 4(f) that the values of $|Z_6|$ for LS, GL, VLS and VGL oscillate in the whole range with difference in magnitude. The values of amplitude ratio for LS are greater than the values for VLS in the domain $30^\circ \leq \theta_0 \leq 60^\circ$. Which shows the effect of viscosity.

Figure 4(g) depicts that the values of $|Z_7|$ for LS, GL, VLS and VGL decrease with slight oscillation in the whole domain. The values of amplitude ratio for VLS are greater than the values for LS in the whole range. From figure 4(h) It is noticed that the variation in the values of $|Z_8|$ for VLS, VGL and GL is similar to the variation in the values of $|Z_7|$ with difference in magnitude.

VIII. CONCLUSION

In this paper reflection and transmission of plane waves at the boundary surface of heat conduction, micropolar viscoelastic solid and micropolar fluid are presented. The amplitude ratio for the incidence of (LD-wave, T-wave and CD-wave) are obtained. These amplitude ratios are function of angle of incidence, frequency, viscosity and micropolarity. A significant micropolarity, viscosity and thermal relaxation effects are observed on the amplitude ratios for incidence of various plane waves (LD-wave, T-wave, and CD-wave). It is observed that the values of $|Z_i|; i = 3, 6$ for LS remain more than the values for VLS when LD-wave is incident. The values of amplitude ratio $|Z_i|; i = 1, 3$, for VLS are greater than the values for LS when T-wave is incident. The values of amplitude ratio $|Z_i|; i = 2, 4, 5, 6$ for LS are greater than the values for VLS when CD-wave is incident. It reveals the effect of viscosity. It is seen that when LD-wave is incident, the values of amplitude ratio $|Z_i|; i = 2, 5$, for VLS are greater as compared to the values for VGL that reveals the effect of micropolarity. It is noticed that the values of $|Z_i|; i = 5, 6$, for VLS remain more than the values for VGL when T-wave is incident. For the incidence of CD-wave the value of $|Z_5|$ for VLS is greater than the value for VGL that reveals the effect of micropolarity. The model considered is one of the more realistic of the earth model and it may be of interest for experimental seismological and geophysics researchers in exploration of valuable material. It is also useful for the investigation of concerning with earthquake other phenomena and seismology.

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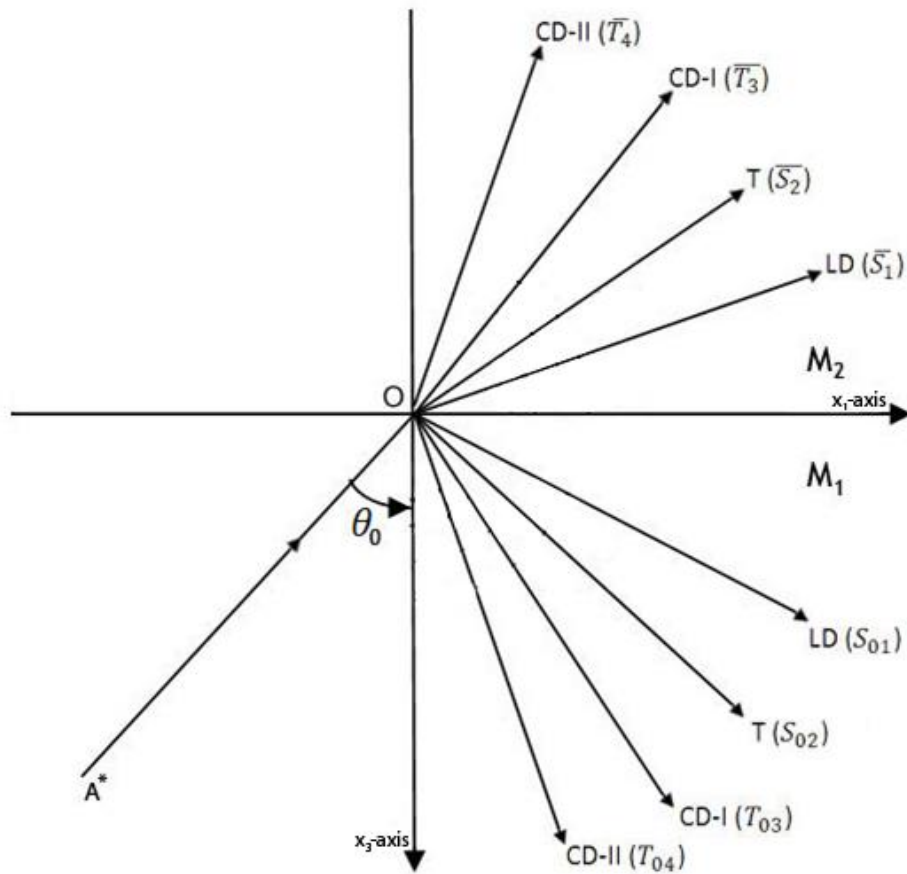


Figure 1. Geometry of the problem

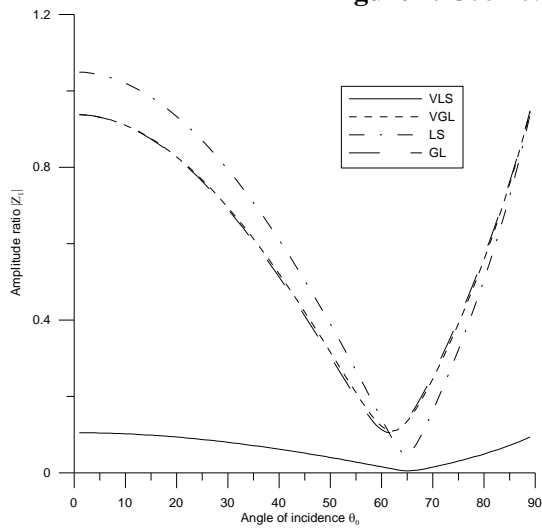


Figure 2(a)

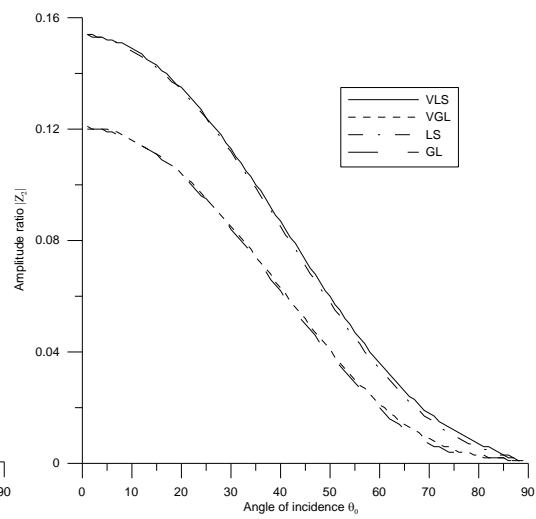


Figure 2(b)

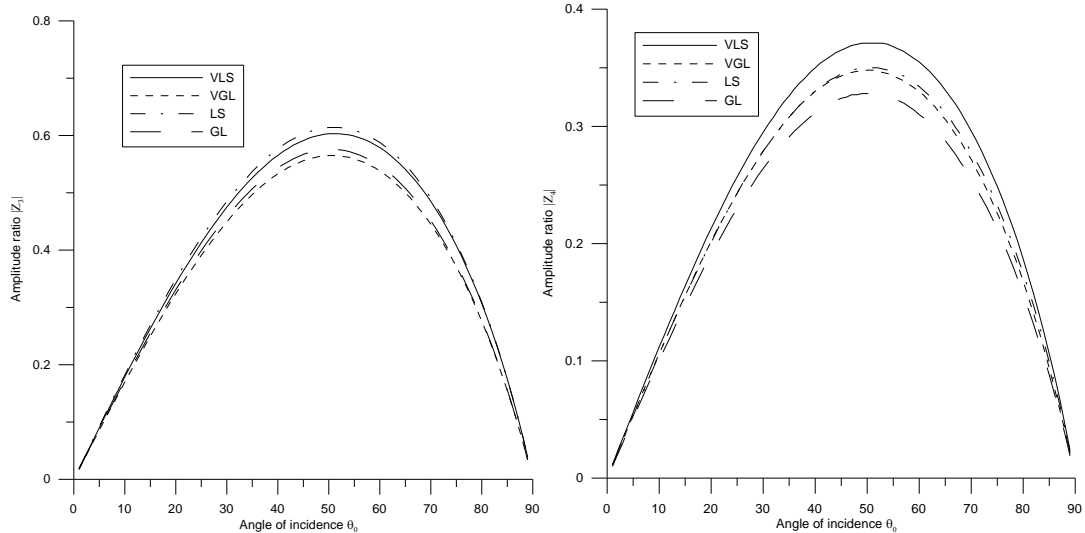


Figure 2(c)

Figure 2(d)

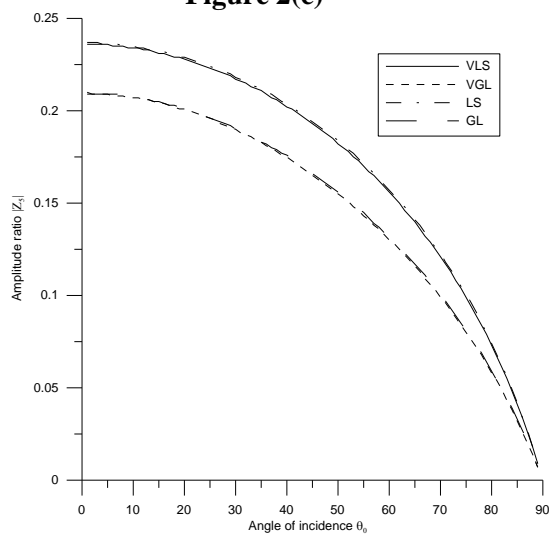


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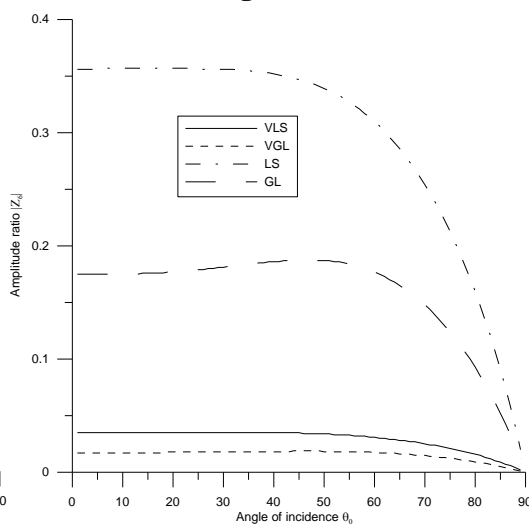


Figure 2(f)

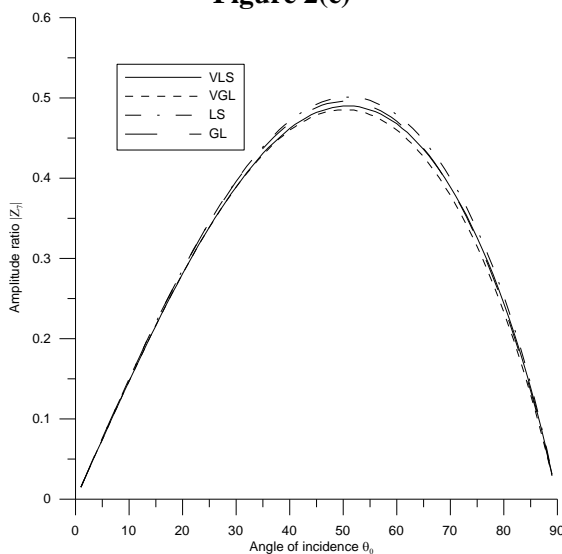


Figure 2(g)

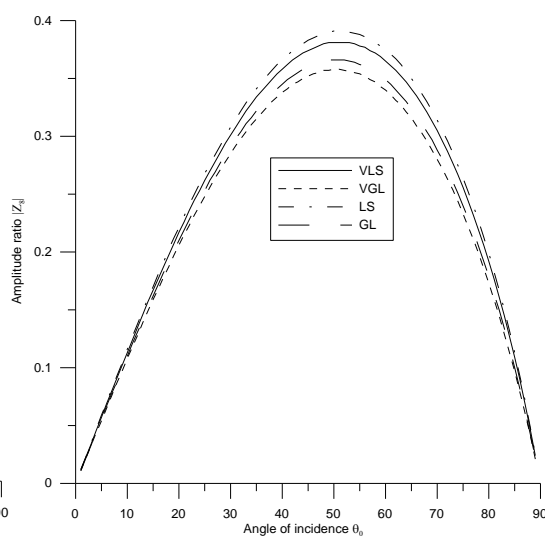


Figure 2(h)

Figure 2. (a)-2(h) Variations of amplitude ratios with angle of incidence for incidence of LD-wave

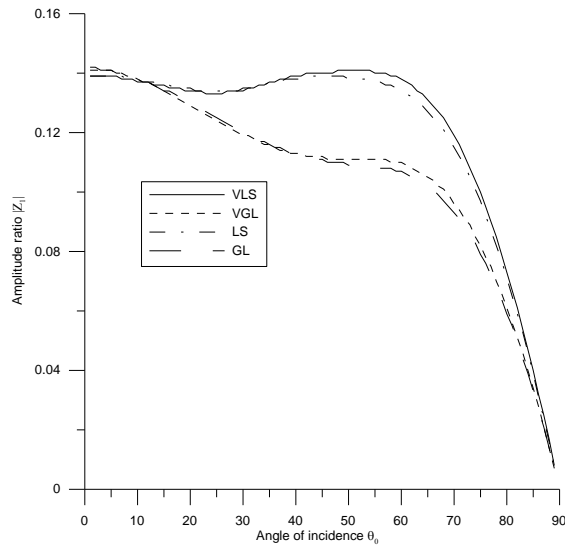


Figure 3(a)

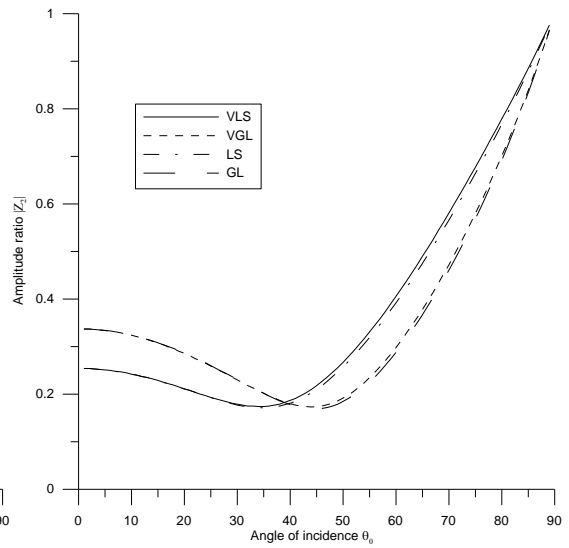


Figure 3(b)

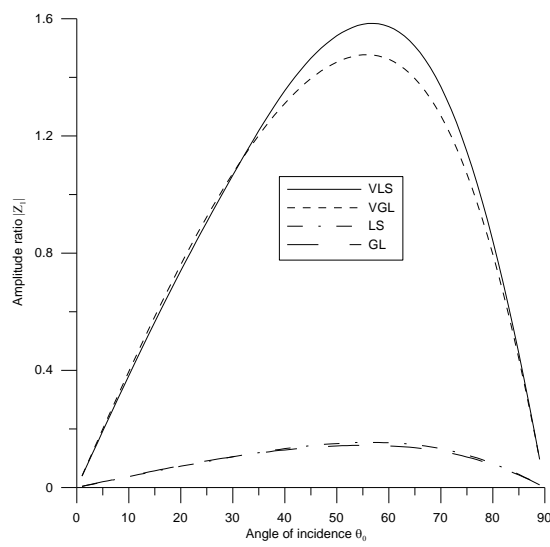


Figure 3(c)

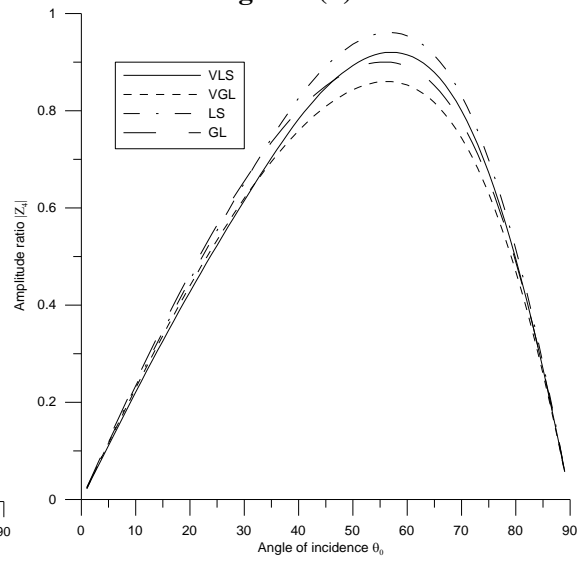


Figure 3(d)

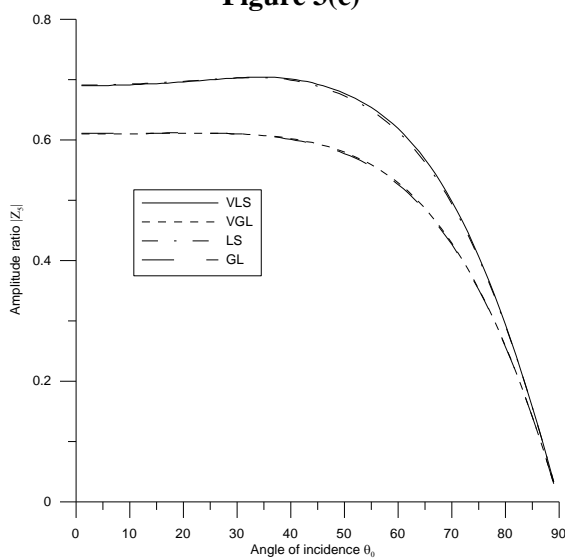


Figure 3(e)

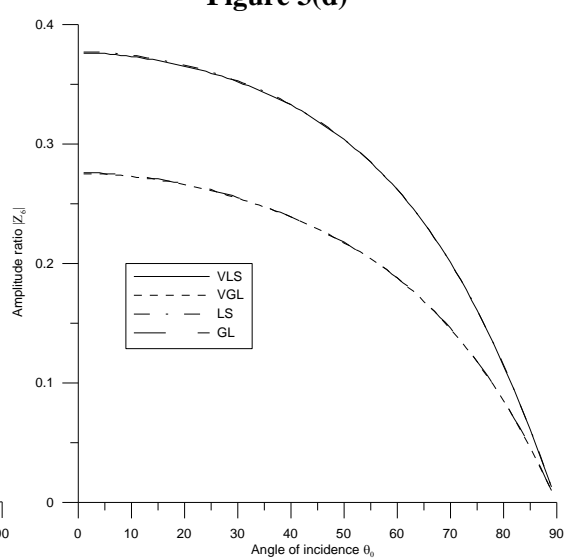


Figure 3(f)

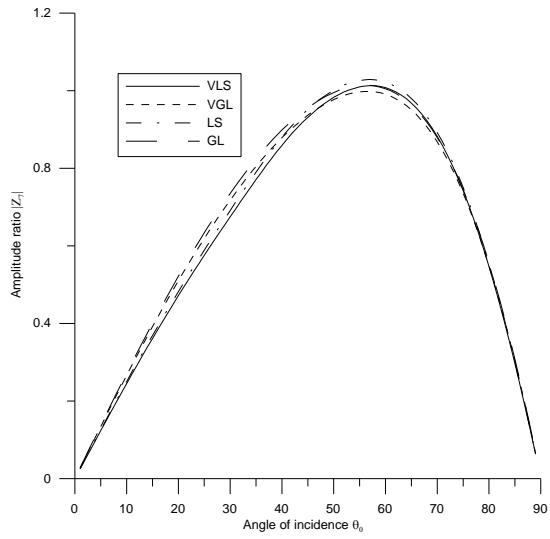


Figure 3(g)

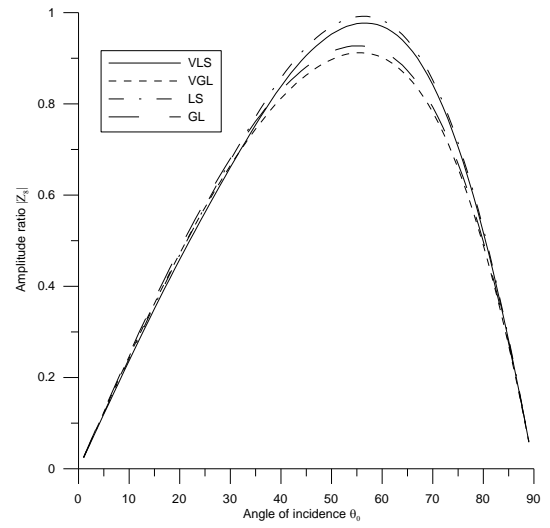


Figure 3(h)

Figure 3. (a)-3(h) Variations of amplitude ratios with angle of incidence for incidence of T-wave

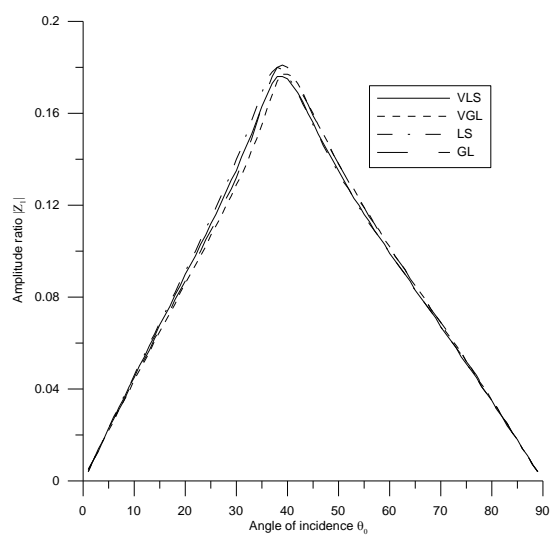


Figure 4(a)

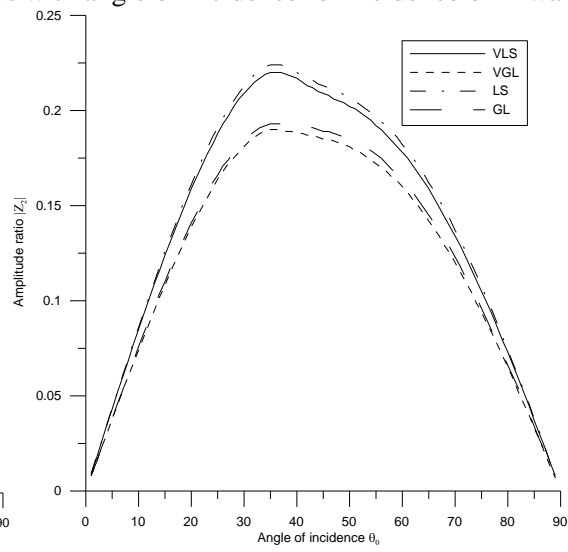


Figure 4(b)

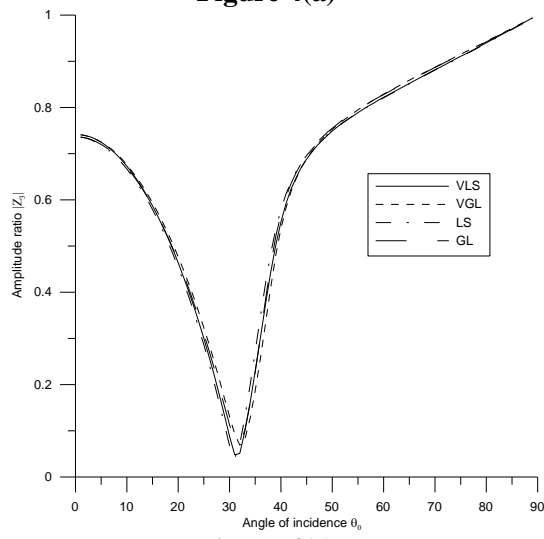


Figure 4(c)

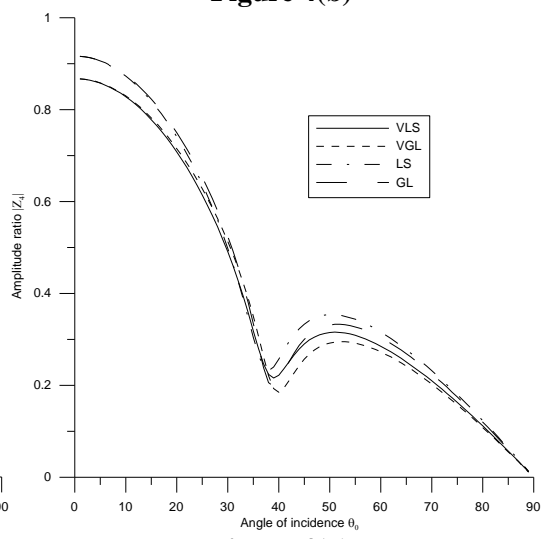


Figure 4(d)

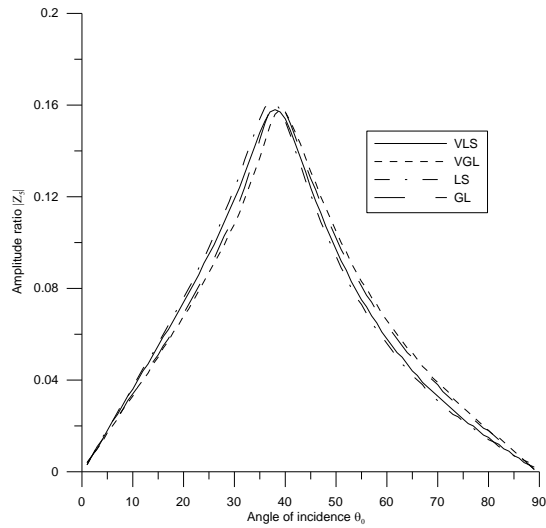


Figure 4(e)

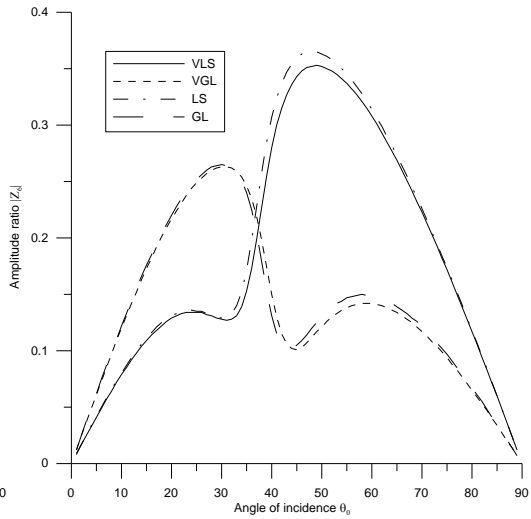


Figure 4(f)

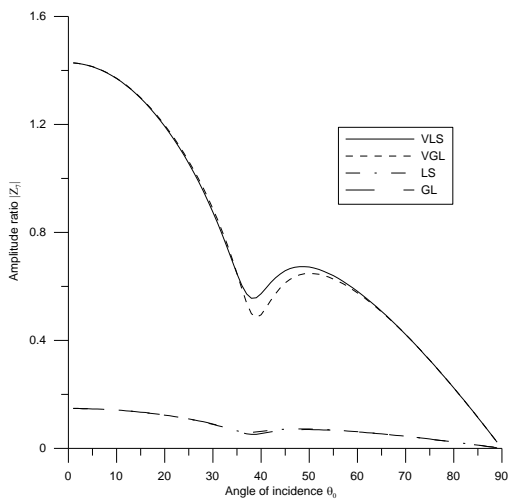


Figure 4(g)

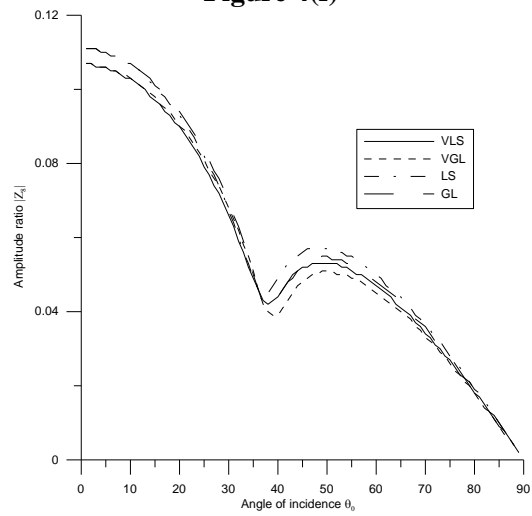


Figure 4(h)

Figure 4. (a)-4(h) Variations of amplitude ratios with angle of incidence for incidence of CD-I wave