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The Enhanced Ensemble Empirical Mode Decomposition for Analyzing Non Linear and Non Stationary Signals

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ABSTRACT

In this paper an algorithm of Enhanced Ensemble Empirical Mode Decomposition (EEEMD) is presented. Empirical Mode Decomposition (EMD) is an adaptive algorithm used for analyzing non linear and non stationary data which works by breaking the signal into a number of amplitude and frequency modulated (AM/FM) zero mean signals which are termed as Intrinsic Mode Functions(IMFs).but EMD experiences "Mode mixing" problem To overcome this problem Ensemble Empirical Mode Decomposition (EEMD) was proposed. The EEMD approach performs the EMD over an ensemble of original signal consists of sifting an ensemble of white noise added signal and treats the mean as the final true result. This approach will put an end to EMD mode mixing problem, however EEMD produced results does not satisfy the strict definition of IMF. To overcome this drawback, in the method here proposed, a unique residue is computed by adding noise at each stage of decomposition to obtain each IMF. The resulting decomposition is complete, with a numerically negligible error. Two examples are presented: a discrete Dirac delta function and an electrocardiogram signal. When compared with EEMD the new method here presented needs lesser number of iterations, thereby reducing the computational cost and an exact signal reconstruction, which is not possible with EEMD.

Keywords: Empirical Mode Decomposition, Ensemble Empirical Mode Decomposition,Mode Mixing Problem

I. INTRODUCTION

Empirical Mode Decomposition (EMD) [1] is an adaptive algorithm used for analysis of non-linear and non-stationary signals. It works by breaking the signal in to a number of amplitude and frequency modulated (AM/FM) zero mean signals which are termed as Intrinsic Mode Functions(IMFs).EMD is found a vast number of diverse applications such as biomedical, watermarking and audio processing to name a few However, EMD experiences "Mode mixing" problem which is defined as either a single IMF consisting of widely different range of frequencies or a signal of similar existing in different IMFs. Due to this problem, the exact signal reconstruction is not possible. To overcome these problems, a new method was proposed: the Ensemble Empirical Mode Decomposition (EEMD) [2]. The main idea of EEMD is, it averages the imfs obtained by several realizations of Gaussian white noise added to the original signal. It eliminates the annoying mode mixing by having filed all the scale space uniformly. However it creates some new ones. Indeed, the reconstructed signal includes residual noise and different realizations of signal plus noise may produce different number of imfs. In order to overcome these conditions, in this paper we proposed an

Improvement of the EEMD algorithm that provides an exact reconstruction of the original signal and a better spectral separation of the modes, with a lower computational cost.

The paper is organized as follows. In Sec. II the main concepts on EMD are recalled Sec.III the EEMD concepts and the proposed method is introduced and the data used for the experiments is described. In Sec. IV the used for the experiments is described. In Sec. V the results obtained by the new method here proposed are presented and compared with EMD and EEMD. Finally, the conclusions are discussed in Sec. V.

II. EMPIRICAL MODE DECOMPOSITION

This section starts with a brief review of the original EMD method. The detailed method can be found in Huang et al. (1998) and Huang et al. (1999). Different to almost all previous methods of data analysis, the EMD method is adaptive, with the basis of the decomposition based on and derived from the data. In the EMD approach, the data ***X(t)*** is decomposed in terms of IMFs.

$$x(t) = \sum_{j=1}^{n} c_j + r_n$$

Where $r_n$is the residue of data **x(t)**, after **n** number of IMFs are extracted. IMFs are simple oscillatory functions with varying amplitude and frequency, and hence have the following properties:

1. Throughout the whole length of a single IMF, the number of extrema and the number of zero crossings must either be equal or differ at most by one (although these numbers could be differ significantly for the original data set);
2. At any data location, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

In practice, the EMD is implemented through a sifting process that uses only local extrema. From any data, the procedure is as follows:

1) Identify all the local extrema (the combination of both maxima and minima) and connect all these local maxima (minima) with a cubic spline as the upper (lower) envelope.
2) Obtain the first component h by taking the difference between the data and the local mean of the two envelopes.
3) Treat h as the data and repeat steps 1 and 2 as many times as is required until the envelopes are symmetric with respect to zero mean under certain criteria. The final h is designated as c_j.

A complete sifting process stops when the residue , becomes a monotonic function from which no more IMF can be extracted .Figure 1 shows the EMD decomposition of a well studied piece-wise regular signal corrupted by white Gaussian noise corresponding 5dB signal to noise power ratio (SNR).

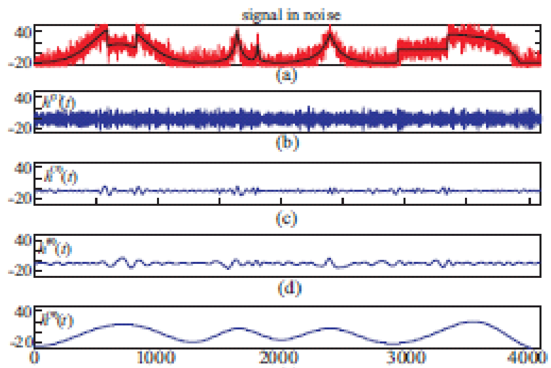

Figure 1. EMD decomposition of a piece-wise regular signal

A.MODE MIXING

As useful as EMD proved to be, it still leaves some difficulties unresolved. One of the major drawbacks of the original EMD is the frequent appearance of mode mixing, which is defined as a single Intrinsic Mode Function (IMF) either consisting of widely disparate scales, or a signal of a similar scale residing in different IMF components When mode mixing occurs, an IMF can cease to have physical meaning by itself, suggesting falsely that there may be different physical processes represented in a mode.

In order to identify the mode mixing problem , we need to use the following algorithm:

1. Normalize the signal to make energy equal to unity.
2. Find IMFS using EMD.
3. Calculate energies of each IMF.
4. Then add all energies distributed in IMFS.
5. Then verify is that energy increasing or not as compared to normalized energy.

When this algorithm is applied to different noises, their energy distribution is shown in the following figure 2.

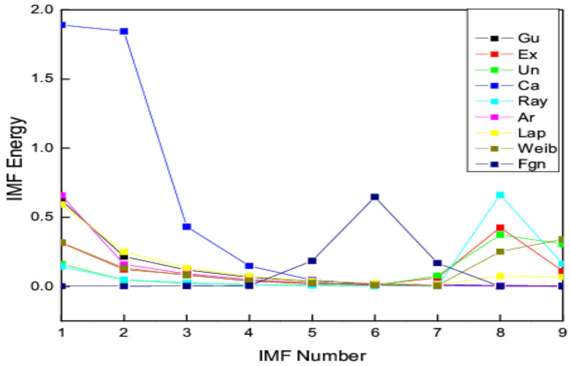

Figure 2. IMFs energy distribution for different noises

All noise distributions showing energy approximately equal to unity except Cauchy noise. Due to mode mixing Cauchy noise shows energy more than unity(2.5). To eliminate this mode mixing problem, a new method called Ensemble Empirical Mode Decomposition was introduced by Huang et al in 2009.

III. ENSEMBLE EMPIRICAL MODE DECOMPOSITION

The word "ensemble" means all parts of something taken together or the total effect of something made up of individual parts.

Ensemble Empirical Mode Decomposition performs EMD over an ensemble of signal plus Gaussian white noise. This approach consists of sifting an ensemble of white noise added signal and treats the mean the final true answer.

EEMD considers the true IMF components (here notated as $\overline{\text{IMF}}$ in what follows) as the mean of ensemble of trails, each consisting of signal plus a white noise of finite amplitude to the original signal.

EEMD algorithm can be described as :

1. Generate different realizations of white Gaussian noise,x^i[n]= x[n] + w^i[n] (i = 1,…,I), here I is the ensemble size.
2. Each x^i [n] (i =1,….,I) is completely decomposed by EMD getting their modes IMF_k^i[n], where k = 1,….,K indicates the modes
3. Assign $\overline{IMF}_k$ as the k-th mode of x[n], which is obtained as the average of the corresponding IMF_k^i:

$$\overline{IMF}_K\,[n] = \frac{1}{I}\sum_{i=1}^{I} IMF_k^i\,[n]$$

When this EEMD algorithm is applied to Cauchy noise, its IMFs energy distribution plot is shown in figure 3

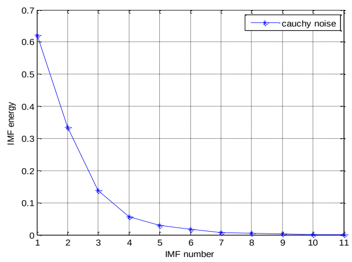

Figure 3. IMFs energy distribution for Cauchy noise

The above figure shows that the total energy of Cauchy noise is equal to unity, which proves that there is no mode mixing.

So By applying EEMD, mode mixing problem in Cauchy can be eliminated. Even though it solves this annoying mode mixing problem, EEMD produced results does not satisfy the strict definition of IMFs, so that the exact signal reconstruction can't be possible. To overcome this drawback, the following algorithm is proposed.

A. ENHANCED ENSEMBLE EMPIRICAL MODE DECOMPOSITION

In EEMD technique a residue r_k^i[n] $= r_{k-1}^i$[n] $-$ IMF_k^i [n] is obtained for each x^i[n] when it is decomposed independently from the other realizations.

In the proposed method, the decomposition modes will be denoted as $\widetilde{IMF}_k$ and we propose to calculate a unique first residue as:

$$[n] = x[n] - \widetilde{IMF}_1[n], \quad (1)$$

where $\widetilde{IMF}_1$[n] is obtained in the same way as in EEMD. Then, compute the $\widetilde{IMF}_2$ by averaging over an ensemble of ofr$_1$[n] plus different realizations of a Gaussian white noise. The next residue is defined as:r_2[n] = r_1[n] $-\widetilde{IMF}_2$[n]. This procedure continues with the rest of the modes until the stopping criterion is reached.

The proposed method can be described by the following algorithm considering x[n] as the targeted data. Let $w^i[n]$ (where i=1,2…..I) are different realizations of Gaussian white noise.

1. Generate different realizations of white Gaussian noise,x^i[n]= x[n] + w^i[n] (i = 1,…,I)
2. Each $x^i[n]$is decomposed by EMD to obtain their first modes and from that calculate the first IMF
$$\widetilde{IMF}_1 [n] = \sum_{i=1}^{I} IMF_1^i [n] = \overline{IMF_1}[n]$$
3. Then in next step, calculate the first residue as in Eq. (1): r_1[n] = x[n] $-\widetilde{IMF}_1$[n].
4. To this residue add white Gaussian noise r_1[n] + w^in, i= 1,…,I, and decompose it by EMD until their first mode to obtain the second mode:
$$\widetilde{IMF}_2[n] = \frac{1}{I}\sum_{i=1}^{I}(r_1 [n] + w^i[n])$$
5. For k = 2,…., K calculate the k-th residue

$$r_k[n] = r_{(k-1)}[n] - \widetilde{IMF}_k [n] \quad (2)$$

6. Decompose realizations r_k [n] + w^i [n]), i = 1,…, I , until their first EMD mode and define the (k + 1)-th mode as
$$\widetilde{IMF}_{(k+1)}[n] = \frac{1}{I}\sum_{i=1}^{I}(r_k [n] + w^i[n]) \quad (3)$$
7. Go to step 4 for next k

Steps 4 to 6 are performed until the obtained residue is no longer feasible to be decomposed (the residue does not have at least two extrema). The residue satisfies :

$$R[n] = x[n] - \sum_{k=1}^{K} \widetilde{IMF}_k \quad (4)$$

With K the total number of IMFs. Therefore , the given signal x[n] can be expressed as

$$x(n) = \sum_{k=1}^{K} \widetilde{IMF}_k + R[n] \quad (5)$$

Eq.(5)makes the proposed method complete and providesan exact reconstruction of the original data.

When considering the amplitude of the added noise, Wu and Huang suggested [2] to use small amplitude values for data dominated by high-frequency signals ,and viceversa. In this paper, a few hundred of realization with same SNR is used for all the stages. The number of realizations and value of SNR might depend upon the application.

B. DATA

Synthetic and real signals are analysed in the present paper. We consider a synthesized signal δ[n]Dirac signal of 512 samples. This dirac delta function was used in[4]to suggest that noise could help data analysis in cases where EMD cannot be performed, giving birth to EEMD in[2].A second example will be explained ,using real data :Electrocardiogram(ECG) signals from the MIT-BIH Normal Sinus Rhythm Database

IV. RESULTS AND DISCUSSIONS

When proposed method is used for denoising of piece-wise regular signal using conventional method there is an increase in SNR and decrease in Mean Square Error(MSE) compared to EMD.

Table 1. compares the SNR and MSE for both EMD and proposed method

Input SNR	EMD		Our method	
	SNR	MSE	SNR	MSE
5dB	15.7900	8	15.78	6.1992
10dB	17.7982	5	17.780	2.1260
0dB	11.8301	21	11.9001	19.1236
-5dB	2.8792	165	2.9011	159.127
-10dB	6.9750	64	7.005	57.157

We apply the proposed method to a dirac delta signal [n].

Figure 4(a) shows the decompositions of the dirac delta obtained by the EEMD method and the decompositions obtained by the proposed method are shown in fig 4(b).In both cases, an ensemble size of I=500 were used, withε_0=0.02,correspondingtoaSNRof34dB. In fig 4(a), it can be seen that EEMD produces thirteen modes, while in the fig 4(b) only nine modes are obtained by the method here proposed.

In decompositions obtained by both methods, the amplitudes of the modes one to five are similar and lessthan10^{-3}fork=6,….,8,with lower energy in the EEMD cases. EEMD modes nine to thirteen have very low amplitude (max($\overline{IMF}_k$) ≤ 2 $\times 10^{-4}$.)Additionally $\overline{IMF}_k$ (k ≥8) are not symmetric as expected. This low energy issue in the case of EEMD is due to the effect of averaging overall realizations, while a large variation in the number of modes could be observed. In order toper form the averaging it is necessary to pad with zeros the missing modes, getting low amplitudes when averaging .A different solution could be to set the number of modes (usuallyat1+[$\log_2$(N)], with N the signal length);in this case the EEMD method would no longer be fully adaptive. The method here proposed does not suffer from this difficultybecause: (i)eachrealizationofresidueplusnoiseisdecomposeduntil thefirstmode is reached, and(ii) for the final mode K we use as stopping criterion the one used in EMD[1].

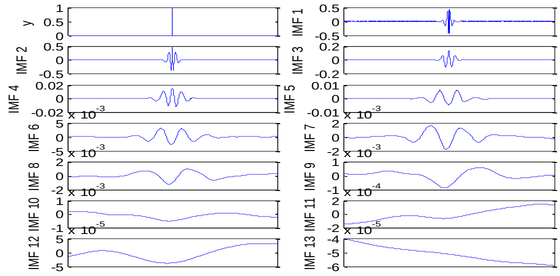

Figure 4(a). decomposition of a 512 sample long delta function by using EEMD.

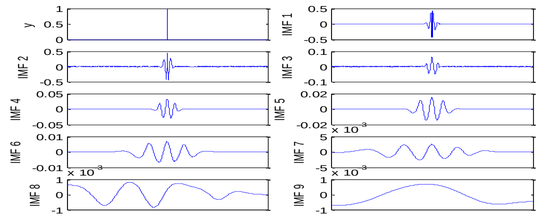

Figure 4(b). decomposition of a 512 sample long delta function by using proposed method.

EMD or EEMD methods are used for denoising of ECG signals[5, 6].These signals are characterized by spike-like events (QRS complexes), similar to discrete Dirac delta functions. Severe mode mixing was observed in ECG signals when decomposing them by EMD. Although EEMD eliminates the mode mixing, it is still too much time consuming because of the large number of sifting iterations required to achieve the decomposition.

In Figure 5(a) decomposition of an ECG signal using EEMD is presented. Figure 5(b) shows the decomposition obtained by using proposed method. In both cases an ensemble size of I = 500, with standard deviation = 0.2 of the added noise (SNR = 14 dB). It can be appreciated in the fig 5(b)that in the seventh mode the fundamental frequency (F) of the signal is clearly captured, while in the case of EEMD, F appears with lower energy in modes seven and eight (left panel). Therefore, a fundamental frequency extraction algorithm could fail to identify the mode that contains it when applied to an EEMD decomposition.

The RR signal, defined as the distance between consecutive R peaks in the ECG, is widely used to study the heart rate variability (HRV) which contains information about the state of the autonomous nervous system (ANS). While this approach provides a non-uniformly and low rate sampled signal, an estimation of the instantaneous heart frequency from the proper mode obtained using our method, would allow uniformly sampled heart rate estimation at a higher frequency.

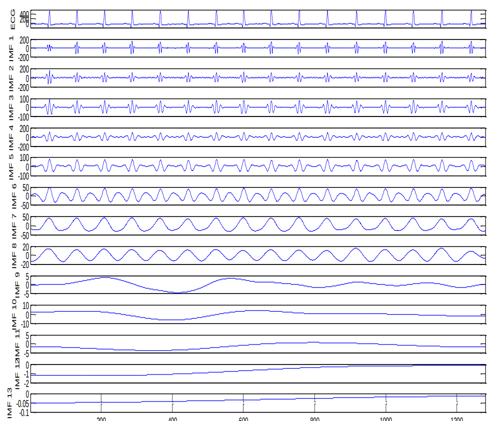

Figure 5(a). decomposition of a 10-second length ECG signal using EEMD

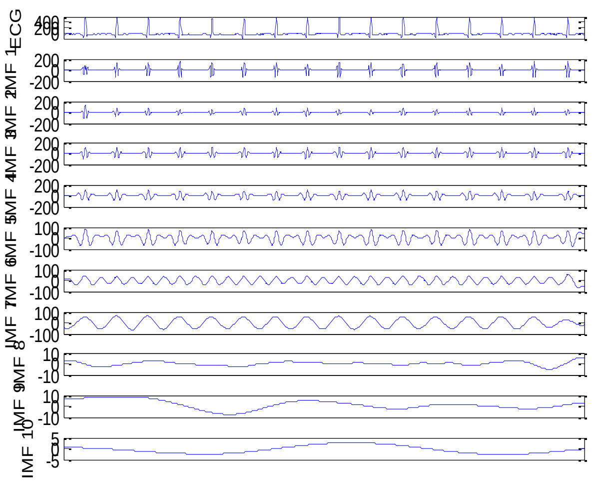

Figure 5(b). decomposition of a 10-second length ECG signal using proposed method

The completeness of the new method is guaranteed by Eq.(5).

This property can be numerically proved by the reconstruction error which is computed the difference between the ECG signal and the sum of modes. The reconstruction errors for ECG using EEMD and proposed method are shown in fig 6(a)and fig 6(b).In the case of our method, the maximum amplitude is less than 2×10^{-15}(the round off error from the precision of the computer) with a standard deviation of2×10^{-14}. To achieve this precision with EEMD (which does not guarantee a complete decomposition) it would be necessary to increase the number of realizations to over10^{29}, considering that in EEMD, the remaining noise has a standard deviation of $\varepsilon_r = \sqrt[\varepsilon]{I}$, turning the process extremely expensive in terms of computational cost.

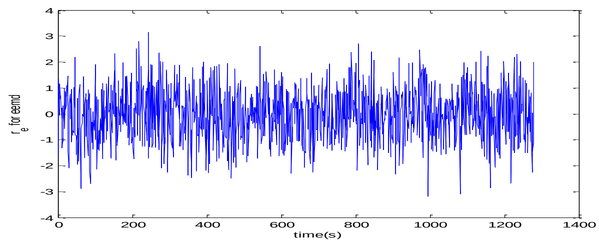

Figure 6(a). reconstructed error for ECG using EEMD

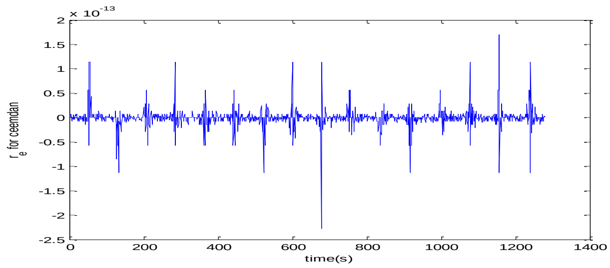

Figure 7. reconstruction error for ECG signal using proposed method:

V.CONCLUSIONS

A new algorithm is presented for analyzing non linear and non stationary signals. The proposed method was successfully tested on artificial and real signals. The new method has the advantages of exact reconstruction of original signal by summing the modes and that of requiring less than half the sifting iterations that EEMD does. Decomposition completeness was theoretically demonstrated and numerically verified in the case of ECG signal. Because of that, a smaller ensemble size is needed resulting in a significant computational cost saving. In that sense, the novel method recovers some of the EMD properties lost by EEMD, such as completeness and a fully data-driven number of modes.

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