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Algebra of Morphological Dilation on Intuitionistic Fuzzy Hypergraphs

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ABSTRACT

Intuitionistic Fuzzy Hypergraphs (IFHG) are those in which the hypernodes are having membership and non membership degrees , so also the hyperedges. The hypernodes with high membership degrees are more relevant with respect to the hypergraph. Likewise the hyperedges with high membership degrees are more important. The purpose of this work is to define algebraic operations like union, intersection and complement of morphological dilation operation on sub hypergraphs of IFHG. De Morgan's law is also applied to IFHG provided the sub hypergraphs considered are having common edge(s).

Keywords : Intuitionistic Fuzzy Hypergraph, Morphology, Dilation.

I. INTRODUCTION

Intuitionistic Fuzzy Sets (IFS) [1] were introduced in 1999, where a non membership degree is also attached with every fuzzy member. A list of open problems [2] using IFS were also discussed. They include constructing an axiomatic system of IFS, extension of IFS modal operators, norm distance, matrices, developing statistical and probabilistic tools for IFS, algorithm for defuzzification etc. Fuzzy traversals of fuzzy hypergraphs, coloring of fuzzy hypergraphs and strongly interconnected hypergraphs were detailed in [3]. Further $\alpha - \beta$ cut [4] on IFHG, incidence matrix of $H_{\alpha-\beta}$ and the dual IFHG were introduced. The authors further propose to show the importance of $\alpha - \beta$ cut in graph partitioning. Operations like complement, join, union, intersection, ringsum, cartesian product, composition are defined for intuitionistic fuzzy graphs [5], where the authors further proposed to apply these operations in clustering techniques. Isomorphism between two intuitionistic fuzzy directed hypergraphs (IFDHG) is also discussed [6], where the authors have introduced IFDHG, its order, in-degree, out-degree, homomorphism, weak isomorphism and co-weak isomorphism between two IFDHGs. Application of IFHG in radio coverage network is also suggested by [7], in which their model can be used to determine station programming and develop marketing strategies. The authors also suggested the use to IFHG for clustering computer networks. An application with Intuitionistic Fuzzy sets for career choice[8] which is a decision making system was developed where the system represented the performance of students using membership μ, non membership υ and hesitation margin π. They applied normalized Euclidian distance to determine the apt career choice.

Few properties of strong IFHGs [9] were discussed and generalized strong spanning IFHG was also introduced. The authors have considered the interrelation between intuitionistic fuzzy vertex and family of intuitionistic fuzzy edges. Morphological operations [10] like dilation, erosion and adjunction on hypergraphs were done. The same authors have also done opening, closing, half opening and half closing and morphological filtering [11] on hypergraphs. Intuitionistic fuzzy transversal [12] of IFDGH was done in which they also introduced minimal transversal, transversal of union of IFDGHs, also their intersection, join structural subtraction, cartesian product and composition. Different types of IFDHGs including core, simple, elementary, sectional elementary IFDHGs, (μ; υ)-tempered IFDHGs [13], properties of (μ;υ)- tempered IFDHGs were also implemented.

This paper is organized as follows: Section 2(**Background**) defines an intuitionistic fuzzy hypergraph with its membership and non membership degrees. Section 3 **(Algebra of graph morphology)** introduces the hypergraph algebra on dilation operation. Section 4**(De Morgan's law applied to IFHG)** test and proves De Morgan's law considering dilation of subgraphs**. Section 5 (Conclusion)** is the concluding part where some applications and future work is mentioned and finally the references are included.

II. BACKGROUND

Let [H_{IF} , (μ_n , γ_n), (μ_e , γ_e), H^n, H^e] be an intuitionistic fuzzy hypergraph with membership degree μ_n and non membership degree γ_n defined on the set of hypernodes H^n and membership degree μ_e and non membership degree γ_e defined on a set of hyperedges H^e of H_{IF} . The sum of the membership degree and non membership degree of the hypernode is 1 [4]. ie, $\mu_n + \gamma_n = 1$. So also the sum of the membership degree and non membership degree of the hyperedge is 1 [4]. ie, $\mu_e + \gamma_e = 1$ .If all the hypernodes in a hyperedge has $\mu_n > 0.5$, then μ_e is the supremum of all μ_n in that edge. In such a case $\gamma_n = 1 - \mu_n$. If there is at least one hypernode with $\gamma_n > 0.5$, then the γ_e of that edge is the supremum of all γ_n in that edge. In such a case $\mu_e = 1 - \gamma_e$. Many morphological operators like dilation with respect to hypernodes, with respect to hyperedges, erosion with respect to hypernodes and erosion with respect to hyperedges can be defined once we have a sub hypergraph X_{IF}.

Let X_{IF} and Y_{IF} be two sub hypergraphs of H_{IF}. We can define them as [X_{IF}, ($\mu_{n'}$, $\gamma_{n'}$) , ($\mu_{e'}$, $\gamma_{e'}$), X^n , X^e] and [Y_{IF} , ($\mu_{n'}$, $\gamma_{n'}$) , ($\mu_{e'}$, $\gamma_{e'}$), Y^n, Y^e]. Let X_{IF} be obtained as a result of $\alpha - \beta$ cut on H_{IF}. ie, $X_{IF} = H_{\alpha-\beta}$ where m < α <= n and β = 1- α . Here α corresponds to the edge membership degree and β corresponds to the edge non membership degree. Let Y_{IF} be obtained as a result of another $\alpha - \beta$ cut on H_{IF}. ie, $Y_{IF} = H_{\alpha-\beta}$ where α >= n and β = 1- α. Here morphological dilation is represented using the operator δ. In this paper we are going to introduce many algebraic operations on morphological dilation.

III. ALGEBRA OF HYPERGRAPH MORHOLOGY

Let us assume m = 0:5; n = 0:8. ie, X_{IF} is a sub hypergraph which consists of edges with membership degree greater than 0.5, but less than or equal to 0.8. ie, $X_{IF} = H_{\alpha-\beta}$ /0:5 < α <= 0:8 ; β = 1- α. Also $Y_{IF} = H_{\alpha-\beta}$ / α >= 0:8; β = 1- α. ie, both X_{IF} and Y_{IF} are two priority sub hypergraphs with different cut offs. Our actual H_{IF} is having many non priority nodes whose μ_n < 0.5 and medium level nodes whose $\mu_n = 0.5$. The hypergraphs H_{IF}, X_{IF} and Y_{IF} are shown in Fig.1.

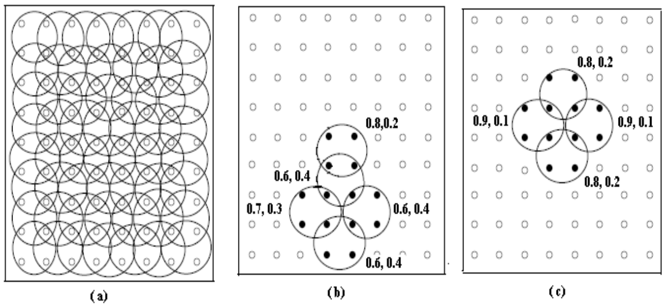

Figure 1 : (a) Hypergraph H_{IF}, (b) X_{IF} and (c) Y_{IF}

The following results hold for the above mentioned sub hypergraphs.

Definition 3.1 Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then node dilation of X ∪ Y is defined as

$$\delta^n(X \cup Y)^e = \delta^n (X^e) \cup \delta^n (Y^e) \qquad (1)$$

Proof: Consider the union operation of two IFHG sub hypergraphs X and Y, where X and Y are defined as in previous section. In L.H.S of (1), consider $X \cup Y = (Z^n, Z^e)$ where $Z^n = X^n \cup Y^n$ and $Z^e = X^e \cup Y^e$ and the union operation is same as the operation on graph.

$$\therefore \delta^n(X \cup Y)^e = Z^n \quad (2)$$

by definition of δ^n [10]

Also $\delta^n(X^e) = X^n$ and $\delta^n(Y^e) = Y^n$

$$\delta^n(X^e) \cup \delta^n(Y^e) = X^n \cup Y^n = Z^n \quad (3)$$

From eq(2) and eq(3), it implies

$\delta^n(X \cup Y)^e = \delta^n(X^e) \cup \delta^n(Y^e)$. Fuzzy membership and non-membership degrees are invariant under this equation. The result of this operation is shown in Fig. 2 .(a). As seen in the figure, this dilation operation will retrieve all nodes within the priority sub graphs. Since the graphs are of higher priority, the hypernodes retrieved are also of high priority.

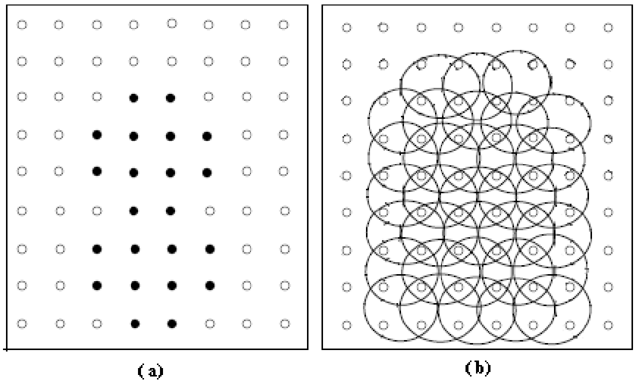

Figure 2 : (a) $\delta^n(X \cup Y)^e$ (b) $\delta^e(X \cup Y)^n$

Example 3.1 : Consider $H = (H^n, H^e)$ as an IFHG. $H^n = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$ be the hypernodes in H. Let $H^e = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the hyperedges in H. Let X be a sub hypergraph, where $X = (X^n, X^e)$; where $X^n = \{n_7, n_8, n_{10}, n_{11}\}$ and $X^e = \{e_5\}$. Let Y be a sub hypergraph, where $Y = (Y^n, Y^e)$;where $Y^n = \{n_8, n_9, n_{11}, n_{12}\}$ and $Y^e = \{e_6\}$. The membership degree and non membership degree of the hypernodes and hyperedges of hypergraph H are given as per the Table I. Here we have set $X = H_{\alpha-\beta} / 0.5 < \alpha <= 0.9; \beta = 1 - \alpha$. Also $Y = H_{\alpha-\beta} / \alpha >= 0.9; \beta = 1 - \alpha$.

TABLE I. DETAILS OF HYPERGRAPH H

Hyper Edges	Membership and non membership degrees				Edge priority
	Hyper nodes				
e1 (0.4,0.6)	n1 (0.5,0.5)	n2 (0.5,0.5)	n3 (0.4,0.6)	n4 (0.5,0.5)	low
e2 (0.5,0.5)	n2 (0.5,0.5)	n4 (0.5,0.5)	n5 (0.5,0.5)	n6 (0.5,0.5)	medium
e3 (0.4,0.6)	n3 (0.4,0.6)	n4 (0.5,0.5)	n7 (0.6,0.4)	n8 (0.8,0.2)	low
e4 (0.9,0.1)	n4 (0.5,0.5)	n8 (0.5,0.5)	n6 (0.5,0.5)	n9 (0.9,0.1)	high
e5 (0.8,0.2)	n7 (0.7,0.3)	n10 (0.5,0.5)	n8 (0.5,0.5)	n11 (0.6,0.4)	high
e6 (0.9,0.1)	n8 (0.5,0.5)	n9 (0.9,0.1)	n11 (0.8,0.2)	n12 (0.9,0.1)	high

The result of the operation $\delta^n(X \cup Y)^e$ is $Z^n = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$. Also same results are obtained from $\delta^n(X^e) \cup \delta^n(Y^e)$ ie, $\{n_7, n_8, n_{10}, n_{11}\} \cup \{n_8, n_9, n_{11}, n_{12}\} = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\} = Z^n$. Here the proof is substantiated with the above example.

Definition 3.2: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then edge dilation of $X \cup Y$ is defined as

$$\delta^e(X \cup Y)^n = \delta^e (X^n) \cup \delta^e(Y^n) \quad (4)$$

Proof: In L.H.S of eq(4), $\delta^e(X \cup Y)^n$ is the collection of all edges which contain hypernodes of $X \cup Y$. As shown in Fig. 2(b), it consists of edges which are of priority and also of non priority. This is because; the hypernodes of sub graphs X and Y are also part of other hyperedges which are of medium priority or low priority. Here we can see that all the edges retrieved by this dilation consist of at least one node of high priority. All other edges are discarded in this operation.

$$\delta^e (X \cup Y)^n = Z^e \quad (5)$$

In R.H.S of eq(4), $\delta^e (X^n) = X^e$ and $\delta^e(Y^n) = Y^e$

Hence

$$\delta^e (X^n) \cup \delta^e(Y^n) = X^e \cup Y^e = Z^e \quad (6)$$

From eq(5) and eq(6), it implies that

$\delta^e (X \cup Y)^n = \delta^e(X^n) \cup \delta^e(Y^n)$

Example 3.2: Consider the same problem defined in example 3.1. Applying it in L.H.S of eq(4), we get $(X \cup Y)^n = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$. Now $\delta^e(X \cup Y)^n = \{ e_3, e_4, e_5, e_6 \}$. Considering R.H.S, we get $\delta^e (X^n) = \{e_3, e_4, e_5, e_6\}$ and $\delta^e(Y^n) = \{ e_3, e_4, e_5, e_6 \}$. Now $\delta^e (X^n) \cup \delta^e (Y^n) = \{e_3, e_4, e_5, e_6\}$.

Definition 3.3: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then node dilation of $X' \cup Y'$ is defined as

$$\delta^n(X' \cup Y')^e = \delta^n(X')^e \cup \delta^n(Y')^e \quad (7)$$

Proof: Let $X' = H - X$, $Y' = H - Y$, $X' = (X^{n'}, X^{e'})$, $Y' = (Y^{n'}, Y^{e'})$. Let $(X' \cup Y')^e$ be the set of all hyperedges not in $X \cup Y$, where $X' \cup Y' = Z'$. Also $Z^{n'}$ and $Z^{e'}$ are the hypernodes and hyperedges of Z'. Hence

$$\delta^n (X' \cup Y')^e = Z^{n'} \quad (8)$$

Also

$$\delta^n (X')^e \cup \delta^n(Y')^e = X^{n'} \cup Y^{n'} \quad (9)$$

From eq(8) and eq(9), it implies that

$\delta^n(X' \cup Y')^e = \delta^n(X')^e \cup \delta^n(Y')^e$. Here $(X' \cup Y')^e$ retrieves all edges which are of high medium and low priority ,so also the dilation operation $\delta^n(X' \cup Y')^e$ retrieves all nodes within these high, medium and low priority hyperedges. The same is shown in Fig.3 (a)

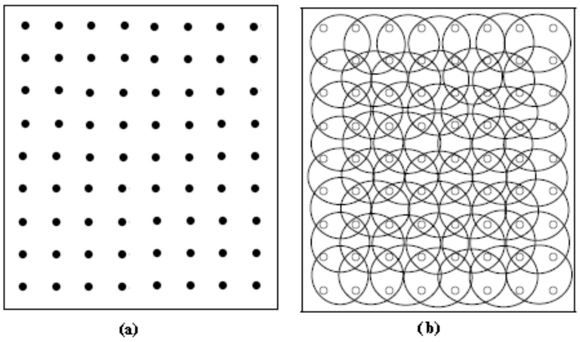

Figure 3: (a) $\delta^n(X' \cup Y')^e$ (b) $\delta^e(X' \cup Y')^n$

Example 3.3. Considering L.H.S of eq(7), we obtain $(X' \cup Y')^e = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. From that we get $\delta^n (X' \cup Y')^e = \{ n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$. Considering R.H.S, we get $\delta^n(X')^e = \{ n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{11}, n_{12}\}$. Also $\delta^n(Y')^e = \{ n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$. Now $\delta^n(X')^e \cup \delta^n(Y')^e = \{ n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$.

Definition 3.4: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then edge dilation of $X' \cup Y'$ is defined as

$$\delta^e(X' \cup Y')^n = \delta^e (X')^n \cup \delta^e (Y')^n \quad (10)$$

Proof: Let $X' = H - X$, $Y' = H - Y$, $X' = (X^{n'}, X^{e'})$, $Y' = (Y^{n'}, Y^{e'})$. $(X' \cup Y')^n$ be the set of all hypernodes not in $X \cup Y$, where $X' \cup Y' = Z'$. Also $Z^{n'}$ and $Z^{e'}$ are the hypernodes and hyperedges of Z'. Hence

$$\delta^e (X' \cup Y')^n = Z^{e'} \quad (11)$$

Also

$$\delta^e (X')^n \cup \delta^e (Y')^n = X^{e'} \cup Y^{e'} \quad (12)$$

From eq(11) and eq(12), it implies that

$\delta^e (X' \cup Y')^n = \delta^e (X')^n \cup \delta^e (Y')^n$.

This dilation operation will retrieve all edges which are of high, medium priority and low priority. The same is shown in Fig.3(b).

Example: Considering L.H.S of eq(10), we obtain the result $(X' \cup Y')^n = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$. Now $\delta^e(X' \cup Y')^n = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Consider R.H.S, where $\delta^e(X')^n = \{e_1, e_2, e_3, e_4, e_6\}$. Now $\delta^e(Y')^n = \{e_1, e_2, e_3, e_4, e_5\}$. From this we get $\delta^e(X')^n \cup \delta^e (Y')^n = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

Definition 3.5: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then node dilation of $X \cap Y$ is defined as

$$\delta^n (X \cap Y)^e = \delta^n (X^e) \cap \delta^n (Y^e) \quad (13)$$

provided there are common edges in X and Y.

Proof: Consider the intersection operation of two IFHG sub hypergraphs X and Y, where X and Y are defined as in previous section. In L.H.S of eq(13), consider $X \cap Y = (Z^n, Z^e)$ where $Z^n = X^n \cap Y^n$ and $Z^e = X^e \cap Y^e$ and the intersection operation is same as the operation on graph.

$$\therefore \delta^n (X \cap Y)^e = Z^n \quad (14)$$

by definition of δ^n

Also $\delta^n (X^e) = X^n$ and $\delta^n (Y^e) = Y^n$

$$\therefore \delta^n(X^e) \cap \delta^n(Y^e) = X^n \cap Y^n = Z^n \quad (15)$$

Eq(13) is implied from eq(14) and eq(15) . Fuzzy membership and non membership degrees are invariant under this equation. The resultant graph is shown in Fig . 4(a). This dilation will retrieve only the priority hypernodes which are found both in X and Y.

Example 3.5. Since this is true only if there are common edges in X and Y, let us modify X by including a common edge with Y so that now new $X=(X^n , X^e)$, where $X^e = \{e_5, e_6\}$ and $X^n = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$.Consider the result of L.H.S of eq(13), where we get $(X \cap Y)^e = \{e_6\}$, so we get $\delta^n(X \cap Y)^e = \{n_8, n_9, n_{11}, n_{12}\}$. Consider R.H.S, where we get $\delta^n(X^e) = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$ and $\delta^n(Y^e) = \{ n_8, n_9, n_{11}, n_{12}\}$. Definition 3.5 is proved with the result $\delta^n(X^e) \cap \delta^n(Y^e) = \{n_8, n_9, n_{11}, n_{12}\}$.

Definition 3.6: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then edge dilation of $X \cap Y$ is defined as

$$\delta^e(X \cap Y)^n = \delta^e (X^n) \cap \delta^e (Y^n) \quad (16)$$

Proof: In L.H.S of eq(16), $(X \cap Y)^n$ is the collection of all hypernodes in $X \cap Y$. ie,

$\delta^e(X \cap Y)^n$ is the collection of all hyperedges which contains these hypernodes.

$$\delta^e (X \cap Y)^n = Z^e \quad (17)$$

In R.H.S of eq(16), $\delta^e (X^n) = X^e$ and $\delta^e (Y^n) = Y^e$

Hence

$$\delta^e (X^n) \cap \delta^e (Y^n) = X^e \cap Y^e = Z^e \quad (18)$$

Eq(16) is implied from eq(17) and eq(18).

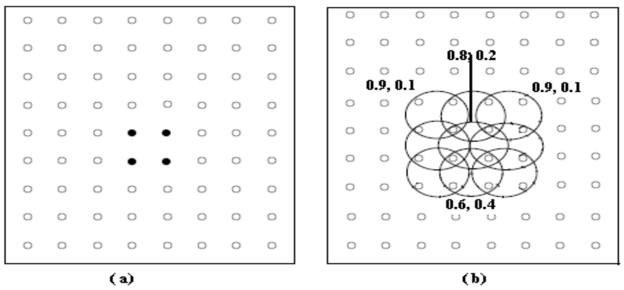

Figure 4: (a) $\delta^n (X \cap Y)^e$ (b) $\delta^e (X \cap Y)^n$

The same result is shown in Fig .4(b). As we see in the figure, not all edges are of high priority. It retrieves all kinds of edges. But it ensures that at least one hypernode in that edge is of high priority.

Example 3.6: Consider the X and Y mentioned in example 3.1. In L.H.S of eq(16), while we get $(X \cap Y)^n = \{n_8, n_9\}$, we get $\delta^e(X \cap Y)^n = \{e_3, e_4, e_5, e_6\}$. Considering R.H.S of eq(16), we obtain $\delta^e(X^n) = \{e_3, e_4, e_5, e_6\}$; $\delta^e(Y^n) = \{e_3, e_4, e_5, e_6\}$. Now $\delta^e(X^n) \cap \delta^e(Y^n) = \{e_3, e_4, e_5, e_6\}$.

Definition 3.7: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then node dilation of $X' \cap Y'$ is defined as

$$\delta^n (X' \cap Y')^e = \delta^n (X')^e \cap \delta^n (Y')^e \tag{19}$$

provided there common edges in X and Y.

Proof: Let $X' = H - X$, $Y' = H - Y$, $X' = (X^{n\prime}, X^{e\prime})$, $Y' = (Y^{n\prime}, Y^{e\prime})$. Let $(X' \cap Y')^e$ be the set of all hyperedges not in $X \cap Y$, where $X' \cap Y' = Z'$. Also $Z^{n\prime}$ and $Z^{e\prime}$ are the hypernodes and hyperedges of Z'. Hence

$$\delta^n (X' \cap Y')^e = Z^{n\prime} \tag{20}$$

Also

$$\delta^n (X')^e \cap \delta^n (Y')^e = X^{n\prime} \cap Y^{n\prime} \tag{21}$$

Eq(19) is implied from eq(20) and eq(21).
Resultant graph obtained is shown in Fig 5(a)

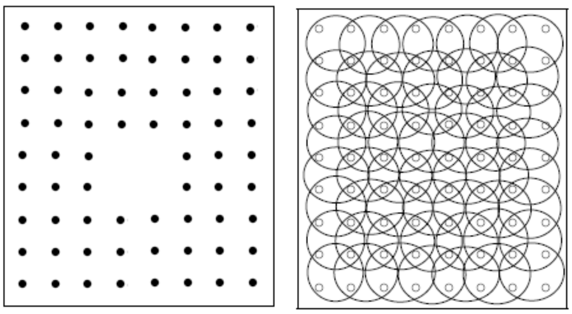
(a) (b)

Figure 5: (a) $\delta^n (X' \cap Y')^e$ (b) $\delta^n(X' \cap Y')^e$

Example 3.7 .Let us consider the modified $X = (X^n, X^e)$, where $X^e = \{e_5, e_6\}$ and $X^n = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$.Now in L.H.S of eq(19), we get $(X' \cap Y')^e = \{e_1, e_2, e_3, e_4\}$. Now $\delta^n(X' \cap Y')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$.Consider R.H.S ,where we get $\delta^n (X')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. $\delta^n(Y')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$. Thus $\delta^n (X')^e \cap \delta^n (Y')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$.

Definition 3.8: Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph. Then edge dilation of $X' \cap Y'$ is defined as

$$\delta^e (X' \cap Y')^n = \delta^e (X')^n \cap \delta^e (Y')^n \tag{22}$$

Proof: Let $X' = H - X$, $Y' = H - Y$, $X' = (X^{n\prime}, X^{e\prime})$, $Y' = (Y^{n\prime}, Y^{e\prime})$. Let $(X' \cap Y')^n$ be the set of all hypernodes not in $X \cap Y$, where $X' \cap Y' = Z'$. Also $Z^{n\prime}$ and $Z^{e\prime}$ are the hypernodes and hyperedges of Z'. Hence

$$\delta^e (X' \cap Y')^n = Z^{e\prime} \tag{23}$$

Also

$$\delta^e (X')^n \cap \delta^e (Y')^n = X^{e\prime} \cap Y^{e\prime} \tag{24}$$

Eq(22) is implied from eq(23) and eq(24).The resultant graph is shown in Fig.5(b).

Example 3.8. Take X and Y defined in example 3.1. Applying it in eq(22), we get $(X' \cap Y')^n = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. So $\delta^e (X' \cap Y')^n = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. $\delta^e (X')^n = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Also $\delta^e (Y')^n = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Thus $\delta^e (X')^n \cap \delta^e (Y')^n = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

3.9.1: Generalized associative law for union

Proposition 1. Let $X_1, X_2, X_3 \ldots\ldots X_n$ be the sub hypergraphs of H_{IF}. Let δ be the node dilation operator defined on $X_1, X_2, X_3 \ldots\ldots X_n$, then

$$\delta^n(X_1 \cup X_2 \cup X_3 \ldots X_n)^e = \delta^n(X_1^e \cup X_2^e \cdots X_n^e) \tag{25}$$

The proof follows from definition 3.1

Similary the edge dilation on $(X_1 \cup X_2 \cup X_3 \ldots X_n)$ is

$$\delta^e(X_1 \cup X_2 \cup X_3 \ldots X_n)^n = \delta^e(X_1{}^n \cup X_2{}^n \cdots X_n{}^n) \quad (26)$$

The proof follows from definition 3.2.

3.9.2: Generalized associative law for intersection

Proposition 2. Let $X_1, X_2, X_3 \ldots X_n$ be the sub hypergraphs of H_{IF}. Let δ be the dilation operator, then node dilation on $(X_1 \cap X_2 \cap X_3 \ldots X_n)$ is

$$\delta^n(X_1 \cap X_2 \cap X_3 \ldots X_n)^e = \delta^n(X_1{}^e \cap X_2{}^e \cdots\cdots X_n{}^e) \quad (27)$$

The proof follows from definition 3.5

Similary the edge dilation on $(X_1 \cap X_2 \cap X_3 \ldots X_n)$ is

$$\delta^e(X_1 \cap X_2 \cap X_3 \ldots X_n)^n = \delta^e(X_1{}^n \cap X_2{}^n \cdots\cdots X_n{}^n) \quad (28)$$

The proof follows from definition 3.6

3.9.3: Distributive law

Proposition 3. Let X, Y and T be three sub hypergraphs of H_{IF}. Then

$$\delta^n((X \cup Y) \cap T)^e = \delta^n(X \cap T)^e \cup \delta^n(Y \cap T)^e \quad (29)$$

The proof follows from definition 3.1 and definition 3.5.

IV. DE MORGAN'S LAW APPLIED TO IFHG

4.1: De Morgan's law applied to dilation with respect to hypernode considering union of subgraphs

Proposition 4 .Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph, then

$$\delta^n(X \cup Y)'^e = \delta^n(X')^e \cap \delta^n(Y')^e \quad (30)$$

provided there are edge(s) in $X \cap Y$.

Proof: $(X \cup Y)'$ be the sub hypergraph with edges which are not present in $X \cup Y$.

Let $(X \cup Y)'^e$ be the edges in that hypergraph . Hence $\delta^n(X \cup Y)'^e$ is the set of all hypernodes in the sub hypergraph $(X \cup Y)'$. Let it be $Z^{n'}$. Also $\delta^n(X')^e$ = set of all hypernodes in X', $\delta^n(Y')^e$ = set of all hypernodes in Y'. Let v be an arbitrary node in $\delta^n(X \cup Y)'^e$, which implies that v belongs to $\delta^n(X')^e$ and v belongs to $\delta^n(Y')^e$.

Hence

$$\delta^n(X \cup Y)'^e \subseteq \delta^n(X')^e \cap \delta^n(Y')^e \quad (31)$$

Let v belongs to $\delta^n(X')^e \cap \delta^n(Y')^e$ which implies that v belongs to X' and v belongs to Y'. Hence $v \notin X$ and $v \notin Y$.

$v \notin X \cup Y$

$\therefore X \in (X \cup Y)'$

$$\delta^n(X')^e \cap \delta^n(Y')^e \subseteq \delta^n(X \cup Y)'^e \quad (32)$$

Eq (30) implied from Eq(31) and Eq(32) .

Example 4.1. Consider the hypergraph H_{IF} given in Table I(also shown in Fig 6(a)). Let X be a sub hypergraph $X = (X^n, X^e)$, where $X^e = \{e_5, e_6\}$ and $X^n = \{n_7, n_8, n_9, n_{10}, n_{11}, n_{12}\}$. Let Y be another sub hypergraph such that $Y = (Y^n, Y^e)$, where $Y^e = \{e_6\}$, $Y^n = \{n_8, n_9, n_{11}, n_{12}\}$. Considering L.H.S of eq (30) we get, $(X \cup Y)'^e = \{e_1, e_2, e_3, e_4\}$. Now $\delta^n(X \cup Y)'^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. Considering R.H.S of eq (30) we get, $\delta^n(X')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. Also $\delta^n(Y')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$. Thus we get $\delta^n(X')^e \cap \delta^n(Y')^e = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$ as shown in Fig 6(b).

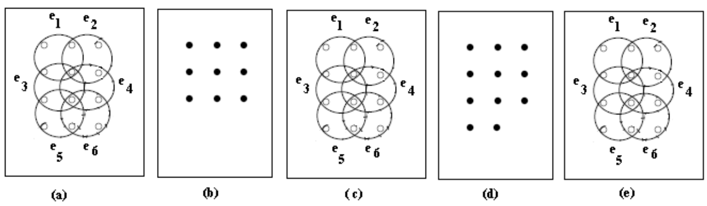

Figure 6:(a) H 6(b) ,6(c) ,6(d) ,6(e) – Results of De Morgan's law

4.2 : De Morgan's law applied to dilation with respect to hyperedge considering union of subgraphs.

Proposition 5. Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph, then

$$\overset{e}{\delta}(X \cup Y)'^{n} = \overset{e}{\delta}(X')^{n} \cap \overset{e}{\delta}(Y')^{n} \quad (33)$$

provided there are edge(s) in $X \cap Y$.

Proof: Let $(X \cup Y)'$ be the sub hypergraph with edges which are not present in $X \cup Y$. Let $(X \cup Y)'^{n}$ be the nodes in that hypergraph . Hence $\overset{e}{\delta}(X \cup Y)'^{n}$ is the set of all hyperedges which include such nodes. Let it be $Z^{e\prime}$. Also $\overset{e}{\delta}(X')^{n}$ = set of all hyperedges in X' , $\overset{e}{\delta}(Y')^{n}$ = set of all hyperedges in Y'. Let v be an arbitrary node in $\overset{e}{\delta}(X \cup Y)'^{n}$, which implies that v belongs to $\overset{e}{\delta}(X')^{n}$ and v belongs to $\overset{e}{\delta}(Y')^{n}$. Hence

$$\overset{e}{\delta}(X \cup Y)'^{n} \subseteq \overset{e}{\delta}(X')^{n} \cap \overset{e}{\delta}(Y')^{n} \quad (34)$$

Let v belongs to $\overset{e}{\delta}(X')^{n} \cap \overset{e}{\delta}(Y')^{n}$ which implies that v belongs to X' and v belongs to Y'. Hence $v \notin X$ and $v \notin Y$.

$\therefore v \notin X \cup Y \therefore v \in (X \cup Y)'$

$$\therefore \overset{e}{\delta}(X')^{n} \cap \overset{e}{\delta}(Y')^{n} \subseteq \overset{e}{\delta}(X \cup Y)'^{n} \quad (35)$$

Eq (33) is implied from Eq(34) and Eq(35) .

Example 4.2. Consider the hypergraphs given in example 4.1. Considering L.H.S of eq (33) we get , $(X \cup Y)'^{n} = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. Thus we get $\overset{e}{\delta}(X \cup Y)'^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Considering R.H.S of eq(33), we get $\overset{e}{\delta}(X')^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $\overset{e}{\delta}(Y')^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Thus $\overset{e}{\delta}(X')^{n} \cap \overset{e}{\delta}(Y')^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ as shown in Fig 6(c).

4.3: De Morgan's law applied to dilation with respect to hypernode considering intersection of subgraphs

Proposition 6. Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph, then

$$\overset{n}{\delta}(X \cap Y)'^{e} = \overset{n}{\delta}(X')^{e} \cup \overset{n}{\delta}(Y')^{e} \quad (36)$$

provided there are edge(s) in $X \cap Y$

Proof: Let $(X \cap Y)'$ be the hypergraph with edges which are not present in $X \cap Y$. Let $(X \cap Y)'^{e}$ be the edges in that hypergraph. Let $\overset{n}{\delta}(X \cap Y)'^{e}$ be the set of all nodes in the Sub hypergraph $(X \cap Y)'$. Let v be a node in $\overset{n}{\delta}(X \cap Y)'^{e}$, then it is not a node of $X \cap Y$ ie $v \notin X \cap Y$.

$$\overset{n}{\delta}(X \cap Y)'^{e} \notin X \cap Y \quad (37)$$

Also $\overset{n}{\delta}(X')^{e}$ is the set of nodes in X' as in fig .2. Also $\overset{n}{\delta}(Y')^{e}$ is the set of nodes in Y' . Let v belongs to $\overset{n}{\delta}(X')^{e} \cup \overset{n}{\delta}(Y')^{e}$ which implies that v either belongs to any node in X' or v belongs to any node as given in ie, $v \notin X \cap Y$.

$$\overset{n}{\delta}(X')^{e} \cup \overset{n}{\delta}(Y')^{e} \notin X \cap Y \quad (38)$$

Eq(36) is implied from eq(37) and eq(38).

Example 4.3. Consider the hypergraphs given in example 4.1. Considering L.H.S of eq (36) , we get $(X \cap Y)'^{e} = \{e_1, e_2, e_3, e_4, e_5\}$. $\overset{n}{\delta}(X \cap Y)'^{e} = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$. Now consider R.H.S , where we get $\overset{n}{\delta}(X')^{e} = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. $\overset{n}{\delta}(Y')^{e} = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$. Now $\overset{n}{\delta}(X')^{e} \cup \overset{n}{\delta}(Y')^{e} = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$ as shown in Fig 6(d).

4.4: De Morgan's law applied to dilation with respect to hyperedge considering intersection of sub graphs

Proposition 7. Let X and Y be the sub hypergraphs of H_{IF} and δ be the dilation operator defined on hypergraph, then

$$\overset{e}{\delta}(X \cap Y)'^{n} = \overset{e}{\delta}(X')^{n} \cup \overset{e}{\delta}(Y')^{n} \quad (39)$$

provided there are edge(s) in $X \cap Y$.

Let $(X \cap Y)'$ be a hypergraph with nodes which are not present in $X \cap Y$. Let $(X \cap Y)'^{n}$ be the nodes in that hypergraph. Let $\overset{e}{\delta}(X \cap Y)'^{n}$ be the set of all edges in the sub hypergraph $(X \cap Y)'$. Let e be an edge in $\overset{e}{\delta}(X \cap Y)'^{n}$, it is not an edge of $X \cap Y$. ie, $e \notin X \cap Y$.

$$\overset{e}{\delta}(X \cap Y)'^{n} \notin X \cap Y \quad (40)$$

Also $\overset{e}{\delta}(X')^{n}$ is the set of all edges in X'. Also $\overset{e}{\delta}(Y')^{n}$ is the set of all edges in Y'.Let e belongs to $\overset{e}{\delta}(X')^{n} \cup \overset{e}{\delta}(Y')^{n}$; which implies that e either belongs to X' or Y'. ie, $e \notin X \cap Y$.

$$\overset{e}{\delta}(X')^{n} \cup \overset{e}{\delta}(Y')^{n} \notin X \cap Y \quad (41)$$

Eq(39) is implied from eq(40) and eq(41).

Example 4.3. Consider the hypergraphs given in example 4.1. Considering L.H.S of eq(39) we get, $\overset{e}{\delta}(X \cap Y)'^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Consider R.H.S of eq(39) where we get $\overset{n}{\delta}(X')^{e} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Now $\overset{e}{\delta}(Y')^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Thus $\overset{e}{\delta}(X')^{n} \cup \overset{e}{\delta}(Y')^{n} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ as shown in Fig 6(e).

V. CONCLUSION

The paper has proved algebraic operations applied to the morphological operation dilation applied to IFHG. The proofs are also substantiated with a sample hypergraph and sub hypergraphs considering their node and edge membership and non-membership degrees. This type of modeling finds applications in the area of computer networks, image processing and text processing. The algebra of morphological erosion is a future enhancement of this paper.

VI. REFERENCES

[1]. Atanassov, Krassimir T. 1999. 'Intuitionistic Fuzzy Sets Theory and Applications' Studies in fuzziness and soft computing , Heidelberg, New York, Physicaverl.

[2]. Atanassov, Krassimir T .2003. 'Intuitionistic fuzzy sets: past, present and future', In proceedings of EUSFLAT , pp. 12-19.

[3]. Mordeson J N, Premchand S N. 2012. 'Fuzzy graphs and fuzzy hypergraphs', Physica, Vol .46. ISBN-978-3-7908-1854-3.

[4]. Parvathi R , Thilagavathi S and Karunambigai M G. 2009. 'Intuitionistic fuzzy hypergraphs', Cybernetics and Information Technologies, Vol. 9, pp.46-53.

[5]. Parvathi R, Thilagavathi S, Karunambigai MG . 2012. 'Operations on intuitionistic fuzzy hypergraphs', International Journal of Computer Applications , Vol .51.5.

[6]. Parvathi R , Thilagavathi S , Atanassov KT.2013, 'Isomorphism on Intuitionistic Fuzzy Directed Hypergraphs', International Journal of Scientific and Research Publications ,pp . 278-285.

[7]. Muhammad Akram , Wieslaw A. Dudek .2013. 'Intuitionistic fuzzy hypergraphs with applications', Information Sciences, Vol .218, pp.182-193.

[8]. Ejegwa P A ,Akubo A J , Joshua OM, 'Intuitionistic fuzzy set and its application in career determination via normalized Euclidean distance method' ,European Scientific Journal, Vol.No .10 , pp.529-536.

[9]. Samanta T K , Mohinta S. 2014. 'Generalized strong intuitionstic fuzzy hypergraph', Mathematica Moravica , Vol .18 , pp .55-65.

[10]. Kannan B, Bino S, Unnikrishnan A, Ramkumar P B .2014. 'Morphological filtering on Hypergraphs ', arXiv :1402. 4258v1.

[11]. Bino S , Unnikrishnan A , Kannan B , Ramkumar P B.2014. 'Mathematical morphology on hypergraphs using vertex-hyperedge correspondence', ISRN Discrete Mathematics, Hindawi publishing corporation, Vol .2014.

[12]. Myithili K K , Parvathi R .2015 .Transversals of intuitionistic fuzzy directed hypergraphs , Notes on IFS, Vol .21.pp .66-79.

[13]. Myithili K K , Parvathi Rangasamy , Akram Muhammad. 2016. 'Certain types of intuitionistic fuzzy directed hypergraphs' International Journal of Machine Learning and Cybernetics , Vol .7 ,pp . 287-295.