Solving an Inventory Models Involving Lead Time Crashing Cost as an Exponential Function in Food Processing and Distribution Industry Using Matlab

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ABSTRACT

Inventory consists of usable but idle resources. The resources may be of any type men, materials, machines etc.. In this paper the Inventory model involving lead time crashing cost as an exponential function in food processing and distribution industry are discussed with Mathematical model. Moreover, a numerical example are presented to illustrate the important issues related to the mathematical modeling using MATLAB

Keywords: Inventory , Lead time, Crashing cost, Food processing, MATLAB.

I. INTRODUCTION

Inventory control is significant in supply chain management. In current years, the majority inventory problems have focused on the integration between the supplier and the retailer. The integrated inventory model has become more and more important, because the supplier and the retailer wish to increase their mutual benefit.

The benefits of just in time purchasing include small lot sizes, frequent deliveries, consistent high quality, reduction in lead times, decrease in inventory levels, lower setup cost and ordering cost, and close supplier ties. In recent years, companies have found that there are substantial benefits from establishing a long-term sole-supplier relationship with supplier. In the just in time environment, a close cooperation exists between supplier and purchaser to solve problems together, and thus maintains stable, long-term relationships. In this paper the Inventory model involving lead time crashing cost as an exponential function in food processing and distribution industry are explained with suitable illustrations

II. ASSUMPTIONS AND NOTATIONS

To develop the mathematical model, we adopt the following variables and parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Demand</td>
</tr>
<tr>
<td>Q</td>
<td>Order Quantity</td>
</tr>
<tr>
<td>C1</td>
<td>Ordering Cost</td>
</tr>
<tr>
<td>C2</td>
<td>Holding Cost</td>
</tr>
<tr>
<td>C3</td>
<td>Inventory cost</td>
</tr>
<tr>
<td>C</td>
<td>Total Cost</td>
</tr>
<tr>
<td>Rp</td>
<td>Reorder Point</td>
</tr>
<tr>
<td>L</td>
<td>Lead time in weeks</td>
</tr>
<tr>
<td>µ</td>
<td>Mean during lead time L</td>
</tr>
<tr>
<td>σL̅</td>
<td>Standard deviation during lead time L̅</td>
</tr>
</tbody>
</table>

III. MATHEMATICAL MODEL

Based on the above notations and assumptions, the total cost is given by

\[ C(Q, L) = \text{Ordering cost} + \text{holding cost} + \text{lead time} \]
crashing cost.

Since \( c_t \) is the ordering cost per order, then the expected ordering cost per year is \( \frac{c_t d}{Q} \).

From assumption (ii), the reorder point, \( R_p = d L_t + a \sigma \sqrt{L_t} \), where \( a \) is known as safety factor. Now we assume a linear decrease over the cycle, then

\[
C_3 = (Q, r) = \frac{Q}{2} + R_p - d L_t
\]  

(1)

Substitute the values of \( R_p \) in equation (1) and we get,

\[
C_3 = \frac{Q}{2} + a \sigma \sqrt{L_t}
\]  

(2)

Lead time crashing cost \( R_p (L_t) \) is given by \( \frac{d}{Q} R_p (L_t) \)

\[
C(Q, L_t) = \frac{d c_1}{Q} + C_2 C_3 + \frac{d}{Q} R_p (L_t)
\]  

(3)

We can substitute the values of \( C_3 \) and \( R_p (L_t) \) in equation (3)

\[
C(Q, L_t) = \frac{c_t d}{Q} + C_2 \left( \frac{Q}{2} + a \sigma \sqrt{L_t} \right) + \frac{d}{Q} e^{\frac{b}{L_t}}
\]  

(4)

Taking partial derivatives of \( C(Q, L_t) \), with respect to \( Q \) and \( L_t \) in each time interval \( L_t \in [L_t^e, L_t^s] \) and equating to zero, we obtain,

\[
\frac{\partial C(Q, L_t)}{\partial Q} = \frac{-C_1 d}{Q^2} + \frac{C_2}{2} - \frac{d e^{\frac{b}{L_t}}}{Q^2}
\]

Now equating the results to zero, we get

\[
\frac{\partial C(Q, L_t)}{\partial Q} = 0 \quad \frac{-C_1 d}{Q^2} + \frac{C_2}{2} - \frac{d e^{\frac{b}{L_t}}}{Q^2} = 0 
\]  

(5)

\[
\frac{\partial C(Q, L_t)}{\partial L_t} = \frac{1}{2} C_2 a \sigma L_t^{-\frac{1}{2}} - \frac{d b e^{\frac{b}{L_t}}}{Q L_t^2}
\]

Now equating the results to zero, we get

\[
\frac{\partial C(Q, L_t)}{\partial L_t} = 0 \text{ and } \frac{1}{2} C_2 a \sigma L_t^{-\frac{1}{2}} - \frac{d b e^{\frac{b}{L_t}}}{Q L_t^2} = 0
\]  

(6)

Notice that for fixed \( L_t \), \( C(Q, L_t) \) is convex in \( Q \), since,
However, for fixed $Q$, $C(Q, L_t)$ is concave in $L_t \in [L_t^L, L_t^U]$

Therefore, for fixed $Q$, the minimum occurs at the end point of the interval.

From equation (5) we have,

$$\frac{\partial C(Q, L_t)}{\partial Q^2} > 0 \quad \text{and} \quad \frac{\partial C(Q, L_t)}{\partial Q^2} = \frac{\partial}{\partial Q}\left(\frac{-C_1 d}{Q^2} + \frac{C_2}{2} - \frac{de^{LT_t}}{Q^2}\right)$$

(7)

$$\frac{\partial C(Q, L_t)}{\partial Q^2} = \frac{2C_1 d}{Q^3} + \frac{2de^{LT_t}}{Q^3}$$

The above solution procedure can be used to find the optimal $Q$ and $L_t \in [L_t^L, L_t^U]$. For each break point $L_t \in [L_t^L, L_t^U]$ we can compute $Q$ using (8) and also can compute the corresponding integrated total cost from equation (4). Finally, the optimal $Q$ and $L_t \in [L_t^L, L_t^U]$ will be the values for which the total cost $C(Q, L_t)$ is minimum.

IV. EXAMPLE

‘The Mummy’s Chips’ is a chips producing shop in Coimbatore. They are making a large quantity of banana chips. There are some data given by them as per the year 2016; the annual demand $d$ is 600 units. They want to know the best supplier by comparing two suppliers namely supplier A and supplier B using the following data

**Supplier A:**
Ordering cost is Rs. 200, Holding cost is 20, Standard deviation $\sigma$ is 6 units per week and the safety factor $a$ value is 2.3

**Supplier B:**
Ordering cost is Rs. 250, Holding cost is 28, Standard deviation $\sigma$ is 6 units per week and the safety factor $a$ value is 2.5

The lead time crashing cost is given below

$$R_p(L_t) = \begin{cases} 
0 & \text{if } L_t = 6 \\
\frac{b}{e^{LT_t}} & \text{if } 1 \leq L_t \leq 5
\end{cases} \quad \text{where, } b=5$$

They want to know the optimal order quantity and related optimal total cost.
SOLUTION
Let us consider the inventory system with the given data and we have lead time crashing cost is \( R_p(L_t) = \begin{cases} b & \text{if } L_t = 6 \\ \frac{b}{e^{L_t}} & \text{if } 1 \leq L_t \leq 5 \end{cases} \) where, \( b = 5 \)

Using equation (8) as the formula to find the optimal order quantity

\[
Q = \sqrt{\frac{(2C_1 d) + (2d e^{L_t})}{C_2}}
\]

and equation (4) as the formula to find the optimal total cost

\[
C(Q, L_t) = \frac{C_1 d}{Q} + C_2 \left( \frac{Q}{2} + a \sigma \sqrt{L_t} \right) + \frac{d}{Q} \cdot b e^{L_t}
\]

Supplier A
d=600 units per week, \( C_1 = 200 \), \( C_2 = 20 \), \( \sigma = 6 \) units per week and \( a = 2.3 \)

We have to obtain optimal order quantity and total cost for supplier A

Supplier B
d=600 units per week, \( C_1 = 250 \), \( C_2 = 28 \), \( \sigma = 6 \) units per week and \( a = 2.5 \)

We have to obtain optimal order quantity and total cost for supplier B

<table>
<thead>
<tr>
<th>( L_t )</th>
<th>Supplier A</th>
<th>Supplier B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>C</td>
<td>Q</td>
</tr>
<tr>
<td>1</td>
<td>145</td>
<td>3168</td>
</tr>
<tr>
<td>2</td>
<td>113</td>
<td>2647</td>
</tr>
<tr>
<td>3</td>
<td>111</td>
<td>2698</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>2762</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>2823</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>2867</td>
</tr>
</tbody>
</table>

Applying by the solution procedure the computational results are presented in table (1).

For supplier A the optimal solution from table (1) can be read off as lead time \( L_t^* = 2 \) weeks, order quantity \( Q^* = 113 \) units and the corresponding integrated total cost \( C^* = 2647 \).

For supplier B the optimal solution from table (1) can be read off as lead time \( L_t^* = 2 \) weeks, order quantity \( Q^* = 106 \) units and the corresponding integrated total cost \( C^* = 3562 \).

Matlab Calculation For Q And C For Supplier A
Matlab Calculation For Q And C For Supplier B

V. CONCLUSION

The integrated inventory model has become more and more important, because the supplier and the retailer wish to increase their mutual benefit. In this Paper we discussed about the Inventory Management which is important to the food processing and distribution industry. Here the solution procedure is developed to find the optimum solution using MATLAB. Both the lead time and optimum order quantity are considered as the decision variable and we consider the crashing cost is an exponential function of lead time. A numerical example demonstrated the effectiveness of the food processing to increase the utility of food and further reduce unnecessary wastage.

VI. REFERENCES


