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Mixed Range-Valued Fuzzy Threshold Graph Colouring

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ABSTRACT

Interval-valued Fuzzy Threshold graph is an extension of Mixed Range-valued Fuzzy Threshold graph Colouring. In this paper, Interval-valued fuzzy threshold graph, splitting graphs are defined and certain properties are studied. Additionally a few vital terms such as Interval-valued fuzzy colouring and chromatic number of Interval-valued fuzzy graphs are depicted and some important theorems are discussed.
Keywords: Interval-Valued Fuzzy Threshold Graph, Split Graph, Chromatic Number, Range-Valued Fuzzy Threshold Graph, Mixed-Range Valued Fuzzy Threshold Graph Colouring.

I. INTRODUCTION

Threshold graph was first introduced by Chavatal and Hammer[1]. G= (V,E) is a threshold graph where V is the set of vertices and E is the set of edges if and only if the total of weights does not exceed a certain threshold. An assignment of colours to the element of a graph under consideration is known as graph colouring. In proper colouring vertices of a graph are coloured in such a way that two adjacent vertices of a graph have different colours. The concept of range-valued fuzzy colouring, using graph colouring by the definition of proper colouring for threshold graph has been introduced.

Interval-valued fuzzy graph theory has been introduced by Zadeh in 1975[7]. Representation of an Interval-valued fuzzy graph is better than a crisp graph version. Butan and Rosenfeld have proved on strong arcs in fuzzy graphs and their properties[2]. Interval-valued fuzzy graph theory has a wide number of areas. Range-valued fuzzy colouring of interval-valued fuzzy graph introduced by Satyanarayana in 2016. We have extended this concept to Interval-valued Fuzzy threshold graph(IVFTG) using Range-valued fuzzy colours.

We have introduced the new concept of "Range-valued fuzzy threshold graph". In 1972, Richard carp Introduced dominating set[14] and chavtel and Hammer introduced threshold graph[4]. We combined these two concepts and extended to "**Mixed range-valued fuzzy threshold graph colouring"** which satisfies all the fuzzy threshold condition.

II. PRELIMERIES

Threshold graph is a graph that can be build from a one-vertex graph by repeated application of the following two operations. (i) Adding pendent vertex to all the vertices of the linear graph (ii) Here, x and y are dominating vertex. We can join pendent vertex of x with dominating vertex y and also we can join pendant vertex of y with dominating vertex X. (iii) Addition of a single dominating vertex to the graph. Ie) single vertex that is connected to all other vertices.

An interval-valued fuzzy threshold graph(IVFTG)[13] G= (V,A,B) is called an interval-valued fuzzy threshold graph with threshold t=$[t^-, t^+]$ such that $\sum_{u \in U} \mu_A^-(u) \leq t^-$ and $\sum_{u \in U} \mu_A^+(u) \leq t^+$iff U(⊆ V) is an independent set in G.

The Range-valued fuzzy graph colouring[10] is a combination of any fundamental colour (C_m)

With a white colour decrease the brightness of that fundamental colour. Here fundamental colours are red, green, yellow, blue, etc. Then the brightness is called fuzzy term.

III. A STUDY ON GRAPHS USING RANGE-VALUED FUZZY COLOURING AND INTERVAL-VALUED FUZZY THRESHOLD GRAPHS

In this section we have discussed about threshold graph with n-dominating set, Converting split graph to threshold graph, Range-valued fuzzy threshold graph colouring

A. Threshold graph with 2- dominating set

Let us consider an example where the two reals $t^-$and t^+ are taken as t^- =0.7 and t^+= 0.9. In this example , we consider two dominating set X, Y and the independent set is U= {a,b,c,d,e,f}.

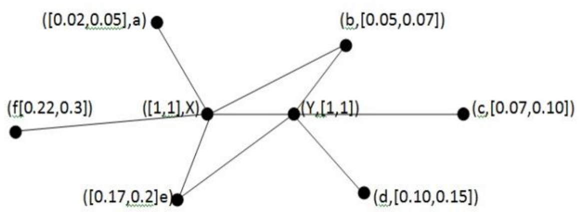

Figure 1. Threshold graph with 2-dominating set and 6-pendant vertices.

$\sum_{u \in U} \mu_A^-(u) \leq t^-$ and $\sum_{u \in U} \mu_A^+(u) \leq t^+$

Consequently, for independent sets S_1 ={a,X} , S_2 ={b,X} , S_3 ={e,X}, S_4 ={f,X}, S_5 ={b,Y}, S_6 ={c,Y}, S_7={d,Y}, S_8={e,Y}

And Satisfies the condition,

(i) S_1= $\mu_A^-(a)+\mu_A^-(X) > t^-$ and $\mu_A^+(a)+\mu_A^+(X) > t^+$

(ii) S_2= $\mu_A^-(b)+\mu_A^-(X) > t^-$ and $\mu_A^+(b)+\mu_A^+(X) > t^+$

(iii) S_3= $\mu_A^-(e)+\mu_A^-(X) > t^-$ and $\mu_A^+(e)+\mu_A^+(X) > t^+$

(iv) S_4= $\mu_A^-(f)+\mu_A^-(X) > t^-$ and $\mu_A^+(f)+\mu_A^+(X) > t^+$

(v) S_5= $\mu_A^-(b)+\mu_A^-(Y) > t^-$ and $\mu_A^+(b)+\mu_A^+(Y) > t^+$

(vi) S_6= $\mu_A^-(c)+\mu_A^-(Y) > t^-$ and $\mu_A^+(c)+\mu_A^+(Y) > t^+$

(vii) S_7= $\mu_A^-(d)+\mu_A^-(Y) > t^-$ and $\mu_A^+(d)+\mu_A^+(Y) > t^+$

(viii) S_8= $\mu_A^-(e)+\mu_A^-(Y) > t^-$ and $\mu_A^+(e)+\mu_A^+(Y) > t^+$

Also this condition satisfies n –dominating sets and n pendant vertices.

B. Converting Split graph to threshold graph

Definition: The **Split graph** is a graph in which the nodes can be divided into a clique and an Intependent set.

Definition: An **Interval-valued fuzzy split graph(IVFSG)** is an IVFG in which the nodes can be divided into a fuzzy cycle and fuzzy independent set.

Theorem: Every Interval-valued fuzzy split graph(IVFSG) is an Interval-valued fuzzy threshold graph(IVFTG).

Proof: Let G=(V,A,B) is a splitting graph. Here x and y are dominating vertices. When we join all the pendant with a dominating vertices x and y of a splitting graph, we can get a threshold graph.

Also it satisfies the condition of the splitting graph.

$\sum_{u \in U} \mu_A^- \leq t^-$, $\sum_{u \in U} \mu_A^+ \leq t^+$,

And $\mu_A^-(x)+\mu_A^-(y) > t^-$, $\mu_A^+(x)+\mu_A^+(y) > t^+$

Hence, we observe that every IVFSG is an IVFTG .

For example,

Let we consider **x** and **y** are dominating vertices. We join a pendant vertex **a** to the dominating set **y** and also join a pendant vertex of **c** to the dominating set **x**.

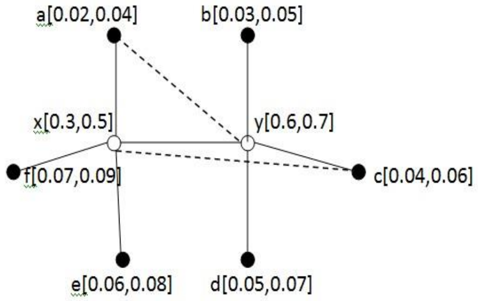

Figure 2. Split graph to threshold graph

The above diagram is converted to threshold graph and also satisfies the threshold condition and split graph condition.

Here $t^- = 0.3, t^+ = 0.5$

$\sum_{u \in U} \mu_A^-(U) \leq t^-, \sum_{u \in U} \mu_A^+(U) \leq t^+$,

And $\mu_A^-(x) + \mu_A^-(y) > t^-, \mu_A^+(x) + \mu_A^+(y) > t^+$.

Threshold conditions are,

S_1 ={a,X} , S_2 ={f,X} , S_3 ={e,X}, S_4 ={c,X}, S_5 ={b,Y}, S_6={a,Y}, S_7={c,Y}, S_8={d,Y}

And Satisfies the condition,

(i) $S_1 = \mu_A^-(a) + \mu_A^-(X) > t^-$ and $\mu_A^+(a) + \mu_A^+(X) > t^+$

(ii) $S_2 = \mu_A^-(f) + \mu_A^-(X) > t^-$ and $\mu_A^+(f) + \mu_A^+(X) > t^+$

(iii) $S_3 = \mu_A^-(e) + \mu_A^-(X) > t^-$ and $\mu_A^+(e) + \mu_A^+(X) > t^+$

(iv) $S_4 = \mu_A^-(c) + \mu_A^-(X) > t^-$ and $\mu_A^+(c) + \mu_A^+(X) > t^+$

(v) $S_5 = \mu_A^-(b) + \mu_A^-(Y) > t^-$ and $\mu_A^+(b) + \mu_A^+(Y) > t^+$

(vi) $S_6 = \mu_A^-(a) + \mu_A^-(Y) > t^-$ and $\mu_A^+(a) + \mu_A^+(Y) > t^+$

(vii) $S_7 = \mu_A^-(c) + \mu_A^-(Y) > t^-$ and $\mu_A^+(c) + \mu_A^+(Y) > t^+$

(viii) $S_8 = \mu_A^-(d) + \mu_A^-(Y) > t^-$ and $\mu_A^+(d) + \mu_A^+(Y) > t^+$

IV. MIXED RANGE-VALUED FUZZY THRESHOLD GRAPH COLOURING

Threshold graph is a graph that can be constructed from a graph having a dominating set by repeated applications of the following two operations

1. Addition of a number of dominating set to threshold graph having the graph.
2. We join a pendant vertex of a dominating set to the any other dominating vertex of the graph.

We can colour all the vertices of threshold graph, by using mixed range values which has degree greater than or equal to 2. Ie) $d_M(I_{ftg}) \geq 2$. Interval-valued fuzzy threshold graph colouring number is denoted by $\chi_M(I_{ftg})$.

This known as **Mixed range-valued fuzzy threshold graph colouring.**

Note: To check threshold conditions of Mixed range-valued fuzzy threshold graph $\mu^-(x,y) = \min\{\sigma^-(x), \sigma^-(y)\}$ and $\mu^+(x,y) = \min\{\sigma^+(x), \sigma^+(y)\}$.

Theorem : The Range-valued of IVFTG has always $\chi(G) = \gamma(G)$.

Proof: Let G=(V,A,B) is said to be range-valued fuzzy threshold graph. Here $\chi(G)$ is denoted by chromatic number of a graph G and $\gamma(G)$ is domination number of a graph G.

We know that the range-valued of IVFTG has always powerless branches. First, we apply the fundamental colour to the central vertex. Then we apply the range-valued of fuzzy fundamental colour.

Example 1:

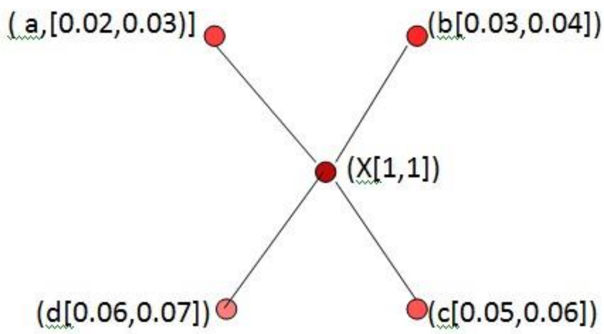

Figure 3. Example of dominating set

From the above figure, here the chromatic number is 1 and also the domination number is 1. Hence $\chi(G) = \gamma(G)$.

Example 2:

We consider a simple threshold graph with 3-dominating set and 8- pendant vertices.

Here, each pendant vertex having the interval values of $[0.s_1, 0.s_2, \dots\dots]$ and dominating set having the interval values $[p_1, p_2, \dots\dots]$ where $(s_1, s_2, \dots\dots, s_n)$ and $(p_1, p_2, \dots\dots, p_n)$ are called natural numbers.

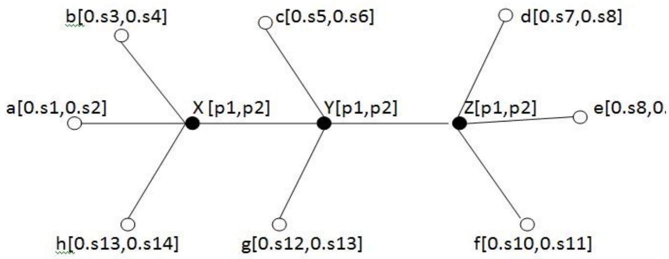

Figure 4. Simple threshold graph

Now, we join one pendant vertices of a dominating set to another dominating set. Hence this is called as mixed range-valued fuzzy threshold graph.

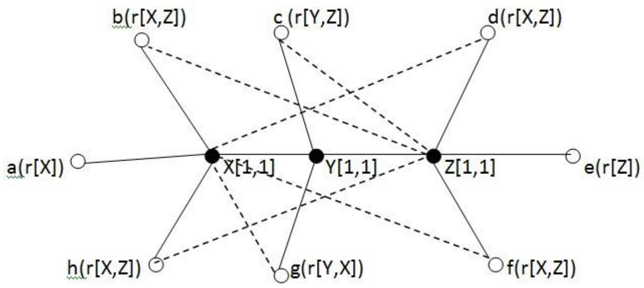

Figure 5. Mixed range-valued fuzzy threshold graph

First, we satisfy the threshold condition. Here $t^- = 0.6, t^+ = 0.8$.

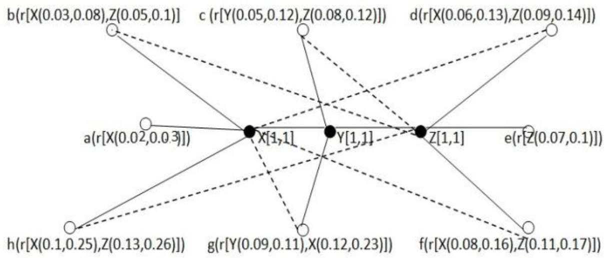

Figure 6. Mixed range-valued fuzzy threshold graph with interval values

$\sum_{u \in U} \mu_A^-(U) \leq t^-$, $\sum_{u \in U} \mu_A^+(U) \leq t^+$

Consequently, for independent sets S_1={R, (range of R)} , S_2={R, Y} , S_3={G,Y}, S_4={R,Y}, S_5={Y,(range of Y)}, S_6={R,Y}, S_7={R, G}, S_8={R ,Y}. And Satisfies the condition,

i. $S_1 = \mu_A^-$(range of R) + $\mu_A^-(R) > t^-$ and μ_A^+(range of R)+ $\mu_A^+(R) > t^+$

ii. $S_2 = \mu_A^-(RY) + \mu_A^-(R) > t^-$ and $\mu_A^+(RY) + \mu_A^+(R) > t^+$

iii. $S_3 = \mu_A^-(GY) + \mu_A^-(G) > t^-$ and $\mu_A^+(GY) + \mu_A^+(G) > t^+$

iv. $S_4 = \mu_A^-(RY) + \mu_A^-(Y) > t^-$ and $\mu_A^+(RY) + \mu_A^+(Y) > t^+$

v. $S_5 = \mu_A^-$ (range of Y)+ $\mu_A^-(Y) > t^-$ and μ_A^+(range of Y)+ $\mu_A^+(Y) > t^+$

vi. $S_6 = \mu_A^-(RY) + \mu_A^-(Y) > t^-$ and $\mu_A^+(RY) + \mu_A^+(Y) > t^+$

vii. $S_7 = \mu_A^-(RG) + \mu_A^-(G) > t^-$ and $\mu_A^+(RG) + \mu_A^+(G) > t^+$

viii. $S_8 = \mu_A^-(RY) + \mu_A^-(R) > t^-$ and $\mu_A^+(RY) + \mu_A^+(R) > t^+$

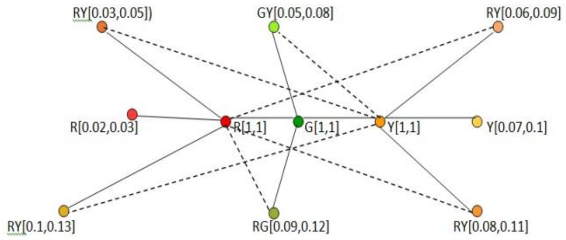

Figure 7. Mixed range-valued fuzzy threshold graph colouring

Next, we consider the chromatic number of these threshold graph.

Chromatic Number $\chi_M(I_{ftg})$=3, degree of dominating set $d_M(I_{ftg}) \geq 2$.

V. APPLICATION

Real life application of range-valued fuzzy threshold graphs are

Paint Manufacturing company

Usually the manufactures are producing many colour shades using range-values. But, if we use threshold condition of range values to get more number of colour shades than usual manner.

For example, we can choose the colour, "Red". By using range valued of the colour red[1,1], many number of colour shades. When we apply the threshold condition, more number of shades can be produced. By using Mixed range valued fuzzy threshold graph(M-IVFTG), we can mix range value of any two colour to produce more and more colour shades.

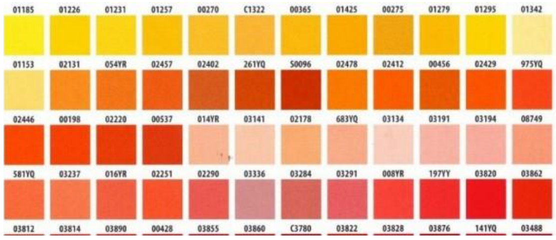

Figure 8. Colour Shades

This process is applicable for also textile industry and beauty cosmetics making company's.

VI. CONCLUSION

In this paper, we have introduced Range-valued fuzzy colouring and Interval-valued fuzzy threshold graph, Converting Split graph to threshold graph and its properties with example. Also, we have derived the theorem related to Interval-valued fuzzy split graph. Then we introduce a new graph namely **Mixed range-valued fuzzy threshold graph** and the parameter.

Finally, we derive the theorem about the graph and notation is denoted by $\chi_M(I_{ftg})$ which has domination number as 3. We can produce new graphs for n number of dominating set in future.

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