

A Markov Decision Model for Hospital Ward Admission Scheduling

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ABSTRACT

Hospitals continually face the challenge of planning and allocating hospital wards to incoming patients in a stochastic demand environment. In this paper, a finite horizon Markov decision process model is developed and analyzed for the hospital ward allocation activity. The model focuses explicitly on developing policies for determining when to admit additional patients using weekly data from incoming patients and the availability of vacancies in hospital wards. Adopting a Markov decision process approach, the states of a Markov chain represent possible states of demand for ward occupancy. The decision of whether or not to admit additional patients is made using dynamic programming over a finite period planning horizon. A numerical example demonstrates the existence of an optimal state-dependent admission policy as well as the corresponding total operational costs in hospital wards.

Keywords: *Admission, Hospital ward, markov decision*

I. INTRODUCTION

Planning and managing ward allocations in hospitals are still great challenges in hospital administration; especially when demand for hospital wards follows a stochastic trend. It requires a good understanding of the environment in which the hospital is operating and the development of a vision regarding future ward allocation decisions. Proper ward allocation must therefore ensure that the requirement of wards by patients are matched with the ward availability so that healthcare delivery is not hindered by lack of wards or the hospital does not have idle, excess wards on hand. Two major problems are usually encountered:

- (i) Determining the most desirable period during which to admit additional patients
- (ii) Determining the optimal operating costs corresponding to the admission policy given a periodic review ward allocation planning system when demand for wards is uncertain.

It is essential for the hospital to gauge and know its reasonably or optimum ward capacity as no single capacity is applicable to the provision of healthcare

services. In this paper, a Markov decision model is proposed whose goal is to optimize the ward allocation decisions for incoming patients when demand for wards is uncertain.

The paper is organized as follows. After reviewing the previous work done, a mathematical model is proposed where consideration is given to the process of estimating model parameters. The model is then solved and applied to a special case study. Thereafter, final remarks follow. Muligan J [1] used Queuing theory to empirically approximate the potential for hospital scale economies due to stochastic nature of hospitals demand and service. The results suggest that scale economies due to stochastic demand and service are likely to be important only for consolidation of small, specialized hospital units. According to Carey [2], the issue of hospital bed capacity can be examined by considering stochastic demand for United States hospitals. An equilibrium condition for the optimal number of "excess beds" can be derived and applied using cost function estimated with a set of data. Results indicate that it may be difficult to justify the costliness of existing levels of empty hospital beds. In the article of Harper and Bagust

[3], the daily bed requirements are examined arising from the flow of emergency admissions to an acute hospital. The paper identifies the implications on fluctuating and unpredictable demands for emergency admission for the management of hospital bed capacity, and to quantify the daily risk of insufficient capacity for patients requiring immediate change to admission.

According to Harper and Shahani [4], a simulation model can be developed that considers various types of patient inflows at the individual patient level and the resulting bed needs over time. The sequence of changes in capacity planning policies and the management of existing capacities can be readily examined. The work further highlights the need for evaluating hospital bed capacities in light of both occupancies and refused admission rates. In the article of Hughes D and McGuire A [5], the authors consider production responses to demand uncertainty with the hospital sector. It is noted that such responses have an impact on hospital cost structures. An empirical model is specified and estimation is undertaken on a sample of UK hospitals over the period 1993-1995, differentiating between hospital output which arises from uncertain demand and output considered to be predictable. It is found that demand uncertainty impose a direct cost equivalent to around 5% of the total cost of emergency admission. In the article of Green [6], the general background involved in hospital capacity planning is described. Examples of how OR models can be used to provide important insights into operational strategies and practices and to identify opportunities and challenges for future research are explored. Additional work by Smet M [7] shows the relationship between uncertain demand, excess bed capacity, hospital costs and performance. A waiting time indicator to proxy hospital standby capacity is incorporated into a multi-product translog cost function for the Belgian general hospitals. This allows calculation of cost elasticity and marginal cost of seven hospital departments as well as the degree of economies of scale and enables identification of differences in efficiencies. The problem was further examined by Lovell, Rodriguez and Wall [8]. In their article, demand uncertainty is incorporated into the system to account for the hospital activity of standby capacity or insurance against unexpected demand. It is found that demand uncertainty in Spanish hospitals affects hospital production and increases costs. Results also show that over capitalization in such hospitals can be explained by hospitals providing insurance demand when faced with

demand uncertainty. In the article of Boutsiolis [9], the expected demand of Greek public hospitals is estimated where a multi variable model with four explanatory variables is used. The forecasted residuals of hospital regressions for each year give the estimated stochastic demand. It is shown that demand varies both among hospitals and over the five-year time period under investigation. Further research by Ayvaz, Nur Huh and Woonghee [10] focuses on a hospital setting and a general model is formulated that is applicable to various resource allocation problems in a hospital. A multiple of customer classes that display different reactions to the delays in service is considered. By adopting a dynamic programming approach, it is shown that the optimal policy for a system involving both lost sales and backorders is not simple but exhibits desirable properties.

In this paper, healthcare system is considered whose goal is to optimize the admission decisions of patients and the total operational costs of wards. At the beginning of each period, a major decision has to be made, namely whether to admit additional patients or not to admit for optimal utilization of the healthcare available resources.

II. METHODS AND MATERIAL

2. MODEL DEVELOPMENT

2.1 Notation and Assumptions

- i, j = states of demand
- P^Z_{ij} = Number of patients
- A = Very High state
- O^Z_{ij} = Operational costs
- B = High state
- n, N = Stages
- C = Moderate state
- V^Z_i = Expected operational costs
- D = Low state
- a^Z_i = Accumulated operational costs
- E = Very low state
- w = Hospital ward
- O^Z = Operational cost matrix
- P^Z = Patient matrix
- Q^Z = Demand transition matrix
- Q^Z_{ij} = Demand transition probability
- Z = Admission policy

$i, j \in \{A, B, C, D, E\}$ $Z \in \{0, 1\}$ $w \in \{1, 2, 3\}$
 $N = 1, 2, \dots, \dots, N$

period n is $i \in \{A, B, C, D, E\}$ the recursive relation relating g_n and g_{n+1} is:

We consider a healthcare system whose demand for hospital wards during a chosen period over a fixed planning horizon is classified as *Very high*(state A), *High*(state B), *Moderate*(state C), *Low*(state D) or *Very low*(state E) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. The representation assumes the following correspondence between the ward occupancy and the states of the chain:

| Ward Occupancy (%) | State of demand | State code |
|--------------------|-----------------|------------|
| 85-100 | Very high | A |
| 70-84 | High | B |
| 55-69 | Moderate | C |
| 40-54 | Low | D |
| 0-39 | Very low | E |

Suppose one is interested in determining the optimal course of action, namely to admit additional patients (a decision denoted by $Z=1$) or not to admit (a decision denoted by $Z=0$) during each time period over the planning horizon where Z is a binary decision variable. Optimality is defined such that the minimum expected operational costs are accumulated at the end of N consecutive time periods spanning the planning horizon. In this paper, a two ward ($W=2$) and two-period ($N=2$) planning horizon are considered.

2.2 Finite Dynamic Programming Formulation

Recalling that the demand can be in state A, B, C, D or E the problem of finding an optimal admission decision can be expressed as a finite period dynamic programming model.

Let $g_n(i, w)$ denote the expected operational costs accumulated in ward w during periods $n, n+1, \dots, N$ given that the state of the system at the beginning of

$$g_n(i, w) = \min_Z [Q_{iA}^Z(w)O_{iA}^Z(w) + g_n(A, w)] \\ + \min_Z [Q_{iB}^Z(w)O_{iB}^Z(w) + g_n(B, w)] \\ + \min_Z [Q_{iC}^Z(w)O_{iC}^Z(w) + g_n(C, w)] \\ + \min_Z [Q_{iD}^Z(w)O_{iD}^Z(w) + g_n(D, w)] \\ + \min_Z [Q_{iE}^Z(w)O_{iE}^Z(w) + g_n(E, w)] \quad (1)$$

together with the conditions

$$g_{N+1}(A) = g_{N+1}(B) = g_{N+1}(C) = g_{N+1}(D) = g_{N+1}(E) = 0$$

This recursive equation may be justified by noting that the cummulation total operational costs $O_{ij}^Z(w) + g_{n+1}(i, w)$ resulting from reaching state $j \in \{A, B, C, D, E\}$ at the start of period $n+1$ from state $i \in \{A, B, C, D, E\}$ at the start of period n occurs with probability $Q_{ij}^Z(w)$. Clearly, $V^Z(w) = Q^Z(w)[O^Z(w)]^T$, $Z \in \{0, 1\}$ where "T" denotes matrix transposition, and hence the dynamic programming recursive equations

$$g_n(i, w) = \min_Z [V_i^Z(w) + Q_{iA}^Z(w)g_{n+1}(A) + Q_{iB}^Z(w)g_{n+1}(B)] \\ + \min_Z [V_i^Z(w) + Q_{iC}^Z(w)g_{n+1}(C) + Q_{iD}^Z(w)g_{n+1}(D)] \\ + \min_Z [V_i^Z(w) + Q_{iE}^Z(w)g_{n+1}(E)] \\ i \in \{A, B, C, D, E\}, n=1, 2, \dots, \dots, N-1, \\ Z \in \{0, 1\}$$

$$g_N(i, w) = \min[V_i^Z(w)] \quad (2) \\ i \in \{A, B, C, D, E\}$$

2.3 Computing $Q^Z(w)$

The demand transition probability from state $i \in \{A, B, C, D, E\}$ to state $j \in \{A, B, C, D, E\}$ given admission policy $Z \in \{0, 1\}$ may be taken as the number of patients observed at ward w when demand is initially in state i and later with demand changing to state j , divided by the number of patients over all states. That is.

$$Q_{ij}^Z(w) = P_{ij}^Z(w) / [P_{iA}^Z(w) + P_{iB}^Z(w) + P_{iC}^Z(w) + P_{iD}^Z(w) + P_{iE}^Z(w)] \\ i \in \{A, B, C, D, E\} \quad Z \in \{0, 1\} \quad (3)$$

3. COMPUTING AN OPTIMAL ADMISSION POLICY

The optimal admission policy for patients to wards is found in this section for each time period separately. We recall that the model assumes two wards over a two-period planning horizon

3.1 Optimization during period 1

When demand is very high (state A), the optimal admission policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } V_A^1(w) < V_A^0(w) \\ 0 & \text{if } V_A^1(w) \geq V_A^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_1(A, w) = \begin{cases} V_A^1(w) & \text{if } Z = 1 \\ V_A^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is high (state B), the optimal admission policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } V_B^1(w) < V_B^0(w) \\ 0 & \text{if } V_B^1(w) \geq V_B^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_1(B, w) = \begin{cases} V_B^1(w) & \text{if } Z = 1 \\ V_B^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is moderate (state C), the optimal admission policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } V_C^1(w) < V_C^0(w) \\ 0 & \text{if } V_C^1(w) \geq V_C^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_1(C, w) = \begin{cases} V_C^1(w) & \text{if } Z = 1 \\ V_C^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is moderate (state D), the optimal admission policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } V_D^1(w) < V_D^0(w) \\ 0 & \text{if } V_D^1(w) \geq V_D^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_1(D, w) = \begin{cases} V_D^1(w) & \text{if } Z = 1 \\ V_D^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is moderate (state E), the optimal admission policy during period 1 is

$$Z = \begin{cases} 1 & \text{if } V_E^1(w) < V_E^0(w) \\ 0 & \text{if } V_E^1(w) \geq V_E^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_1(E, w) = \begin{cases} V_E^1(w) & \text{if } Z = 1 \\ V_E^0(w) & \text{if } Z = 0 \end{cases}$$

3.2 Optimization during period 2

Using dynamic programming recursive equation in (2), and recalling that $a_i^z(w)$ denotes the already accumulated operational costs at the end of period 1 as a result of decisions made during that period, it follows that

$$\begin{aligned} a_i^z(w) = & V_i^z(w) + Q_{iA}^z(w) \min[V_A^1(w), V_A^0(w)] + Q_{iB}^z(w) \min[V_B^1(w), V_B^0(w)] \\ & + Q_{iC}^z(w) \min[V_C^1(w), V_C^0(w)] + Q_{iD}^z(w) \min[V_D^1(w), V_D^0(w)] \\ & + Q_{iE}^z(w) \min[V_E^1(w), V_E^0(w)] \end{aligned}$$

Therefore, when demand is very high (state A), the optimal admission policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_A^1(w) < a_A^0(w) \\ 0 & \text{if } a_A^1(w) \geq a_A^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_2(A, w) = \begin{cases} a_A^1(w) & \text{if } Z = 1 \\ a_A^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is high (state B), the optimal admission policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_B^1(w) < a_B^0(w) \\ 0 & \text{if } a_B^1(w) \geq a_B^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_2(B, w) = \begin{cases} a_B^1(w) & \text{if } Z = 1 \\ a_B^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is moderate (state C), the optimal admission policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_C^1(w) < a_C^0(w) \\ 0 & \text{if } a_C^1(w) \geq a_C^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_2(C, w) = \begin{cases} a_C^1(w) & \text{if } Z = 1 \\ a_C^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is low (state D), the optimal admission policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_D^1(w) < a_D^0(w) \\ 0 & \text{if } a_D^1(w) \geq a_D^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_2(D, w) = \begin{cases} a_D^1(w) & \text{if } Z = 1 \\ a_D^0(w) & \text{if } Z = 0 \end{cases}$$

When demand is very low (state E), the optimal admission policy during period 2 is

$$Z = \begin{cases} 1 & \text{if } a_E^1(w) < a_E^0(w) \\ 0 & \text{if } a_E^1(w) \geq a_E^0(w) \end{cases}$$

The associated total operational costs for managing hospital wards are

$$g_2(E, w) = \begin{cases} a_E^1(w) & \text{if } Z = 1 \\ a_E^0(w) & \text{if } Z = 0 \end{cases}$$

4. CASE STUDY

4.1 Case Description

In order to demonstrate the use of model in §2-3, real case applications from Case Medical Hospital, a Hospital in Uganda is presented in this section. The demand for hospital wards fluctuates on a weekly basis. The hospital wants to avoid admissions of excess patients when wards are at full capacity, and hence seeks decision support in terms of an optimal admission policy for patients and the associated total operational costs for managing hospital wards over a two-week planning horizon.

4.2 Data Collection

A sample of 40 transitions for the demand of hospitals wards was used at three separate wards and the associated total operational costs were noted when additional patients were admitted ($Z=1$) versus when additional patients were not admitted ($Z=0$).

When additional patients were admitted ($Z=1$), the following matrices were obtained.

Ward 1:

$$N^1(1) = \begin{bmatrix} 20 & 10 & 5 & 3 & 2 \\ 10 & 15 & 8 & 4 & 3 \\ 8 & 6 & 20 & 4 & 2 \\ 6 & 8 & 4 & 15 & 7 \\ 9 & 6 & 5 & 2 & 18 \end{bmatrix}$$

$$P^1(1) = \begin{bmatrix} 50 & 25 & 10 & 17 & 13 \\ 18 & 40 & 10 & 18 & 15 \\ 15 & 19 & 18 & 14 & 18 \\ 12 & 14 & 18 & 20 & 11 \\ 8 & 10 & 12 & 18 & 20 \end{bmatrix}$$

Ward 2:

$$N^1(2) = \begin{bmatrix} 18 & 12 & 6 & 2 & 2 \\ 7 & 18 & 6 & 4 & 5 \\ 9 & 7 & 16 & 5 & 3 \\ 8 & 6 & 5 & 14 & 7 \\ 10 & 5 & 8 & 2 & 15 \end{bmatrix}$$

$$P^1(2) = \begin{bmatrix} 42 & 28 & 12 & 14 & 13 \\ 15 & 45 & 7 & 18 & 18 \\ 20 & 22 & 12 & 16 & 20 \\ 13 & 10 & 20 & 17 & 11 \\ 11 & 8 & 14 & 18 & 16 \end{bmatrix}$$

When additional patients were *not* admitted ($Z=0$), the matrices below follow:

Ward 1:

$$N^0(1) = \begin{bmatrix} 22 & 3 & 8 & 5 & 2 \\ 9 & 16 & 5 & 7 & 3 \\ 8 & 5 & 18 & 6 & 3 \\ 7 & 10 & 3 & 16 & 4 \\ 10 & 5 & 3 & 4 & 18 \end{bmatrix}$$

$$P^0(1) = \begin{bmatrix} 38 & 20 & 12 & 16 & 20 \\ 10 & 40 & 8 & 10 & 12 \\ 6 & 20 & 10 & 16 & 18 \\ 8 & 12 & 20 & 18 & 12 \\ 8 & 10 & 12 & 24 & 20 \end{bmatrix}$$

Ward 2:

$$N^0(2) = \begin{bmatrix} 20 & 7 & 4 & 6 & 3 \\ 10 & 12 & 8 & 9 & 1 \\ 7 & 6 & 20 & 2 & 5 \\ 6 & 4 & 8 & 14 & 8 \\ 12 & 7 & 4 & 1 & 16 \end{bmatrix}$$

$$P^0(2) = \begin{bmatrix} 35 & 18 & 16 & 10 & 8 \\ 12 & 42 & 8 & 12 & 11 \\ 4 & 22 & 8 & 10 & 12 \\ 6 & 14 & 18 & 16 & 14 \\ 10 & 12 & 14 & 20 & 18 \end{bmatrix}$$

4.3 Computation of Model Parameters

Using (5), the state transition matrices for wards1 and 2 are

Ward 1:

$$Q^1(1) = \begin{bmatrix} 0.5 & 0.25 & 0.125 & 0.075 & 0.05 \\ 0.25 & 0.375 & 0.20 & 0.10 & 0.075 \\ 0.20 & 0.15 & 0.50 & 0.10 & 0.05 \\ 0.15 & 0.20 & 0.10 & 0.375 & 0.175 \\ 0.225 & 0.15 & 0.125 & 0.05 & 0.45 \end{bmatrix}$$

$$Q^0(1) = \begin{bmatrix} 0.55 & 0.20 & 0.075 & 0.125 & 0.05 \\ 0.225 & 0.40 & 0.125 & 0.175 & 0.075 \\ 0.20 & 0.125 & 0.45 & 0.15 & 0.075 \\ 0.175 & 0.25 & 0.075 & 0.40 & 0.10 \\ 0.25 & 0.125 & 0.075 & 0.10 & 0.45 \end{bmatrix}$$

Ward 2

$$Q^1(2) = \begin{bmatrix} 0.45 & 0.30 & 0.15 & 0.05 & 0.05 \\ 0.175 & 0.45 & 0.15 & 0.10 & 0.125 \\ 0.225 & 0.175 & 0.40 & 0.125 & 0.075 \\ 0.20 & 0.15 & 0.125 & 0.35 & 0.175 \\ 0.25 & 0.125 & 0.20 & 0.05 & 0.375 \end{bmatrix}$$

$$Q^0(2) = \begin{bmatrix} 0.50 & 0.175 & 0.10 & 0.15 & 0.075 \\ 0.25 & 0.30 & 0.20 & 0.225 & 0.025 \\ 0.175 & 0.15 & 0.50 & 0.05 & 0.125 \\ 0.15 & 0.10 & 0.20 & 0.35 & 0.20 \\ 0.25 & 0.125 & 0.075 & 0.10 & 0.45 \end{bmatrix}$$

Using (2), the expected operational costs are computed and results are summarized in Table I below:

Table I: Expected Operational Costs for states of demand in hospital wards given different admission policies

| Ward (w) | State of Demand (i) | Expected Operational costs $V_i^Z(w)$ | |
|----------|---------------------|---------------------------------------|------|
| | | Z=1 | Z=0 |
| 1 | A | 34.4 | 28.8 |
| | B | 24.4 | 20.9 |
| | C | 17.2 | 12.0 |
| | D | 15.8 | 14.3 |
| | E | 14.7 | 15.6 |
| 2 | A | 30.5 | 24.4 |
| | B | 27.9 | 20.2 |
| | C | 16.7 | 10.0 |
| | D | 14.5 | 14.3 |
| | E | 13.5 | 15.2 |

The accumulated operational costs are similarly computed and results are summarized in Table II below:

TABLE II : Accumulated operational costs for states of demand in hospital wards given different admission policies

| Ward (w) | State of Demand (i) | Accumulated Operational costs $a_i^Z(w)$ | |
|----------|---------------------|--|-------|
| | | Z=1 | Z=0 |
| 1 | A | 53.33 | 52.24 |
| | B | 44.37 | 40.85 |
| | C | 34.26 | 29.02 |
| | D | 33.44 | 32.66 |
| | E | 33.15 | 33.50 |
| 2 | A | 50.43 | 44.29 |
| | B | 45.88 | 37.92 |
| | C | 32.53 | 24.70 |
| | D | 31.03 | 29.69 |
| | E | 29.90 | 32.08 |

4.4 The Optimal Admission Policy

4.4.1 Results - Ward 1 (Week 1)

Since $28.8 < 34.4$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$28.8 when demand is Very High (state A). Since $20.9 < 24.4$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$20.9 when demand is High (state B).

Since $12.2 < 17.2$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$12.2 when demand is Moderate (state C). Since $14.3 < 15.8$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$14.3 when demand is Low (state D).

Since $14.7 < 15.6$, it follows that $Z=1$ is an optimal admission policy with expected operational costs of \$14.7 when demand is Very Low (state E).

4.4.2 Results - Ward 2 (Week 1)

Since $24.4 < 30.5$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$24.4 when demand is Very High (state A). Since $20.2 < 27.9$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$20.2 when demand is High (state B).

Since $10.0 < 16.7$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$10.0 when demand is Moderate (state C). Since $14.3 < 14.5$, it follows that $Z=0$ is an optimal admission policy with expected operational costs of \$14.3 when demand is Low (state D).

Since $13.5 < 15.2$, it follows that $Z=1$ is an optimal admission policy with expected operational costs of \$13.5 when demand is Very Low (state E).

4.4.3 Results - Ward 1 (Week 2)

Since $52.24 < 53.33$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$52.24 when demand is Very High (state A). Since $40.85 < 44.37$, it follows that $Z=0$ is an optimal admission

policy with accumulated operational costs of \$40.85 when demand is High (state B).

Since $29.02 < 34.26$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$29.02 when demand is Moderate (state C). Since $32.66 < 33.44$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$32.66 when demand is Low (state D).

Since $33.15 < 33.50$, it follows that $Z=1$ is an optimal admission policy with accumulated operational costs of \$33.15 when demand is Very Low (state E).

4.4.4 Results - Ward 2 (Week 2)

Since $44.29 < 50.43$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$44.29 when demand is Very High (state A). Since $37.92 < 45.88$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$37.92 when demand is High (state B).

Since $24.70 < 32.53$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$24.70 when demand is Moderate (state C). Since $26.69 < 31.03$, it follows that $Z=0$ is an optimal admission policy with accumulated operational costs of \$26.69 when demand is Low (state D).

Since $29.90 < 32.08$, it follows that $Z=1$ is an optimal admission policy with accumulated operational costs of \$29.90 when demand is Very Low (state E).

III. CONCLUSION

Markov decision processes can be very useful in optimizing decisions pertaining to hospital ward admissions scheduling of patients under demand uncertainty. This is possible provided the problem is formulated as a multi-period decision problem using dynamic programming over a finite period planning horizon. It would however be worthwhile to extend the research and examine the behaviour of admission policies under non stationary demand conditions. In the same spirit, the model developed raises a number of salient issues to consider: Admission of patients under emergency conditions and capacity management for

medical personnel to handle unexpected patients. Finally, special interest is sought in further extending the research by considering admission policies in the context of Continuous Time Markov Chains (CTMC).

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