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# Effects of Radiating Oblate Spheroid and Triaxial Rigid Body in the Restricted Three Body Problem

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**ABSTRACT**

This paper deals with finding stationary solutions and periodic orbits of the planar restricted three body problem, when the smaller primary is a triaxial rigid body with one of the axes as the axis of symmetry and the bigger primary an oblate spheroidand the source of radiationwith equatorial plane of the primaries coinciding with the plane of the motion. It is further observed that the collinear points are unstable, while the triangular equilibrium points are stable for the range of mass parameter $0 \leq \mu < \mu_{crit}$ (the critical mass parameter). It is also observed that the triangular equilibrium points have long or short periodic elliptical orbits in the same range of μ.

**Keywords:** Restricted Problem; Libration Points; Rigid Body; Oblate Spheroid; Periodic orbits.

## I. INTRODUCTION

The present paper deals with the stationary solutions of the planar restricted three body problems when the bigger primary is taken as radiating oblate spheroid, where as the smaller primary is a triaxial rigid body[while considering the bigger primary as a oblate radiating spheroid whose equatorial plane is coincided with the plane of motion, the smaller primary is considered to be a triaxial rigid body whose one of the axis as the axis symmetry whose equatorial plane is considered with the plane of motion]. In restricted three body problem five equilibrium points exist, out of which two haveequilateral triangular configuration with the primaries and the remaining three are collinear with the primaries. The triangularlibration points $L_{4,5}$ are stable for mass ratio i.e$\mu<0.03852$ . The stability occur around the triangular Libration points due to the fact that potential energy of infinitesimal has a maxima rather than a minima at $L_{4,5}$ .The stability of infinitesimal around triangular equilibrium points is achieved due to the influence of the Coriolis force, because of the rotating coordinate system.

Ingeneral classical restricted three body problem, apart from gravitational attraction, other perturbing forces have been ignored. These Perturbations arisedue to lack of the sphericity, the triaxiality, oblateness, radiation forces of the bodies, variation of the masses, the atmospheric drag, the solar wind, the Poynting Robertson effect and the action of other bodies. The perturbation produced by Jupiter on the asteroids known as the Kirkwood gaps,[Kirkwood gaps is the ring of the asteroids orbits lying between the orbits of the Mars and Jupiter] are due to perturbative forces. This enables many researchers to study the restricted problem by taking into account the effect of small perturbations in the Coriolis and the centrifugal forces, radiation, oblateness and triaxiality of the bodies. Perturbations in the Coriolis and centrifugal forces means small changes caused by minor disturbances of rotating coordinate axes which tending to perturb the motion of the infinitesimal body. These effects have been discussed [Szebehely(1967a); Subbarao and Sharma (1975); Bhatnagar andhallan(1978); Sharma et al. (2001); Abdul Raheem and Singh (2006), Singh and Ishwar(2009)].

This pater has attempted to analyze the motion of an infinitesimal mass in the gravitational field of two primaries in the presence of small perturbation given in the Coriolis and centrifugal forces. The primaries are modeled as a triaxial rigid body and an oblate and radiating spheroid with one of the axes as the axis of symmetry and their equatorial plane coinciding with the plane of motion. Also, the infinitesimal body is assumed having no influence on the motion of these primaries. We have used the methods given by Ravinder Sharma and Mona Khanna.

The force of radiation

$$F = F_g - F_p = \left(1 - \frac{F_p}{F_g}\right) F_g = qF_g$$

Where $F_q$= The gravitational attraction force

$F_p$= The radiation pressure

Q = The mass reduction factor

The massive primary is supposed to be oblate spheroid as well as the source of radiation and smaller one is considered to be atriaxial rigid body.

## II. EQUATION OF MOTION

Considering the synodic co-ordinate system (x,y) with the origin at the center of mass of the primariesszebehely(1967a). The x-axis is taken along the line joining the primaries. Let $m_1$ and $m_2$ be the masses of bigger and smaller primaries respectively, and where $r_1$ and $r_2$ be the distance of the infinitesimal mass m from$m_1$ and $m_2$respectively. The unit of distancebetween the primaries is taken as unity; the unit of masses is taken so that the sum of the masses of the primaries is unity. The unit of time is so chosen as to make the gravitational constant unity following the book of Szebehely(1967a).

$$\frac{\partial \Omega}{\partial x} \text{ and } \frac{\partial \Omega}{\partial y} \quad \ldots(1)$$

$$r_2^2 = (x + 1 - \mu)^2 + y^2 \quad r_1^2 = (x - \mu)^2 + y^2$$

and $\mu = \dfrac{m_2}{m_1 + m_2}$

With $m_1 \geq m_2$ being the masses of the primaries.

Where $A_1 = \dfrac{a^2}{5R^2}$, $A_2 = \dfrac{b^2}{5R^2}$, $A_3 = \dfrac{c^2}{5R^2}$,

$A_1^1 = \dfrac{a_1^2}{5R^2}$, $A_3^1 = \dfrac{c_1^2}{5R^2}$,

Here R is the distance between the primaries.

$$\Omega = \frac{1}{2} n^2 \left[(1-\mu) r_1^2 + \mu r_2^2\right] + \frac{q(1-\mu)}{r_1} + \frac{q(1-\mu) A_1}{2r_1^3}$$

$$+ \frac{\mu}{r_2} + \frac{\mu}{2r_2^3}(2\sigma_1 - \sigma_2) - \frac{3\mu}{2r_2^5}(\sigma_1 - \sigma_2) y^2 \ldots(2)$$

Where

$\sigma_1 = A_1 - A_3,$ $\sigma_2 = A_2 - A_3,$ and $\sigma_1^1 = A_1^1 - A_3^1.$

It is assumed that the value of $\sigma_1, \sigma_2$ and$a_1$are less than unity.,

Where $n^2 = 1 + \frac{3}{2}(2\sigma_1 - \sigma_2) + \frac{3}{2} A_1$ …(3)

## III. LOCATION OF LIBRATION POINTS

Jacobi integral of equation no (1) is given by

$$C = 0$$

The equation of manifold is given by

$$F(x, y, \ldots C = 0 \quad \ldots..(4)$$

Now the libration points are the singularities of the manifold and hence are the solution of

$\Omega_x = 0, \Omega_y = 0.$ …..(5)

Since, there are five equilibrium points, three are collinear and two triangular points. So , considering two different cases we have:-

Case (a)- Triangular Libration Points: $(y \neq 0)$

$\Omega_x = 0$ , $\Omega_y = 0$

The triangular libration points are the solutions of the equations

$$n^2-\frac{1}{r_2^3}+\frac{3}{r_2^5}(\sigma_1-\sigma_2)(x-\mu)-\frac{3}{2r_2^5}(2\sigma_1-\sigma_2)$$

$$+\frac{15\mu}{2r_2^7}(\sigma_1-\sigma_2)y^2=0$$

$$-n^2(1-\mu)+\frac{q(1-\mu)}{r_1^3}+\frac{3(1-\mu)qA_1}{2r_1^5}$$

$$+\frac{3\mu}{r_2^5}(\sigma_1-\sigma_2)(x+1-\mu)=0\ldots(6)$$

Substituting the value of $r_1, r_2$ the coordinates of libration points $L_{4,5}$ are given as:-

$$x=\mu-\frac{1}{2}+\frac{\varepsilon}{3}+\frac{A_1}{2}(\varepsilon-1)+\left[\frac{-\mu}{2(1-\mu)}-\frac{3}{8}\right]\sigma_1$$

$$+\left[\frac{7}{8}+\frac{\mu}{2(1-\mu)}\right]\sigma_2$$

And

$$y=\frac{\sqrt{3}}{2}\left[1+\frac{2}{3}\left\{\frac{-\varepsilon}{2}-\frac{A_1}{2}(1+\varepsilon)+\left[\frac{\mu}{2(1-\mu)}-\frac{19}{8}\right]\sigma_1\right.\right.$$

$$\left.\left.+\left[\frac{15}{8}-\frac{\mu}{2(1-\mu)}\right]\sigma_2\right\}\right]\ldots.(7)$$

Which is confirmation with the result given by Khanna and Bhatnagar(1998), when ε=0 where (q=1-ε).

Now analyzing the case for collinear points:

Case (b) – Collinear librationpoints :

The collinear equilibrium points are the solutions of the equationsy=0 in $\Omega_y=0$ i.e

$$f(x)=n^2x-\frac{q(1-\mu)(x-\mu)}{r_1^3}-\frac{3(1-\mu)(x-\mu)qA_1}{2r_1^5}$$

$$-\frac{\mu(x+1-\mu)}{r_2^3}-\frac{3\mu}{2r_2^5}(2\sigma_1-\sigma_2)(x+1-\mu)=0$$

..(8)

Where

$$r_1=|x-\mu|\ r_2=|x+1-\mu|.$$

Since y=0 hence, then they lie on the x-axis and their abscissa are the roots of the equation (8).

Again , if $f(x)>0$ in each of the open intervals $(-\infty,\mu-1)$, $(\mu-1,\mu)$ and $(\mu,\infty)$, the function $f$ is strictly increasing in each of them.

Also $f(x)\to-\infty$ as $x\to-\infty$ , $(\mu-1)+0$ or $\mu+0$ , $f(x)\to\infty$ as $x\to\infty$ , $(\mu-1)-0$ or $\mu-0$.

Therefore, there exist one and only one value of x in each of the above interval such that $f(x)=0$ . Further, $f(\mu-2)<0$ , $f(0)\ge0$ and $f(\mu+1)>0$. Therefore, there are only three real roots of equation (12), one lying in each of the intervals $(\mu-2,\mu-1),(\mu-1,0)$ and $(\mu,\mu+1)$ . Thus there are three collinear libration points. [Ravinder Sharma et al]

## IV. STABILITY OF LIBRATION POINTS

Case (a) - Stability of Triangular Libration Points

To examine the stability of triangular equilibrium points, taking the variational equations of motion.Substituting $x=a^1+\xi\ y=b^1+\eta$

In the equation of motion (1) , where $(a^1,b^1)$ are the co-ordinates of $L_4$ (or $L_5$) and $\xi,\eta<<1$ are the small displacement in x ,y. Then by applying Taylor's theorem and retaining first order terms in the infinitesimal.

The variationalequation of motion are can be written as:-

$$\ddot{\xi} \ [illegible]\ \Omega^{\circ}_{xy}\eta,$$

And $\ddot{\eta}$ [illegible] $\Omega^{\circ}_{yy}\eta.$ …(9)

Here, the subscript in Ω indicates the second partial derivatives of Ω and superscript '0' indicates that the derivative is to be evaluated at the librationpoint $(a^1,b^1)$ . The characteristic equation corresponding to equation (13) is :-

$$\lambda^4+\left(4n^2-\Omega^{\circ}_{xx}-\Omega^{\circ}_{yy}\right)\lambda^2+\Omega^{\circ}_{xx}\Omega^{\circ}_{yy}-\left(\Omega^{\circ}_{xy}\right)^2=0$$

….(10)

Where

$$\Omega^{\circ}_{xx}=\frac{3}{4}\left[1+(\mu-1)\varepsilon+\frac{5}{2}(\mu-1)A_1\right.$$

$$+\left\{\frac{11}{2}+\frac{37}{8}\mu+\frac{2\mu}{(1-\mu)}-\frac{4\mu^2}{(1-\mu)}\right\}\sigma_1$$

$$\left.+\left\{\frac{-5}{2}+\frac{49}{8}\mu-\frac{2\mu}{(1-\mu)}+\frac{4\mu^2}{(1-\mu)}\right\}\sigma_2-\frac{47}{2}A_1\varepsilon\mu-4A_1\varepsilon\right]$$

$$\Omega^{\circ}_{xy} - \frac{\sqrt{3}}{2}\Bigg[3\mu-\frac{3}{2}+\frac{5}{2}\varepsilon+\frac{(11\mu-19)}{4}A_1$$
$$+\left\{\frac{5}{4}-\frac{407}{8}\mu-\frac{2\mu}{1-\mu}+\frac{\mu^2}{1-\mu}\right\}\sigma_1$$
$$+\left\{\frac{3}{4}+\frac{255}{4}\mu+\frac{2\mu}{1-\mu}-\frac{\mu^2}{1-\mu}\right\}\sigma_2 -3\varepsilon\mu+\left(\frac{37}{4}-\frac{33}{4}\mu\right)A_1\varepsilon\Bigg]$$

And $$\Omega^{\circ}_{yy} - \frac{\alpha}{4}+\left\{-\frac{33}{8}+\frac{123}{32}\mu+\frac{3\mu}{2(1-\mu)}-\frac{\mu^2}{(1-\mu)}\right\}\sigma_1$$
$$+\left\{\frac{33}{8}+\frac{9}{32}\mu-\frac{3\mu}{2(1-\mu)}+\frac{\mu^2}{(1-\mu)}\right\}\sigma_2$$
$$-\frac{(11+5\mu)}{4}\varepsilon+\frac{33}{8}A_1(1-\mu)+\frac{3}{4}A_1\varepsilon(5\mu-21)$$
(11)

Replacing $\lambda^2$ by $\lambda$ in equation(10), we get

$$\Lambda^2+p\Lambda+q=0 \qquad \text{....(12)}$$

Where

$$p=1+(-9+15\mu)\frac{A_1}{2}+\frac{(7+\mu)}{2}\varepsilon+\frac{(47-77\mu)}{4}A_1\varepsilon$$
$$+\left[12-\frac{73}{16}\mu-\frac{3\mu}{1-\mu}+\frac{4\mu^2}{1-\mu}\right]\sigma_1$$
$$+\left[-\frac{33}{4}+\frac{3}{4}\mu+\frac{3\mu}{1-\mu}-\frac{4\mu^2}{1-\mu}\right]\sigma_2>0$$

$$q=\frac{27}{4}\mu-\frac{27}{4}\mu^2+\frac{3(29-11\mu)}{8}\varepsilon$$
$$-\frac{9(3+\mu)}{8}A_1+\frac{9(4-7\mu)}{4}A_1\varepsilon$$
$$+\Bigg[\frac{159}{16}-\frac{3457}{32}\mu+\frac{2187}{16}\mu^2-\frac{3\mu}{4(1-\mu)}$$
$$+\frac{15\mu^2}{2(1-\mu)}-\frac{9\mu^3}{2(1-\mu)}\Bigg]\sigma_1$$
$$+\Bigg[-\frac{27}{8}+\frac{109}{16}\mu-\frac{765}{2}\mu^2+\frac{3\mu}{4(1-\mu)}$$
$$-\frac{15\mu^2}{2(1-\mu)}+\frac{9\mu^3}{2(1-\mu)}\Bigg]\sigma_2$$
...(13)

And $$\Lambda_{1,2}=\frac{1}{2}\left[-p\pm\sqrt{p^2-4q}\right] \qquad \text{...(14)}$$

Where, the roots $\lambda_1=+\Lambda_1^{1/2}, \lambda_2=-\Lambda_1^{1/2}$, $\lambda_3=+\Lambda_2^{1/2}$ and $\lambda_4=-\Lambda_2^{1/2}$ depend, on the value of the mass parameter, $q,\sigma_1,\sigma_2,A_1$

Now the discriminant of equation (16) is zero if

$$p^2-4q=0. \qquad \text{...(15)}$$

I.e
$$1-27\mu(1-\mu)-\frac{(73-35\mu)}{2}\varepsilon$$
$$-\frac{(18+33\mu)}{2}A_1-\frac{(25-49\mu)}{2}A_1\varepsilon$$
$$+\left[-\frac{63}{4}+423\mu-\frac{2187}{4}\mu^2-\frac{3\mu}{1-\mu}-\frac{12\mu^2}{(1-\mu)}+\frac{18\mu^3}{(1-\mu)}\right]\sigma_1$$
$$+\Bigg[-3-\frac{103}{4}\mu+1530\mu^2+\frac{6\mu}{1-\mu} \qquad \text{...(16)}$$
$$+\frac{12\mu^2}{(1-\mu)}-\frac{18\mu^3}{(1-\mu)}\Bigg]\sigma_2=0$$

If $\sigma_1,\sigma_2$ and $A_1$ are equal to zero then $\mu=\mu_0$ is a root of equation (16), where $\mu_0=0.03852...$(Szebehely)[8].

But, When $\sigma_1\neq 0,\sigma_2\neq 0,\ A_1\neq 0$ we suppose $\mu_{crit}=\mu_0+\mu^1$ as the root of the equation (16), where the value of $\mu^1$ is to be determined such that condition (20) is satisfied.

Where

$$\mu^1=\frac{p_1\sigma_1+p_2\sigma_2+p_3A_1}{2(14-27\mu_0)} \qquad \text{...(17)}$$

Where

$$p_1=\Bigg[-\frac{63}{4}+423\mu-\frac{2187}{4}\mu^2-\frac{3\mu}{1-\mu}$$
$$-\frac{12\mu^2}{(1-\mu)}+\frac{18\mu^3}{(1-\mu)}\Bigg]=-0.740226$$

$$p_2=\Bigg[-3-\frac{103}{4}\mu+1530\mu^2+\frac{6\mu}{1-\mu}$$
$$+\frac{12\mu^2}{(1-\mu)}-\frac{18\mu^3}{(1-\mu)}\Bigg]=-1.503$$

And $$p_3=-\frac{(27+33\mu)}{2}A_1=-9.63558$$

Now, taking three regions for the value of $\mu$ separately:-

$$0\le\mu<\mu_0+\mu^1=\mu_{crit}$$

We have

$$-\frac{p}{2}<\Lambda_1<-\frac{p}{2}+\frac{1}{2}+\frac{3}{2}\sigma_1-\frac{3}{4}\sigma_2-\frac{3}{4}A_1\le 0$$

And $$-\frac{p}{2}>\Lambda_2>-\frac{p}{2}-\frac{1}{2}-\frac{3}{2}\sigma_1+\frac{3}{2}\sigma_2+\frac{3}{4}A_1$$

Since, p>0 ∴ we have the negative values of $\Lambda_1$ and $\Lambda_2$ . Then, in that case, the four roots of the characteristics equation are written as :

$$\lambda_{1,2} = \pm i\left(-\Lambda_1\right)^{1/2} = \pm i s_1 ,$$

And $\lambda_{3,4} = \pm i\left(-\Lambda_2\right)^{1/2} = \pm i s_2$ . ...(18)

Hence, confirming the stability of libration points.

Now from equation(1), the value of Ω around equilibrium points, applying Taylos's series, can be written as:-

$$\Omega = \Omega^\circ + \Omega^\circ_{xx}\frac{\xi^2}{2!} + \Omega^\circ_{xy}\xi\eta + \Omega^\circ_{yy}\frac{\eta^2}{2!} + O(\xi^3,\eta^3),$$

...(19)

$$\text{Or}\ \Omega = \frac{3}{2} + \left[\frac{3}{2} - \frac{5}{8}\mu + \frac{\mu}{2(1-\mu)} - \frac{\mu^2}{2(1-\mu)}\right]\sigma_1$$

$$+\left[-\frac{3}{4} + \frac{9}{8}\mu - \frac{\mu}{2(1-\mu)} + \frac{\mu^2}{2(1-\mu)}\right]\sigma_2$$

$$+\left[\frac{5}{4} - \frac{\mu}{2}\right]A_1 + \frac{3}{8}\xi^2 - \frac{3}{8}\xi^2(1-\mu)\varepsilon$$

$$+\frac{15}{16}\xi^2(1-\mu)A_1 + \frac{141}{16}\xi^2 A_1\varepsilon\mu - \frac{3}{2}\xi^2 A_1\varepsilon$$

$$+\frac{3}{8}\left[\frac{11}{2} + \frac{37}{8}\mu + \frac{2\mu}{(1-\mu)} - \frac{4\mu^2}{(1-\mu)}\right]\sigma_1\xi^2$$

$$+\frac{3}{8}\left[-\frac{5}{2} + \frac{49}{8}\mu - \frac{2\mu}{(1-\mu)} + \frac{4\mu^2}{(1-\mu)}\right]\sigma_2\xi^2$$

$$+\frac{3\sqrt{3}}{2}\xi\eta\left[3\mu - \frac{3}{2} + \left(\frac{5}{2} - 2\mu\right)\varepsilon\right.$$

$$\left. + \left(\frac{37}{4} - \frac{33}{4}\mu\right)A_1\varepsilon + \frac{1}{4}(11\mu - 19)A_1\right]$$

$$+\frac{3\sqrt{3}}{2}\xi\eta\left[\frac{5}{4} - \frac{407}{8}\mu - \frac{2\mu}{(1-\mu)} + \frac{\mu^2}{(1-\mu)}\right]\sigma_1$$

$$+\frac{3\sqrt{3}}{2}\xi\eta\left[\frac{3}{4} + \frac{255}{4}\mu + \frac{2\mu}{(1-\mu)} - \frac{\mu^2}{(1-\mu)}\right]\sigma_2$$

$$+\frac{9}{8}\eta^2 - \frac{9(11+5\mu)}{8}\varepsilon\eta^2 + \frac{33}{16}(1-\mu)A_1\eta^2$$

$$+\frac{3}{8}(5\mu - 21)A_1\varepsilon\eta^2$$

$$+\left[-\frac{33}{16} + \frac{123}{64}\mu + \frac{3\mu}{4(1-\mu)} - \frac{\mu^2}{2(1-\mu)}\right]\sigma_1\eta^2$$

$$+\left[\frac{33}{16} + \frac{9}{64}\mu - \frac{3\mu}{4(1-\mu)} + \frac{\mu^2}{2(1-\mu)}\right]\sigma_2\eta^2$$

Now, introducinga rotating coordinate system so, the variables $\bar{\xi}$ and $\bar{\eta}$ bythe transformation

$$\xi = \bar{\xi}\cos\alpha - \bar{\eta}\sin\alpha,$$

And $\eta = \bar{\xi}\sin\alpha + \bar{\eta}\cos\alpha,$

This will rotate the system by an angle α. The value of α is so chosen, such that term containing $\bar{\xi}$ and $\bar{\eta}$ is equal to zero in the expression of Ω.

Again, The value of equation (19) can be written as :-

$$\Omega = \bar{l}\,\bar{\xi}^2 + \bar{m}\bar{\eta}^2 + \bar{n}$$

$$\text{Where}\ \bar{l} = \frac{3}{8} + \frac{3}{4}\sin^2\alpha - \frac{3\sqrt{3}}{4}(1-2\mu)\cos\alpha\sin\alpha$$

$$+\left[\left\{-\frac{33}{16} + \frac{111}{64}\mu + \frac{33}{8}\cos^2\alpha + \frac{3}{16}\mu\sin^2\alpha\right.\right.$$

$$\left. + \frac{3}{4}\left(\frac{\mu}{1-\mu}\right) - \frac{3}{2}\left(\frac{\mu^2}{1-\mu}\right) + \left(\frac{\mu^2}{1-\mu}\right)\sin^2\alpha\right\}$$

$$\left. + \frac{3\sqrt{3}}{2}\left\{\frac{15}{12} - \frac{407}{24}\mu - \frac{2}{3}\left(\frac{\mu}{1-\mu}\right) + \frac{1}{3}\left(\frac{\mu^2}{1-\mu}\right)\right\}\cos\alpha\sin\alpha\right]\sigma_1$$

$$\left[\left\{-\frac{15}{16} + \frac{147}{16}\mu + 3\sin^2\alpha - \frac{579}{64}\mu\sin^2\alpha\right.\right.$$

$$\left. - \frac{3}{4}\left(\frac{\mu}{1-\mu}\right) + \frac{3}{2}\left(\frac{\mu^2}{1-\mu}\right) - \left(\frac{\mu^2}{1-\mu}\right)\sin^2\alpha\right\}$$

$$\left. \frac{3\sqrt{3}}{2}\left\{\frac{1}{4} + + \frac{255}{12}\mu + \frac{2}{3}\left(\frac{\mu}{1-\mu}\right) - \frac{1}{3}\left(\frac{\mu^2}{1-\mu}\right)\right\}\cos\alpha\sin\alpha\right]\sigma_2$$

$$+\left[\frac{15}{16}(1-\mu) + \frac{9}{8}(1-\mu)\sin^2\alpha\right.$$

$$\left. + \frac{3\sqrt{3}}{8}\left(\frac{11}{3}\mu - \frac{19}{3}\right)\cos\alpha\sin\alpha\right]A_1$$

$$+\left[\frac{3}{8}(1-\mu)\cos^2\alpha - \frac{1}{8}(1+5\mu)\sin^2\alpha\right.$$

$$\left. + \frac{3\sqrt{3}}{2}\left(\frac{5}{6} - \frac{2\mu}{3}\right)\cos\alpha\sin\alpha\right]\varepsilon$$

$$\left. \frac{3\sqrt{3}}{2}\left\{\frac{1}{4} + + \frac{255}{12}\mu + \frac{2}{3}\left(\frac{\mu}{1-\mu}\right) - \frac{1}{3}\left(\frac{\mu^2}{1-\mu}\right)\right\}\cos\alpha\sin\alpha\right]\sigma_2$$

$$+\left[\frac{15}{16}(1-\mu)+\frac{9}{8}(1-\mu)\sin^2\alpha\right.$$

$$\left.+\frac{3\sqrt{3}}{8}\left(\frac{11}{3}\mu-\frac{19}{3}\right)\cos\alpha\sin\alpha\right]A_1$$

$$+\left[\frac{3}{8}(1-\mu)\cos^2\alpha-\frac{1}{8}(1+5\mu)\sin^2\alpha\right.$$

$$\left.+\frac{3\sqrt{3}}{2}\left(\frac{5}{6}-\frac{2\mu}{3}\right)\cos\alpha\sin\alpha\right]\varepsilon$$

$$\bar{m}=\frac{3}{8}+\frac{3}{4}\cos^2\alpha+\frac{3\sqrt{3}}{4}(1-2\mu)\cos\alpha\sin\alpha$$

$$+\left[\left\{-\frac{33}{16}+\frac{111}{64}\mu+\frac{33}{8}\sin^2\alpha+\frac{3}{16}\mu\cos^2\alpha\right.\right.$$

$$\left.+\frac{3}{4}\left(\frac{\mu}{1-\mu}\right)-\frac{3}{2}\left(\frac{\mu^2}{1-\mu}\right)+\left(\frac{\mu^2}{1-\mu}\right)\cos^2\alpha\right\}$$

$$\left.-\frac{3\sqrt{3}}{2}\left\{\frac{5}{12}-\frac{407}{24}\mu-\frac{2}{3}\left(\frac{\mu}{1-\mu}\right)+\frac{1}{3}\left(\frac{\mu^2}{1-\mu}\right)\right\}\cos\alpha\sin\alpha\right]\sigma_1$$

$$\left[\left\{-\frac{15}{16}+\frac{147}{16}\mu+\frac{9}{8}\cos^2\alpha-\frac{69}{32}\mu\cos^2\alpha\right.\right.$$

$$\left.-\frac{3}{4}\left(\frac{\mu}{1-\mu}\right)+\frac{3}{2}\left(\frac{\mu^2}{1-\mu}\right)-\left(\frac{\mu^2}{1-\mu}\right)\cos^2\alpha\right\}$$

$$\left.+\frac{3\sqrt{3}}{2}\left\{\frac{1}{4}++\frac{255}{12}\mu+\frac{2}{3}\left(\frac{\mu}{1-\mu}\right)-\frac{1}{3}\left(\frac{\mu^2}{1-\mu}\right)\right\}\cos\alpha\sin\alpha\right]\sigma_2$$

$$+\left[\frac{15}{16}(1-\mu)+\frac{9}{8}(1-\mu)\cos^2\alpha\right.$$

$$\left.+\frac{3\sqrt{3}}{8}\left(\frac{11}{3}\mu-\frac{19}{3}\right)\cos\alpha\sin\alpha\right]A_1$$

$$+\left[\frac{3}{8}(1-\mu)\sin^2\alpha-\frac{1}{8}(1+5\mu)\cos^2\alpha\right.$$

$$\left.-\frac{3\sqrt{3}}{2}\left(\frac{5}{6}-\frac{2\mu}{3}\right)\cos\alpha\sin\alpha\right]\varepsilon$$

$$\bar{n}=\frac{3}{2}+\left[\frac{3}{2}-\frac{5}{8}\mu+\frac{\mu}{2(1-\mu)}-\frac{\mu^2}{2(1-\mu)}\right]\sigma_1+$$

$$+\left[-\frac{3}{4}+\frac{9}{8}\mu-\frac{\mu}{2(1-\mu)}+\frac{\mu^2}{2(1-\mu)}\right]\sigma_2+\left(\frac{5}{4}-\frac{\mu}{2}\right)A_1 \quad ..(20)$$

$$\tan 2\alpha=\frac{N}{D}$$

And where

$$N=\frac{3\sqrt{3}}{2}\left[\frac{(1-2\mu)}{2}-\left\{\frac{15}{12}-\frac{407}{24}\mu-\frac{2\mu}{3(1-\mu)}+\frac{\mu^2}{3(1-\mu)}\right\}\sigma_1\right.$$

$$-\left\{\frac{1}{4}+\frac{255}{12}\mu+\frac{2\mu}{3(1-\mu)}-\frac{\mu^2}{3(1-\mu)}\right\}\sigma_2$$

$$\left.-\left(\frac{11}{3}\mu-\frac{19}{3}\right)A_1-\left(\frac{5}{6}-\frac{2}{3}\mu\right)\varepsilon\right]$$

$$D=\frac{3}{4}+\left[-\frac{33}{8}+\frac{3}{16}\mu-\frac{\mu^2}{2(1-\mu)}\right]\sigma_1$$

$$+\left[\frac{9}{8}-\frac{69}{32}\mu-\frac{\mu^2}{(1-\mu)}\right]\sigma_2+\frac{9}{8}(1-\mu)A_1-\frac{1}{8}(1+5\mu)\varepsilon \quad (21)$$

Now, from equation(7), thejacobian constant can be written as:-

$$C=2\Omega=2\bar{l}\,\bar{\xi}^2+2\bar{m}\bar{\eta}^2+2\bar{n}. \quad \ldots(22)$$

Hence, it follows that the above curve is an ellipse and the direction of its major axis α, which is given by equation (21). The lengths of the semi major and semi minor axes are given by equation (21) and (22) respectivey:-

$$a_1=\frac{C-2\bar{n}}{2\bar{l}} \qquad b_1=\frac{C-2\bar{n}}{2\bar{m}},$$

Now, taking the second region for value of μ as:-

$\mu_c<\mu<\frac{1}{2}$: When $\mu_c<\mu<\frac{1}{2}$, the discriminant of the characteristic equation is negative.

Also, $\Lambda_{1.2}=\frac{1}{2}\left[-p\pm\sqrt{d}\right]$

Where, p is given by equation (17) and $d=p^2-4q$.

Therefore,

$$\Lambda_{1.2}=\frac{1}{2}\left[-p\pm i\delta\right],$$

Where $0<\delta=+d^{1/2}$

The characteristic equation (16) has the following roots:-

$$\lambda_{1,2}=\pm\Lambda_1^{1/2}, \qquad \lambda_{3,4}=\pm\Lambda_2^{1/2}$$

$$\lambda_1=\frac{1}{\sqrt{2}}(-p+i\delta)^{1/2}=\alpha_1+i\beta_1,$$

Or

$$\lambda_2=\frac{-1}{\sqrt{2}}(-p+i\delta)^{1/2}=\alpha_2+i\beta_2,$$

$$\lambda_3=\frac{1}{\sqrt{2}}(-p-i\delta)^{1/2}=\alpha_3+i\beta_3,$$

$$\lambda_1=\frac{-1}{\sqrt{2}}(-p-i\delta)^{1/2}=\alpha_4+i\beta_4.$$

And

The lengths of these roots are equal and are given by

$$|\lambda| = |\lambda_{1,2,3,4}| = \frac{1}{\sqrt{2}}\left(p^2+\delta^2\right)^{1/2},$$

The principal argument of the first root is

$$\theta = \theta_1 = \arctan\left[\frac{p \pm \sqrt{p^2+\delta^{\to 2}}}{\delta}\right].$$

The arguments of the four roots are related by

$$\theta = \theta_1 = \theta_2 - \pi = 2\pi - \theta_3 = \pi - \theta_4.$$

The real and imaginary parts of the roots, $\alpha_1$ and $\beta_1$,are related by

$$\alpha = \alpha_1 = -\alpha_2 = -\alpha_3 = -\alpha_4$$
$$and\, \beta = \beta_1 = -\beta_2 = -\beta_3 = \beta_4,$$

Where,

$$\alpha = \frac{\delta}{2\left[2|\lambda|^2+p\right]^{1/2}} \quad \beta = \frac{\left[p+2|\lambda|^2\right]^{1/2}}{2}$$

And $\alpha>0$ if $\beta>0$.

Also, here the real parts of two of the characteristic roots are positive (and equal) and so the equilibrium point in this case is unstable.

Now, considering the region for $\mu$ as:-

(iii) $\mu = \mu_c$ ;When $\mu = \mu_c$, d=0

Consequently, $\Lambda_{1,2} = -\frac{p}{2}$

And $\lambda_1 = \lambda_3 = i\left[\frac{p}{2}\right]^{1/2}, \lambda_2 = \lambda_4 = -i\left[\frac{p}{2}\right]^{1/2}.$

Indicating that the double roots have secular terms in the solution of the equation of motion and so the equilibrium points is unstable in this case.

Case (b) – Stability of Collinear Libration Points

Considering the point lying in ($\mu$-2,$\mu$-1). For thesepoints ,since $r_2<1$, we have:-

$$\Omega_{xy}^{\circ} = 0\ \Omega_{xx}^{o} > 0\ \Omega_{yy}^{o} < 0$$

Similarly, for the points lying in($\mu$-1,0) and ($\mu$,$\mu$+1), $\Omega_{xy}^{\circ} = 0, \Omega_{xx}^{\circ} > 0$ and $\Omega_{yy}^{\circ} < 0$.

From the above equation it is clear that, $\Omega_{xx}^{\circ}\Omega_{yy}^{\circ} - (\Omega_{xy}^{\circ})^2 < 0$, the discriminant is positive and the four roots of the characteristic equation can be written as $\lambda_1$=s, $\lambda_1$=-s, $\lambda_1$=it, and $\lambda_1$=-it,where s and t are real, which indicates the unbounded motion around the collinear equilibrium points. As a consequence the collinear equilibrium points are stable.

## V. CONCLUSION

The stationary solution has been found the restricted three body problem when bigger primary as an oblate spheroid and source of radiation and smaller primary a triaxial rigid body with one of the axes as axis of symmetry. The equatorial plane of both primaries coincides with the plane of motion. The existence of five libration points have been found, two triangular and three collinear libration points respectively. The stability been analysed for triangular points in three different regions as follows:- (i)When $0 < \mu < \mu_{crit}$ , the triangular equilibrium points are stable.

(ii)When $\mu = \mu_{crit}$ , since the solution contain secular terms, indicating the unstability of triangular points.

(iii)When $\mu_{crit} \le \mu \le \frac{1}{2}$ , all four characteristic roots are imaginary and two of them have positive real part leading to instability of triangular points.

Also for collinear points, out of fourroots two are real and two imaginary making the motion around it unbounded and hence, leading to instability.

It is possible to find periodic orbits around the triangular equilibrium points. However, these points are unstable when the solution contain secular terms.

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