Some Classes of Operators of Order ‘N’ on Hilbert Space and Complex Hilbert Space
K. M. Manikandan¹, P. Suganya²

¹Assistant Professor, Department of Mathematics, Bharathiar University/Dr.S.N.S. Rajalakshmi college of Arts & Science, Coimbatore, Tamilnadu, India
²Research Scholar Department of Mathematics, Bharathiar University/Dr.S.N.S. Rajalakshmi college of Arts & Science, Coimbatore, Tamilnadu, India

ABSTRACT

In this paper we introduce n-power hypo-normal operator of order-n, n-power quasi-normal operator of order-n, quasi parahyponormal operator of order-n on a Hilbert space H. we give some properties of these operators.

Keywords: n-power hypo-normal, n-power quasi-normal, parahyponormal, quasi parahyponormal operator.

I. INTRODUCTION

SECTION 1.0

Let B(H) denotes the algebra of all bounded linear operators acting on a complex Hilbert space H. An operator T ∈ B(H) is said to be self adjoint if T* = T, isometry if T*T = I. The operator T ∈ B(H) is called normal if T T* = T* T, Quasinormal if, T(T* T) = (T* T)T. An operator T on H is called hyponormal if T T* ≤ T* T.

SECTION 2.0

In this section we introduce n-power quasi-normal operator of order-n.

Definition: 2.1 n-power Quasi-normal operator of order-n
An operator T is called n-power Quasi-normal operator of order-n if,

T^n(T^n T) = (T^n T)T^n

Theorem: 2.2
Let T₁, T₂, ..., T_k be n-power quasi-normal operators of order-n in B(H). Then the direct sum (T₁ ⊕ T₂ ⊕ ... ⊕ T_k) and tensor product (T_1 ⊗ T_2 ⊗ ... ⊗ T_k) are n-power quasi-normal operators of order-n.

Proof:
From the definition of n-power quasi-normal operators of order-n, we have

T^n(T^n T) = (T^n T)T^n

(T₁ ⊕ T₂ ⊕ ... ⊕ T_k)^n[(T₁ ⊕ T₂ ⊕ ... ⊕ T_k)ⁿ(T₁ ⊕ T₂ ⊕ ... ⊕ T_k)] = (T₁ⁿ ⊕ T₂ⁿ ⊕ ... ⊕ T_kⁿ)[(T₁ⁿ ⊕ T₂ⁿ ⊕ ... ⊕ T_kⁿ)]

= (T₁ⁿ ⊕ T₂ⁿ ⊕ ... ⊕ T_kⁿ)(T₁ ⊕ T₂ ⊕ ... ⊕ T_k)

Since, T₁, T₂, ..., T_k be n-power quasi-normal operator of order-n, then

= T₁ⁿ(T₁ⁿ T₁) ⊕ T₂ⁿ(T₂ⁿ T₂) ⊕ ... ⊕ T_kⁿ(T_kⁿ T_k)

Also

(T₁ ⊗ T₂ ⊗ ... ⊗ T_k)^n[(T₁ ⊗ T₂ ⊗ ... ⊗ T_k)^n(T₁ ⊗ T₂ ⊗ ... ⊗ T_k)]

= (T₁ⁿ ⊗ T₂ⁿ ⊗ ... ⊗ T_kⁿ)(T₁ ⊗ T₂ ⊗ ... ⊗ T_k)

= (T₁⊗ T₂ ⊗ ... ⊗ T_k)^n

Since, T₁, T₂, ..., T_k be n-power quasi-normal operator of order-n, then

= T₁ⁿ(T₁ⁿ T₁) ⊗ T₂ⁿ(T₂ⁿ T₂) ⊗ ... ⊗ T_kⁿ(T_kⁿ T_k)

Also

(T₁⊗ T₂ ⊗ ... ⊗ T_k)^n

Proof:
From the definition of n-power quasi-normal operators of order-n, we have

T^n(T^n T) = (T^n T)T^n
Section 3.0:
In this section we introduce n-power Hypo-normal operator of order-n.

Definition: 3.1 n-power Hypo-normal operator of order-n
An operator T is called n-power Hypo-normal operator of order n if,
\[ T^nT^n \geq T^nT^n \]

Theorem 3.2:
If S&T are doubly commuting n-power hypo-normal operators of order -n and S^t=T^tS,then ST is an n-power hypo-normal operator of order-n.

Proof:
Since ST=TS
\[ S^nT^n = (ST)^n \text{ and } S^nT^n = T^nS^n \]
\[ \text{[\therefore ST]} \]
\[ S^nT^n = T^nS^n \]
\[ \text{[\therefore ST]} \]
\[ S^nT^n = T^nS^n \]

Hence, ST is parahyponormal operator of order-n.

Section 4.0:
In this section we introduce parahyponormal operator of order-n.

Definition: 4.1 Parahyponormal operator of order-n
An operator T \in B(H) is said to be parahyponormal operator of order-n, if
\[ \|Tx\|^2 \leq \|TT^n x\| \]

Theorem 4.2:
If S,T \in B(H) are doubly commuting parahyponormal operators of order-n and S^nT^n = T^nS^n then ST is para-hyponormal operator of order-n.

Proof:
\[ S^nT^n = (ST)^n \quad \text{[\therefore ST]} \]
\[ S^nT^n = T^nS^n \quad \text{[\therefore ST]} \]

Hence, ST is parahyponormal operator of order-n.

Section 5.0:
In this section we introduce quasi parahyponormal operator of order-n.

Definition: 4.1 Quasi-Parahyponormal operator of order-n
An operator T \in B(H) is said to be quasi para-hyponormal operator of order-n, if
\[ \|TT^n x\|^2 \leq \|T^2T^n x\| \]

Theorem 5.2:
Let T \in B(H) be a quasi parahyponormal operator of order-n, if T commutes with isometric operator S then TS is quasi parahyponormal operator of order-n.

Proof:
Let A=TS for all real number \lambda.
(A^2A^{*2n})^2 + 2\lambda(AA'^{*n})^2 + \lambda^2 \geq 0.
[(TS)^2(TS')^{*2n}]^2 + 2\lambda[(TS)(TS')^{*n}]^2 + \lambda^2 \geq 0.

\[ (T^2S^2)(S'^{2n}T'^{*2n})^2 + 2\lambda[(TS)(S'^{n}T')^{*n}]^2 + \lambda^2 \geq 0. \]
\[
\begin{bmatrix}
TS = ST
S'S = I
\end{bmatrix}
\]

Therefore, A is quasi parahyponormal operator of order-n.

**Theorem 5.3:**

If a quasi parahyponormal operator of order-n T commutes with an isometric operator S then \( T^S \) is quasi parahyponormal operator of order-n.

**Proof:**

Let \( A=T^S \)

We have for any real number \( \lambda \).

\[(A^2A'^{*2n})^2 + 2\lambda(AA'^{*n})^2 + \lambda^2 \geq 0.
\]
\[
[(T^2S^2)(T'^{*2n})^2 + 2\lambda[(T^2S'^{*n})(T'^{*n}S'^*n)]^2 + \lambda^2 \geq 0.
\]
\[
[S^2T^2(S'^{*n}T'^*n)]^2 + 2\lambda[(S^2T^2)(S'^{*n}T'^*n)]^2 + \lambda^2I \geq 0.
\]
\[
\begin{bmatrix}
S^2T^2 = S^2T^2
S'S'^*n = I, S'S'^*n = I
\end{bmatrix}
\]

Therefore, A is quasi parahyponormal of order-n.

**II. REFERENCES**

[10]. On parahyponormal and quasi parahyponormal operators, Sivakumar N, Dhiyva G (2016)