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Cocentroidal Matrices in JS Metric Space

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ABSTRACT

In this note we attempt unfold gates of a classical unit – Cocentroidal Matrices. Its fundamental structure is in R^n but in this note we deal in R^3. If not linearly dependent then a unique plane contains all the three column vector points corresponding to a matrix. We, after defining the notion of centroid of a matrix and distance between two matrices, search for an infinite set of matrices so that all of them (1) lie on the same plane in R^3 (2) correspond to unique **centroid**. [* The basic matrix is called a root matrix.]
The members of the set exhibit many characteristics that parallel to some of those of Euclidean geometry and to also of topological space.

Keywords: Co-centroidal Matrices, JS Metric Space, Angle between Matrices, Centro-normal Matrices and Centro - linear Matrices.

I. INTRODUCTION

We begin our discussion with fundamental notion of centroid; an important concept which can be discussed using primary Euclidean geometry. Then we define the space, JS Metric Space, with axioms and some salient characteristics. Then we define Co-centroidal matrices and its properties.

II. CENTROID OF A MATRIX

We consider an n×n matrix,

$A = [A_1 \quad A_2 \quad \dots \quad \dots \quad A_n]_{1\times n}$

Where each $A_i = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ \vdots \\ a_{ni} \end{bmatrix}_{n\times i}$ for a fixed i=1 to n.

These entries correspond to i^th^ variable addressed in i^th^ column that associates all numeric values corresponding to each row/variable of the second type. As a result we have $\bar{A}_i \in R^n$ continuing on the same line, we have $[A_1 \quad A_2 \quad \dots \quad \dots \quad A_n]$ - as a set of n distinct (say) vectors in R^n.

The vector

$\overline{OG} = \left(\frac{\sum a_{1i}}{n}, \frac{\sum a_{2i}}{n}, \dots \dots \frac{\sum a_{ni}}{n}\right)$ for all i = 1 to n is a virtual centroid (G) of the virtual plane containing the points

$A_1, A_2, \dots\dots A_n$ of R^n[11].

We consider a case in R^3.

Let $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$ which corresponds to three

Vectors $\overrightarrow{OP} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$, $\overrightarrow{OQ} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$,

and $\overrightarrow{OR} = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix}$ in R^3.

We assume that these vectors are linearly independent; i.e. det. A ≠ 0

As defined, Centroid of the system given by A is G $= \left(\frac{\sum a_j}{3}, \frac{\sum b_j}{3}, \frac{\sum c_j}{3}\right)$ for j = 1, 2, 3. [13]

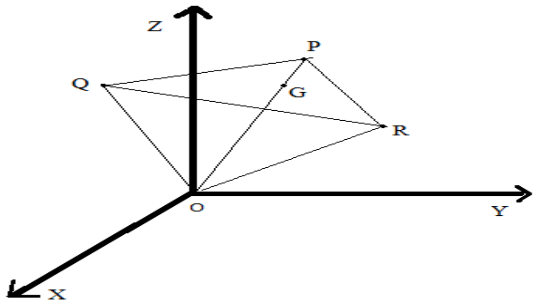

Figure 1: Spotting the Centroid.

III. EUCLIDEAN STRUCTURE

Each matrix A_i(i = 1 to n) corresponds a vector $\overline{OG_i}$ defined in the above format. Let us denote $\overline{OG_i}$ = u_i for each i = 1 to n.

Assuming that each $u_i \in R^n$, define the Euclidean norm as

$\|u_i\|$ = < u_i , u_i > and $\|u_i\| \geq 0$ (1)

Members of this structure with regular binary operations and an inner product < > defined over them fulfils conditions which are required for inner product space.

IV. COCENTROIDAL MATRICES IN JS METRIC SPACE

We define the metric between two matrices A_i and A_j.

Where A_i and A_j are defined in the above section 2.

d : $M_n \times M_n \to R^+ \cup\{0\}$ such that for

A_i , $A_j \in M_n$

$$d(A_i , A_j) = d(G_i, G_j) = \left(\sum(\boldsymbol{g_i} - \boldsymbol{g_i^*})^2\right)^{\frac{1}{2}} \quad \textbf{(2)}$$

It states that the distance between two matrices A_i and A_j of M_n is the distance between their centroids $G_i$and G_j[11].

With following axioms;

(1) d(A_i , A_j)≥ 0
(2) d(A_i , A_j) = d(A_j , A_i)
(3) d(A_i , A_j) = 0 ⇒ A_i = A_j **
(4) For the matrices A_i , A_j, and A_k ; all distinct and having distinct centroids G_i, G_j and G_k.

$$d(A_i , A_j) + d(A_j , A_k) \geq d(A_i , A_k)$$

For a given matrix A_i for some i ∈ N, there is an infinite set of matrices A_j ∀ j ∈ N such that

$A_i \neq A_j \Rightarrow d(A_i , A_j) = 0$

For example, Let, A1= $\begin{bmatrix} 11 & 2 & -4 \\ 12 & 1 & -5 \\ 14 & -3 & -13 \end{bmatrix}$, and A2 = $\begin{bmatrix} 8 & 2 & -1 \\ 9 & 1 & -2 \\ 9 & -3 & -8 \end{bmatrix}$, both are different matrices with the same centroid $\left(3,\ \frac{8}{3},\ \frac{-2}{3}\right)$.

Let, distance between them

$$d(A_1, A_2)=d(G_1,G_2)=(\sum(g_1 - g_2^*)^2)^{\frac{1}{2}}= \sqrt{(3-3)^2 + (\tfrac{8}{3} - \tfrac{8}{3})^2 + (\tfrac{-2}{3} + \tfrac{2}{3})^2} = 0$$

[It means that if the matrices are not same yet the distance between them may be zero.]

We call all such matrices lying on the same plane (in which the vector points of $\boldsymbol{A_i}$ are lying) as **Cocentroidal Matrices.**

All the above properties together on a set of matrices in $M_{n\times n}$with the distance function defined as

d(A_i , A_j) = d(G_i, G_j) = $\left(\sum(\boldsymbol{g_i} - \boldsymbol{g_i^*})^2\right)^{\frac{1}{2}}$ generates a space – We call it **'JS' Metric space**[11].

V. ANGLE BETWEEN TWO MATRICES

The next important concept which is going to play an important role for further work is about 'angle between two matrices'.

We define it as follows.

Let A_i , and $A_j$be the two matrices with their centroids $G_1$and G_2 defined as follows.

Let $G_1=\left(\frac{\sum a_{1i}}{n}, \quad \frac{\sum a_{2i}}{n}, \quad \ldots \quad \ldots \quad \frac{\sum a_{ni}}{n}\right)$

$= (g_1, g_2, \ldots \ldots .., g_n) = U_1$

$G_2=\left(\frac{\sum b_{1j}}{n}, \quad \frac{\sum b_{2j}}{n}, \quad \ldots \quad \ldots \quad \frac{\sum b_{nj}}{n}\right)$

$= (g_1^*, g_2^*, \ldots \ldots, g_n^*) = V_1$

The angle between these matrices is defined as the angle between two centroid vectors U_1 and V_1.

It is denoted as

$(A \wedge B) = \beta$, with $\cos \beta = \frac{<U_1, V_1>}{||U_1|| \, ||V_1||}$ (3)

with $0 \le \beta \le \pi$

On the basis of this, we have some important derivations as follows.

A. Centro-normal Matrices

If two matrices one from each infinite set of Co-centroidal matrices are such that

$U_1 \perp V_1$ then $\beta = \pi/2$.

Such matrices are said to be Centro-normal matrices[12].

The matrices A and B given below are the members of the different sets of

co-centroidal matrices.

$A = \begin{pmatrix} -9 & 2 & -2 \\ 5 & 4 & -3 \\ 4 & 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 & 1 \\ -8 & -1 & 0 \\ 4 & 4 & 1 \end{pmatrix}$ with U_1 = (-3, 2, 4) and V_1 = (2, -3, 3)

and $\langle U_1, V_1 \rangle = 0$

It is important to note the two facts

(1) That these matrices are in the different planes and these planes are not perpendicular to each other.

(2) We have $\|U_1\|^2 + \|V_1\|^2 = \|G_1G_2\|^2$; this verification sounds the notion of normality of matrices A and B.

B. Centro-Linear Matrices

In continuation of the above notion, if we have a case,

When $U_1 = KV_1$ with $K \neq 0$, then the sets of co-centroidal matrices from which these two member matrices belong are on the same line joining their centroids G_1 and G_2[12].

The matrices $A = \begin{pmatrix} -9 & 2 & -2 \\ 5 & 4 & -3 \\ 4 & 3 & 5 \end{pmatrix}$ and

$B = \begin{pmatrix} 9 & 2 & -2 \\ -5 & 4 & -3 \\ 4 & -3 & -5 \end{pmatrix}$ with U_1 = (-3, 2, 4) and

V_1 = (3, -2,-4) in this case we have k = -1 and so the vectors U_1 and V_1 are linearly dependent.

VI. CONCLUSION

This note is an indication to that what we planned in the near future. Its principal area of co-centroidal matrices and related concepts play very important role on the member matrices of JS Metric Space.

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