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Selection of Reinforcement Steel Bars with Markovian Weldability Distribution against their Boron Content

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ABSTRACT

Deciding the functional suitability of reinforcement steel bars for welding operations has a dependence on boron content especially if such bars are made from recycled steel. In this study, a mathematical model is developed to optimize the selection decision of recycled steel bars from steel manufacturers considering a Markovian weldability distribution since the boron distribution in steel is predominantly random. In the given model, a building/fabrication contractor intends to select one of two manufacturers of recycled steel bars basing on the weldability of steel as determined by their boron content selected in equal monthly intervals. A Markov decision process approach is adopted where five states of a Markov chain represent possible states of weldability for steel bars. The boron content is minimized in order to achieve maximum weldability capacity where the decision to select the best steel is made using dynamic programming over a finite period planning horizon. A numerical example demonstrates the existence of an optimal state-dependent selection decision and boron composition over the planning horizon.
Keywords: *Boron, Reinforcement Bar, Weldability, Markovian*

I. INTRODUCTION

The ease with which a metal or alloy can be joined by fusion is termed weldability. It envisages the metallurgical compatibility of the metal with a specific welding process, its ability to be joined with mechanical soundness and the capacity of the resulting weld to perform satisfactorily under the intended service conditions (Jyoti*et al*., 2012). Steel owes much of its versatility to the ease with which it can be joined with fusion welding. The weldability of plain carbon steel thermo-mechanically treated (TMT) bars has also given them an edge over the rest of concrete reinforcement bar varieties.

The major limitation of welded joints, however, is the frequent tendency to fusion defects which include any cracks, flaws or discontinuities that compromise the usefulness of the finished weld. The most insidious discontinuities are those that cause brittle cracking especially if they are of a metallurgical origin.

Cracks in welds are formed either when the weld pool is in the process of cooling, which is a case of hot cracking or after it has cooled; resulting in cold cracking. Most forms of cracks occur when the weld is in the process of cooling as a result of shrinking strains (Thomas, 2011). The stresses caused by the shrinking metal and the rigidity of the base metal which provides the restraints are key precursors in the eventual crack formation. Even when the case of cold cracking occurs as in hydrogen cracking, the sensitivity of the microstructure of heat affected zone is the major underlying factor (Carl*et al*, 2011).

The conditions that lead to the evolution of cracks are therefore dependent on the composition of the base metal which when heated, transforms into austenite and on cooling, forms varying levels of martensite depending on the steel carbon equivalent (CE) which in turn depends on the types and amounts of alloying (tramp) elements in the base metal (Eq.*i*).

$$CE = C + A(C) * \{Si/24 + Mn/6 + Cu/15 + Ni/20 + (Cr + Nb + V + Mo)/5 + 5B\}$$ ---i

where:
$A(C) = 0.75 + 0.25\tanh\{20(C - 0.12)\}$ (Yurioka,1985).

Similarly, equation ii) from the work of Ito and Bessyo depicts the critical metal parameter Pcm for weld cracking and the influence of major alloying elements on weld crack formation.

$$P_{cm} = \%C + \frac{\%Mn + \%Cu + Cr}{20} + \frac{Ni}{60} + \frac{\%Mo}{15} + \frac{\%V}{10} + 5B \ldots\ldots\ldots\ldots\ldots\ldots\ldots..ii)$$

The majority of steel reinforcement bars worldwide are made from recycled steel. The fact that the alloy content of a recycled component is quite difficult to predict and control makes the properties of the steel bars hard to predetermine.

Importantly, alloying/tramp elements have individual effects on steel strength that are additive and increase with particular alloying element content (Grange*et al*, 1977).Both equation *i)*and *ii)* allude to the effect of boron in steel which even when only present in small percentages, strongly alters the CE of steel and influence its inclination crack.

Boron comes into recycled steel from boron containing steel scrap articles but more predominantly as a result of the induction furnace and continuous casting lining (Tupkary*et al*, 2008). In both forms, the boron content is not easy to control since its economic industrial chemical regulation is still not viable.

Weldability, a crucial value of thermo-mechanically treated bars, is strongly affected by the presence of boron, since it majorly influences the CE and thus the strength of the base metal (Saeed*et al*, 2012). Because of this and other reasons, the control and predictability of boron and its influence in respect major product functionalities have become an issue of vital importance.

In this research, a mathematical model is developed to depict the relationship between the steel boron content and reinforcing bar weldability using a stochastic approach based on a Markovian weldability distribution since it has also been established that the incidence and effect of tramp elements in steel has a random distribution (Senfuka*et al*, 2013).

II. METHODS AND MATERIAL

2. Model Formulation

2.1 Notation and assumptions

i,j = States of demand
A = Excellent state
B = Very good state
C = Good state
D = Fair state
E = Poor state
n,N = Stages
Z = Selection decision
NZij = Number of transitions
WZ = Weldability transition matrix
WZij = Weldability transition probability
RZ = Boron content matrix
RZij = Boron composition due to state transition
eZi = Expected boron composition
aZi = Accumulated boron composition
m = Manufacturer

i,j ε {A,B,C,D,E} m ε {1,2} Z ε {1,2} n=1,2, …………………….N

Consider a production system consisting of two manufacturing plants producing recycled steel bars in batches for a designated number of customers. The weldability state of steel bars during each time period over a fixed planning horizon is classified as *Excellent* (denoted by state A), *Very good*, (denoted by state B), *Good* (denoted by state C), *Fair*(denoted by state D) and *Poor*(denoted by state E). The transition probabilities for weldability capacity over the planning horizon from one state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to select bars from manufacturer 1(a decision denoted by Z=1) or to select bars from manufacturer 2 (a decision denoted by Z=2) during each time period over the planning horizon. Optimality is defined such that the expected boron content is accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-period (N=2) planning horizon is considered.

2.2 Finite period dynamic programming problem formulation

Recalling that weldability capacity can be in states A, B,C,D and E, the problem of finding an optimal selection decision among the manufacturers may be expressed as a finite period dynamic programming model.

Let $g_n(i)$ denote the optimal expected boron composition accumulated during the periods *n,n+1,......,**N*** given that the state of the system at the beginning of period *n* is *i*ϵ{ A,B,C,D,E }.The recursive equation relating g_n and g_{n+1} is:

$$g_n(i) = min_Z[W^Z_{iA}(m)R^Z_{iA}(m) + g_{n+1}(A), W^Z_{iB}(m)R^Z_{iB}(m) + g_{n+1}(B)]$$

$$+min_Z[W^Z_{iC}(m)R^Z_{iC}(m)+g_{n+1}(C), W^Z_{iD}(m)R^Z_{iD}(m)+g_{n+1}(D), W^Z_{iE}(m)R^Z_{iE}(m)+ g_{n+1}(E)] \quad (1)$$

iε**{**A,B,C,D,E **}**, m={1,2}, n= 1,2,……………………N

together with the final conditions
$g_{N+1}(A) = g_{N+1}(B) = g_{N+1}(C) = g_{N+1}(D) = g_{N+1}(E) = 0$

This recursive relationship may be justified by noting that the cumulative boron composition $R^Z_{ij}(m)+ g_{N+1}(j)$ resulting from reaching state j ϵ{ A,B,C,D,E} at the start of period n+1 from state i ϵ { A,B,C,D,E} at the start of period *n* occurs with probability $R^Z_{ij}(m)$.

Clearly, $e^Z(m) = [W^Z_{ij}(m)]\ [\ R^Z_{ij}(m)\ ]^T$, Z ϵ{1,2} , m ϵ{1,2} (2)

where 'T' denotes matrix transposition, and hence the dynamic programming recursive equations

$$g_N(i) = min_Z[e^Z_i(m)+W^Z_{iA}(m)g_{N+1}(A)+W^Z_{iB}(m)g_{N+1}(B)+W^Z_{iC}(m)g_{N+1}(C)$$

$$+min_Z[W^Z_{iD}(m)g_{N+1}(D) + W^Z_{iE}(m)g_{N+1}(E)] \quad (3)$$

$$g_N(i,m) = min_Z[e^Z_i(m)] \quad (4)$$

result where (4) represents the Markov chain stable state.

2.2.1 Computing $W^Z(m)$

The transition probability for weldability capacity from state iε{A,B,C,D,E} to state j ϵ{A,B,C,D,E},given selection decision Z ϵ{ 1,2 } may be taken as the number of state transitions observed at manufacturing plant m with weldability capacity initially in state i and later with weldability capacity changing to state j, divided by the sum of transitions over all states. That is,

$$W^Z_{ij}(m) = N^Z_{ij}(m)/[N^Z_{iA}(m) + N^Z_{iB}(m) + N^Z_{iC}(m) + N^Z_{iD}(m) + N^Z_{iE}(m)]$$

i.e. **{**A, B, C, D, E**}**, Z ϵ{1,2} , m= {1, 2} (5)

3. Optimization

The optimal selection decision and boron content are found in this section for each period separately.

3.1 Optimization during period 1

When weldability capacity is Excellent (i.e. in state A), the optimal selection decision during period 1 is

$$Z = \begin{cases} 1 & if \quad e^1_A(m) < e^2_A(m) \\ 2 & if \quad e^1_A(m) \geq e^2_A(m) \end{cases}$$

The associated boron composition is :

$$g_1(A,m) = \begin{cases} e^1_A(m) & if \quad Z = 1 \\ e^2_A(m) & if \quad Z = 2 \end{cases}$$

Similarly, when weldability capacity is Very good (i.e. in state B), the optimal selection decision and associated boron composition during period 1 are

$$Z = \begin{cases} 1 & if \quad e^1_B(m) < e^2_B(m) \\ 2 & if \quad e^1_B(m) \geq e^2_B(m) \end{cases}$$

and

$$g_1(B,m) = \begin{cases} e^1_B(m) & if \quad Z = 1 \\ e^2_B(m) & if \quad Z = 2 \end{cases}$$

respectively.

When weldability capacity is good (i.e. in state C), the optimal selection decision and associated boron composition during period 1 are:

$$Z = \begin{cases} 1 & if \quad e^1_C(m) < e^2_C(m) \\ 2 & if \quad e^1_C(m) \geq e^2_C(m) \end{cases}$$

and

$$g_1(C,m) = \begin{cases} e^1_C(m) & if \quad Z = 1 \\ e^2_C(m) & if \quad Z = 2 \end{cases}$$

respectively.

When weldability capacity is fair (i.e. in state D), the optimal selection decision and associated boron composition during period 1 are:

$$Z = \begin{cases} 1 & if \quad e^1_D(m) < e^2_D(m) \\ 2 & if \quad e^1_D(m) \geq e^2_D(m) \end{cases}$$

and

$$g_1(D,m) = \begin{cases} e^1_D(m) & if \quad Z = 1 \\ e^2_D(m) & if \quad Z = 2 \end{cases}$$

respectively.

When weldability capacity is poor (i.e. in state E), the optimal selection decision and associated boron composition during period 1 are:

$$Z = \begin{cases} 1 & if \quad e_E^1(m) < e_E^2(m) \\ 2 & if \quad e_E^1(m) \geq e_E^2(m) \end{cases}$$

and

$$g_1(E,m) = \begin{cases} 1 & if \quad e_E^1(m) < e_E^2(m) \\ 2 & if \quad e_E^1(m) \geq e_E^2(m) \end{cases}$$

respectively.

3.2 Optimization during period 2

Using dynamic programming recursive equation (1) and recalling that $a^Z_i(m,2)$ denotes the already accumulated boron content at the end of period 1 as a result of decisions made during that period, when weldability capacity is Excellent(i.e. in state A), the optimal selection decision and the associated boron composition during period 2 are:

$$Z = \begin{cases} 1 & if \quad a_A^1(m) < a_A^2(m) \\ 2 & if \quad a_A^1(m) \geq a_A^2(m) \end{cases}$$

and

$$g_2(A,m) = \begin{cases} a_A^1(m) & if \quad Z = 1 \\ a_A^2(m) & if \quad Z = 2 \end{cases}$$

respectively.

Similarly, when weldability capacity is very good (i.e. in state B), the optimal selection decision and the associated weldability capacity are:

$$Z = \begin{cases} 1 & if \quad a_B^1(m) < a_B^2(m) \\ 2 & if \quad a_B^1(m) \geq a_B^2(m) \end{cases}$$

and

$$g_2(B,m) = \begin{cases} a_B^1(m) & if \quad Z = 1 \\ a_B^2(m) & if \quad Z = 2 \end{cases}$$

respectively.

When weldability capacity is good (i.e. in state C), the optimal selection decision and the associated weldability capacity are:

$$Z = \begin{cases} 1 & if \quad a_C^1(m) < a_C^2(m) \\ 2 & if \quad a_C^1(m) \geq a_C^2(m) \end{cases}$$

And

$$g_2(C,m) = \begin{cases} a_C^1(m) & if \quad Z = 1 \\ a_C^2(m) & if \quad Z = 2 \end{cases}$$

respectively.

When weldability capacity is fair (i.e. in state D), the optimal selection decision and the associated weldability capacity during period 2 are:

$$Z = \begin{cases} 1 & if \quad a_D^1(m) < a_D^2(m) \\ 2 & if \quad a_D^1(m) \geq a_D^2(m) \end{cases}$$

And

$$g_2(D,m) = \begin{cases} a_D^1(m) & if \quad Z = 1 \\ a_D^2(m) & if \quad Z = 2 \end{cases}$$

respectively.

When weldability capacity is poor (i.e. in state E), the optimal selection decision and the associated weldability capacity during period 2 are:

$$Z = \begin{cases} 1 & if \quad a_E^1(m) < a_E^2(m) \\ 2 & if \quad a_E^1(m) \geq a_E^2(m) \end{cases}$$

And

$$g_2(E,m) = \begin{cases} a_E^1(m) & if \quad Z = 1 \\ a_E^2(m) & if \quad Z = 2 \end{cases}$$

respectively.

III. RESULTS AND DISCUSSION

4. Case Study

In order to demonstrate use of the model in sections 3 to 4, real case applications from rolling mills 1 and 2 in Uganda are presented in this section. Steel bars are manufactured for fabrication shops and the degree of weldability capacity varies for the two manufacturers. The fabrication shop wants to avoid high boron composition when the state of weldability capacity is Excellent (state A), Very good (state B), Good (state C) or Fair (state D) in order to utilize steel for welding at lower levels of boron composition. Hence, decision support is sought for the fabrication shop in terms of an optimal selection decision and the associated boron composition in a two-month planning period for the two competing manufacturers.

4.1 Data Collection

Past data revealed the following patterns of weldability capacity and boron composition over 30 days.

Manufacturer 1

Days	$W^1_{ij}(1),C_{eq}$	$R^1_{ij}(1)$	Days	$W^1_{ij}(1),C_{eq}$	$R^1_{ij}(1)$
1	0.5279	0.001	16	0.4741	0.0012
2	0.5046	0.001	17	0.5236	0.0015
3	0.4604	0.001	18	0.5635	0.002
4	0.5376	0.001	19	0.3941	0.0007
5	0.3801	0.0012	20	0.3757	0.0008
6	0.4537	0.0012	21	0.3896	0.0012
7	0.3801	0.0012	22	0.4584	0.002
8	0.4537	0.0012	23	0.3354	0.001
9	0.5179	0.0012	24	0.4143	0.0008
10	0.4590	0.0012	25	0.4171	0.0022
11	0.4047	0.0007	26	0.4137	0.0006
12	0.4172	0.0013	27	0.6683	0.0018
13	0.3990	0.0009	28	0.6657	0.0013
14	0.4619	0.0008	29	0.4741	0.0012
15	0.6683	0.0018	30	0.3812	0.0008

Manufacturer 2

Days	$W^2_{ij}(2)$	$R^2_{ij}(2)$	Days	$W^2_{ij}(2)$	$R^2_{ij}(2)$
1	0.4460	0.0015	16	0.4261	0.002
2	0.4856	0.0015	17	0.4935	0.0006
3	0.5430	0.0007	18	0.3426	0.0007
4	0.5280	0.0018	19	0.4340	0.0008
5	0.4697	0.0009	20	0.5177	0.0013
6	0.4427	0.0013	21	0.3441	0.0014
7	0.6400	0.0018	22	0.6241	0.0012
8	0.4586	0.0014	23	0.3398	0.0011
9	0.4457	0.0011	24	0.5381	0.0012
10	0.3960	0.0011	25	0.4537	0.0012
11	0.5116	0.002	26	0.3480	0.0007
12	0.4732	0.003	27	0.4402	0.0014
13	0.5268	0.002	28	0.4170	0.0021
14	0.4143	0.001	29	0.4199	0.0013
15	0.5307	0.0015	30	0.6234	0.0007

4.2 Determining $W^Z(m)$ and $R^Z(m)$

4.2.1 Estimating Elements of $W^1(1)$ and $R^1(1)$

State Transition (i,j)	No. of Transitions	Weldability CE	Boron Content	Weldability Transition Probability, $W^1_{ij}(1)$	Boron content due to state transition $R^1_{ij}(1)$
AA	0	0	0	0	0
AB	0	0	0	0	0
AC	1	0.335	0.0008	(1/1) = 1	(0.003/2)=0.0015
	0	0.414	0.0022		
AD	0	0	0	0	0
AE	0	0	0	0	0
TOTALS	**1**			**1**	
BA	0	0	0	0	0
BB	2	0.394	0.0007		
		0.376	0.0008	(2/6)=0.333	(0.0027/3)=0.0009
		0.390	0.0012		
BC	3	0.380	0.0012		
		0.454	0.0012		
		0.405	0.0007	(3/6)=0.500	(0.0074/6)=0.0012
		0.417	0.0013		
		0.390	0.002		
		0.458	0.001		
BD	1	0.399	0.0009	(1/6)=0.167	(0.0017/2)=0.0009
		0.462	0.0008		
BE	0	0	0	0	0
TOTALS	**6**			**1**	

State Transition (i,j)	No. of Transitions	Weldability CE	Boron Content (B)	Weldability Transition Probability, $W^1_{ij}(1)$	Boron content due to state transition $R^1_{ij}(1)$
CA	1	0.458	0.001		
		0.335	0.0008	(1/7) = 0.143	(0.0032/2)=0.0016
CB	3	0.454	0.0012		
		0.380	0.0012		
		0.459	0.0012		
		0.405	0.0007	(3/7)=0.428	(0.0065/6)=0.0011
		0.417	0.0013		
		0.399	0.0009		
CC	1	0.414	0.0006		
		0.417	0.0018	(1/7)=0.143	(0.0024/2)=0.0012
CD	0	0	0	0	0
CE	2	0.454	0.0012		
		0.518	0.0012		
		0.414	0.0006	(2/7)=0.286	(0.0048/4)=0.0012
		0.668	0.0018		
TOTALS	7			**1**	

State Transition (i,j)	No. of Transitions	Weldability CE	Boron Content	Weldability Transition Probability, $W^{1}_{ij}(1)$	Boron content due to state transition $R^{1}_{ij}(1)$
DA	0	0	0	0	0
DB	1	0.474	0.0012		
		0.381	0.0008	(1/4) =	(0.003/2)=0.0015
DC	0	0	0	0.250	0
DD	0	0	0	0	0
DE	3	0.460	0.0008	0	
		0.534	0.0018		
		0.462	0.0012		
		0.668	0.0015		(0.0073/3)=0.0024
		0.474	0.001	(3/4)=0.750	
		0.524	0.001		
TOTALS	**4**			**1**	
EA	0	0	0	0	0
EB	0	0	0	0	0
EC	1	0.517	0.0012		
		0.459	0.0012	(1/7)=0.143	(0.0024/2)=0.0012
ED	3	0.405	0.001		
		0.460	0.001		
		0.668	0.0018		
		0.474	0.0012	(3/7)=0.429	(0.007/6)=0.0012
		0.666	0.0012		
		0.474	0.0008		
EE	3	0.527	0.001		
		0.505	0.001		
		0.524	0.002		
		0.564	0.0007	(3/7)=0.429	(0.0067/6)=0.0011
		0.668	0.0012		
		0.665	0.0008		
TOTALS	**7**			**1**	

4.2.2 Estimating Elements of $W^2(2)$ and $R^2(2)$

State Transition (i,j)	No. of Transitions	Weldability CE	Boron Content	Weldability Transition Probability, $W^{1}_{ij}(2)$	Boron content due to state transition $R^{2}_{ij}(2)$
AA	0	0	0	0	0
AB	0	0	0	0	0
AC	2	0.342	0.0007		
		0.434	0.0008		
		0.348	0.0007	(2/3) = 0.667	(0.0036/4)=0.0009
		0.440	0.0014		
AD	0	0	0	0	0
AE	1	0.339	0.0014		
		0.538	0.0012	(1/3)=0.333	(0.0026/2)=0.0013

TOTALS	**3**			**1**	
BA	0	0	0	0	0
BB	0	0	0	0	0
BC	0	0	0	0	0
BD	0	0	0	0	0
BE	1	0.396	0.0011		
		0.512	0.002	(1/1)=1	(0.0031/2)=0.0016
TOTALS	**1**			**1**	

State Transition (I,j)	No. of Transitions	Boron Content (B)	Weldability Transition Probability, $W^{1}_{ij}(1)$	Boron content due to state transition $R^{2}_{ij}(2)$
CA	1	0.0012		
		0.0007	(1/10) =	(0.0019/2)=0.0001
CB	1	0.0011	0.100	
		0.0011		(0.0022/2)=0.001
CC	3	0.0011	(1/10) =	
		0.0011	0.100	
		0.0014		
		0.0021		(0.007/3)=0.0023
		0.0013		
CD	2	0.0015	(3/10)=0.300	
		0.0015		
		0.002		(0.0056/4)=0.0014
		0.0006		
CE	3	0.001	(2/10)=0.200	
		0.0015		
		0.0008		
		0.0013		(0.0066/6)=0.0011
		0.0013		
		0.0007	(3/10)=0.300	
TOTALS	7		**1**	

State Transition (i,j)	No. of Transitions	Weldability CE	Boron Content	Weldability Transition Probability, $W^1_{ij}(2)$	Boron content due to state transition $R^2_{ij}(2)$
DA	1	0.494	0.0006		
		0.343	0.0007	(1/4)=0.250	(0.0013/2)=0.0007
DB	0	0	0	0	0
DC	0	0	0	0	0
DD	0	0	0	0	0
DE	3	0.486	0.0015		
		0.543	0.0007		
		0.470	0.0009		
		0.640	0.0013	(3/4)=0.750	(0.0094/5)=0.0019
		0.474	0.003		
		0.527	0.002		
TOTALS	**4**			**1**	
EA	2	0.597	0.0013		
		0.344	0.0014		
		0.624	0.0012	(2/7)=0.222	(0.005/4)=0.0013
		0.340	0.0011		
EB	0	0	0	0	0
EC	5	0.640	0.0018		
		0.459	0.0014		
		0.527	0.002		
		0.414	0.001	(5/9)=0.555	(0.0097/6)=0.0016
		0.531	0.0015		
		0.426	0.002		
ED	2	0.528	0.0018		
		0.470	0.0009		
		0.512	0.002	(2/9)=0.222	(0.0077/4)=0.0019
		0.473	0.003		
EE	1	0.543	0.0007		
		0.528	0.0018	(1/9)=0.111	(0.0025/2)=0.0013
TOTALS	**10**			**1**	

Manufacturer 1: Manufacturer 2:

State-transition matrices

$$N^1(1) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 3 & 1 & 0 \\ 1 & 3 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 & 3 \end{bmatrix} N^2(2) = \begin{bmatrix} 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 3 & 2 & 3 \\ 1 & 0 & 0 & 0 & 3 \\ 2 & 0 & 4 & 2 & 1 \end{bmatrix}$$

Manufacturer 1:
Manufacturer 2:

Weldability transition matrices

$$W^1(1) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.33 & 0.50 & 0.17 & 0 \\ 0.14 & 0.43 & 0.14 & 0 & 0.29 \\ 0 & 0.25 & 0 & 0 & 0.75 \\ 0 & 0 & 0.14 & 0.43 & 0.43 \end{bmatrix} W^2(2) = \begin{bmatrix} 0 & 0 & 0.67 & 0 & 0.33 \\ 0 & 0 & 0 & 0 & 1 \\ 0.10 & 0.10 & 0.30 & 0.20 & 0.30 \\ 0.25 & 0 & 0 & 0 & 0.75 \\ 0.22 & 0 & 0.55 & 0.22 & 0.11 \end{bmatrix}$$

Matrices for boron content

$$R^1(1) = \begin{bmatrix} 0 & 0 & 0.0015 & 0 & 0 \\ 0 & 0.0009 & 0.0012 & 0.0009 & 0 \\ 0 & 0.0011 & 0.0012 & 0 & 0.0012 \\ 0 & 0.001 & 0 & 0 & 0.0024 \\ 0 & 0 & 0.0012 & 0.0012 & 0.0011 \end{bmatrix} R^2(2) = \begin{bmatrix} 0 & 0 & 0.0009 & 0 & 0.0013 \\ 0 & 0 & 0 & 0 & 0.0016 \\ 0.0001 & 0.0001 & 0.0023 & 0.0014 & 0.0013 \\ 0.0007 & 0 & 0 & 0 & 0.0023 \\ 0.0013 & 0 & 0.0016 & 0.0019 & 0.0013 \end{bmatrix}$$

4.3 Calculating $e_i^Z(m)$ and $a_i^Z(m)$

When steel bars are selected from manufacturer 1(m=1, Z=1), the matrices $W^1(1)$ and $R^1(1)$ yield the following expected boron composition:

$$e_A^1(1) = (0)(0) + (0)(0) + (1)(0.0015) + (0)(0) + (0)(0) = 0.0015$$
$$e_B^1(1) = 0 + (0.33)(0.0009) + (0.5)(0.0012) + (0.17)(0.0009) + 0 = 0.0011$$
$$e_C^1(1) = (0.14)(0.0016{+}0.0012){+}(0.43)(0.0011){+}(0.29)(0.0012) = 0.0012$$
$$e_D^1(1) = 0 + (0.25)(0.001) + 0 + 0 + (0.75)(0.0024) = 0.0021$$
$$e_E^1(1) = 0 + 0 + (0.14)(0.0012) + (0.43(0.0011)) = 0.0012$$

When steel bars are selected from manufacturer 2(m=2, Z=2), the matrices $W^2(2)$ and $R^2(2)$ yield the following expected boron composition:

$$e_A^2(2) = 0 + 0 + (0.67)(0.0009) + 0 + (0.33)(0.0013) = 0.0010$$
$$e_B^2(2) = 0 + 0 + 0 + 0 + (1)(0.0016) = 0.0016$$
$$e_C^2(2) = (0.10)(0.0002) + (0.3)(0.0023 + 0.0011) + (0.2)(0.0011) = 0.0013$$
$$e_D^2(2) = (0.25)(0.0007) + 0 + 0 + 0 + (0.75)(0.0023) = 0.0019$$
$$e_E^2(2) = (0.22)(0.0013) + 0 + (0.55)(0.0016) + (0.22)(0.0019) = 0.0017$$

4.4 Optimal Decisions for Steel Selection against Weldability states

4.4.1 Month 1 Decisions

Excellent state

Since 0.0010< 0.0015, it follows that Z=2 is an optimal decision for steel selection in month 1 with associated boron composition of 0.0010 when weldability of steel bars is Excellent.

Very good state:

Since 0.0011<0.0016, it follows that Z=1 is an optimal decision for steel selection in month 1 with associated boron composition of 0.0011 when weldability of steel bars is Very good.
Good state:

Since 0.0012 < 0.0013, it follows that Z=1 is an optimal decision for steel selection in month 1 with associated boron composition of 0.0012 when weldability of steel bars is Good.

Fair state:

Since 0.0019 < 0.0021, it follows that Z=2 is an optimal decision for steel selection in month 1 with associated boron composition of 0.0019 when weldability of steel bars is Fair.

Poor state:

Since 0.0012 < 0.0017, it follows that Z=1 is an optimal decision for steel selection in month 1 with associated boron composition of 0.0012 when weldability of steel bars is Poor.

Hence, in month 1, the optimal selection criterion is in favor of manufacturer 1 when weldability of steel bars is very good, good or poor. Otherwise manufacturer 2 can be selected when weldability of steel bars is excellent or fair.

The accumulated boron composition is computed for manufacturer 1 when weldability of steel bars is Excellent, Very good, Good, Fair or Poor and the following results are obtained:

$a^1_A(1) = 0.0015 + (0)(0.0010) +(0)(0.0011) +(1)(0.0012) +(0)(0.0019) +(0)(0.0012) = 0.0027$
$a^1_B(1) = 0.0011 + (0)(0.0010) +(0.33)(0.0011) +(0.5)(0.0012) +(0.17)(0.0019) +(0)(0.0012) = 0.0024$
$a^1_C(1) = 0.0012 + (0.14)(0.0010) +(0.43)(0.0011) +(0.14)(0.0012) +(0)(0.0019) +(0.29)(0.0012) = 0.0023$
$a^1_D(1) = 0.0021 + (0)(0.0010) +(0.25)(0.0011) +(0)(0.0012) +(0)(0.0019) +(0.75)(0.0012) = 0.0033$
$a^1_E(1) = 0.0012 + (0)(0.0010) +(0)(0.0011) +(0.14)(0.0012) +(0.43)(0.0019) +(0.43)(0.0012) = 0.0027$

Similarly, the accumulated boron composition is computed for manufacturer 2 when weldability of steel bars is Excellent, Very good, Good, Fair or Poor and the following results are obtained:

$a^2_A(2) = 0.0010 + (0)(0.0010) +(0)(0.0011) +(0.67)(0.0012) +(0)(0.0019) +(0.33)(0.0012) = 0.0022$
$a^2_B(2) = 0.0016 + (0)(0.0010) +(0)(0.0011) +(0)(0.0012) +(0)(0.0019) +(1)(0.0012) = 0.0028$
$a^2_C(2) = 0.0013 + (0.10)(0.0010) +(0.10)(0.0011) +(0.30)(0.0012) +(0.20)(0.0019) +(0.30)(0.0012) = 0.0026$
$a^2_D(2) = 0.0019 + (0.25)(0.0010) +(0)(0.0011) +(0)(0.0012) +(0)(0.0019) +(0.75)(0.0012) = 0.0031$
$a^2_E(2) = 0.0017 + (0.22)(0.0010) +(0)(0.0011) +(0.55)(0.0012) +(0.22)(0.0019) +(0.11)(0.0012) = 0.0031$

4.4.2 Month 2 Decisions

Excellent state:

Since 0.0022<0.0027, it follows that Z=2 is an optimal decision for steel selection in month 2 with associated accumulated boron composition of 0.0022 when weldability of steel bars is Excellent.

Very good state:

Since 0.0024<0.0028, it follows that Z=2 is an optimal decision for steel selection in month 2 with associated accumulated boron composition of 0.0024 when weldability of steel bars is Very good.

Good state:

Since 0.0023<0.0026, it follows that Z=1 is an optimal decision for steel selection in month 2 with associated accumulated boron composition of 0.0023 when weldability of steel bars is Good.

Fair state:

Since 0.0031<0.0033, it follows that Z=2 is an optimal decision for steel selection in month 2 with associated accumulated boron composition of 0.0031 when weldability of steel bars is Fair.

Poor state:

Since 0.0027<0.0031, it follows that Z=1 is an optimal decision for steel selection in month 2 with associated accumulated boron composition of 0.0027 when weldability of steel bars is Poor.

Hence in month 2, the optimal selection criterion is in favor of manufacturer 2 when weldability of steel bars is Excellent, Very good, or Fair. Manufacturer 1 can be selected when weldability of steel bars is Good or Poor.

IV. CONCLUSION

An optimization model for determining the selection criteria of recycled reinforcement steel bars under Markovian weldability distribution against their boron content was presented in this paper. The decision of selecting better welding steel bars from two competing manufacturers is modeled as a multi-period decision problem using dynamic programming over a finite period planning horizon. The working of the model was demonstrated by means of a real case study as demonstrated in section 4 of the paper.

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