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Sub Compatible and Sub Sequentially Continuous Maps in Intuitionistic Fuzzy Metric Space

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ABSTRACT

The present paper introduces the new concepts of sub compatibility and sub sequential continuity in intuitionistic fuzzy metric spaces which are weaker than occasionally weak compatibility and reciprocal continuity. We also establish a common fixed point theorem four maps using sub compatibility and sub sequential continuity. Our results particularly extend and generalize the result of M. Alamgir Khan et al [28]
Keywords: Fuzzy Sets, Fuzzy Metric, Fuzzy Metric Space, Intuitionistic Fuzzy Sets, T-Conorm, Cauchy Sequence

I. INTRODUCTION

The concept of fuzzy sets was introduced by *zadeh* [13] following the concept of fuzzy sets, fuzzy metric spaces have been introduced by kramosil and michlek [9] and George and veeramani [6] modified the notion of fuzzy metric space with the help of continuous t-norms.

As a generalization of fuzzy sets , Atanassov [14] introduced and studied the concept of intuitionistic fuzzy sets park[11] using the idea of intuitionistic fuzzy sets defined the notion of intuionistic fuzzy metric spaces with the help of continuous t-norm and continuous t co-norm as a generalization of fuzzy metric space due to George & veeramani [6] had showed that every metric induces an intuitionistic fuzzy metric every fuzzy metric space is an intuitionistic fuzzy metric space and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete choudhary[15] introduced mutually contractive sequence of self maps and proved a fixed point theorem kramosil & michlek [9] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well-known fixed point theorem of Banach [4], Turkoglu et al [12] gave the generalization of jungek's common fixed point theorem [19] to intuitionistic fuzzy metric spaces, they first formulate the definition of weakly commuting and R- weakly commuting mapping in intuitionistic fuzzy metric spaces and proved proved the intuitionistic fuzzy version of pant's theorem[20] .

The present paper introduces the new concepts of sub compatibility and sub sequential continuity in intuitionistic fuzzy metric spaces which are weaker than occasionally weak compatibility and reciprocal continuity. We also establish a common fixed point theorem four maps using sub compatibility and sub sequential continuity. Our results particularly extend and generalize the result of M. Alamgir Khan et al [28].

II. METHODS AND MATERIAL

Definition 2.1 [7]. A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if $([0,1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$. Examples of t-norms are $a * b = ab$ and $a * b = \min\{a, b\}$

Definition 2.2 [7]. A binary operation $\lozenge: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-conorm if $\lozenge$ is satisfying the following condition

2.2.1 $\lozenge$ is commutative and associate

2.2.2 $\lozenge$ is continuous

2.2.3 $a \lozenge 0 = a$ for all $a \in [0,1]$

2.2.4 $a \lozenge b \leq c \lozenge d$ Whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 2.3 [4]. the 3-tuple (X, M, $*$) is called a fuzzy metric space (FM-space) if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$.

2.3.1 $M(x, y, 0) > 0$

2.3.2 $M(x, y, t) = 1, \forall\, t > 0$ iff $x = y$

2.3.3 $M(x, y, t) = M(y, x, t)$,

2.3.4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

2.3.5 $M(x, y, \ .): [0, \infty] \rightarrow [0,1]$ is continuous.

Remark 2.1 since $*$ is continuous, it follows from (2.3.4) that the limit of a sequence in FM-space is uniquely determined

Definition 2.4 [16]. A five –tuple (X, M, N, $*$, $\lozenge$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set $*$ is a continuous t – norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

2.4.1 $M(x, y, t) + N(x, y, t) \leq 1$

2.4.2 $M(x, y, t) > 0$

2.4.3 $M(x, y, t) = M(y, x, t)$

2.4.4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

2.4.5 $M(x, y, .): (0, \infty) \rightarrow (0,1)$ is continuous

2.4.6 $N(x, y, t) > 0$

2.4.7 $N(x, y, t) = N(y, x, t)$

2.4.8 $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$

2.4.9 $N(x, y, .): (0, \infty) \rightarrow (0,1]$ is continuous

Then (M, N) is called an intuitionistic fuzzy metric on X , the function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively

Remark 2.1. every fuzzy metric space (X, M, $*$) is an intuitionistic fuzzy metric space form$(X, M, 1 - M, *, \lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$ are associated ie $x \lozenge y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in [0,1]$ but the converse is not true

Example 2.1. Induced intuitionistic fuzzy metric space [1]

Let (X,d) be a metric space denote $a * b = ab$ and $a \lozenge b = \min\{1, a + b\}$ for all $a, b \in [0,1]$ and let M, and N be fuzzy sets on $X^2 \times (0,1)$ defined as follows

$M(x, y, t) = \frac{t}{t + d(x,y)}$

$N(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$ Then

$(X, M, N, *\lozenge)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric

Remark2.2. note that the above example holds even with the t-norm $a * b = \min\{a, b\}$ and the t-conorm $a \lozenge b = \max\{a, b\}$

Example 2.2.. Let $X = N$ define $a * b = \max\{0, a + b - 1\}$ and $a \lozenge b = a + b - ab$ for all $a, b \in [0,1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ as follows

$$M(x, y, t) = \begin{cases} \frac{x}{y} \text{ if } x \leq y \\ \frac{y}{x} \text{ if } y \leq x \end{cases}$$

$$N(x, y, t) = \begin{cases} \frac{y - x}{y} \text{ if } x \leq y \\ \frac{x - y}{x} \text{ if } y \leq x \end{cases}$$

For All $x, y \in X$ and $t > 0$ then $(X, M, N, *, \lozenge)$ is an intuitionistic fuzzy metric space

Remark 2.3. Note that in above example, t-norm $*$ and t- conorm $\lozenge$ are not associated, and there exist no metric d on X satisfying

$M(x, y, t) = \frac{t}{t + d(x, y)}$,

$$N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Where $M(x, y, t)$ and $N(x, y, t)$ are defined in above example, also note that the above functions (M, N) is not an intuitionistic metric with the t-norm and t-conorm defined as $a * b = \min\{a, b\}$ $a \lozenge b = \max\{a, b\}$

Definition 2.5 [1]. let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each $\varepsilon > 0$ and each $t > 0$ there exists $n_0 \in N$ such that

$$M(x_n, x, t) > 1 - \varepsilon$$

and

$$N(x_n, x, t) < \varepsilon \text{ For all } n \geq n_0$$

(b) An intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete

Definition 2.6 [19]. let A and B maps from a Intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$into itself. The maps A and B are said to be compatible (or asymptotically commuting) if for all $t > 0$

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$$

and

$$\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$

Definition 2.7[21]. Two mappings A and B of a intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ into itself are R-weakly commuting provided there exists some real number R such that

$$M(ABx, BAx, t) \geq M(Ax, Bx, t/R)$$

and

$N(ABx, BAx, t) \leq N(Ax, Bx, t/R)$ For each $x \in X$ and $t > 0$

Definition 2.9 . Two self maps A and B on a set X are said to be owc if and only if there is a point $x \in X$ which is a coincidence point of A and B at which A and B commute. i.e., there exists a point $x \in X$ such that $Ax = Bx$ and $ABx = BAx$

Definition 2.10 . Two self maps A and B on an IFM-space $(X, M, N, *, \Diamond)$ are said sub compatible if and only if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$, $z \in X$ and which satisfy $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$ for all $t > 0$

Obviously two occasionally weakly compatible maps are sub compatible maps, however the converse is not true in general as shown in the following example

Exampla 2.11 .Let $X = [0, \infty)$ with usual metric d and define
$M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $x , y \in X$ and $t > 0$.
Define the maps $A, B: X \to X$ by setting
$Ax = \begin{cases} x^2, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$, $Bx = \begin{cases} 3x - 2 , & x < 1 \\ x + 3 & x \geq 1 \end{cases}$
Define a sequence $x_n = 1 - \frac{1}{n}$, then $Ax_n = (1 - \frac{1}{n})^2 \to 1$, $Bx_n = 3\left(1 - \frac{1}{n}\right) - 2 = 1 - \frac{3}{n} \to 1$
$ABx_n = A\left(1 - \frac{3}{n}\right) = (1 - \frac{1}{n})^2 = 1 + \frac{9}{n^2} - \frac{6}{n}$ and
$BAx_n = B(1 - \frac{1}{n})^2 = 3(1 - \frac{1}{n})^2 - 2 = 3\left[\left(1 + \left(\frac{1}{n}\right)^2 - \frac{2}{n}\right)\right] - 2 = 1 + \left(\frac{1}{n}\right)^2 - \frac{6}{n}$ and

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$$

and

$\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$
Thus, A and B are sub compatible but A and B are not owc maps as ,
$A(4) = 7 = B(4)$and $AB(4) = A(7) = 13 \neq BA(4) = 10$.
Now , we aim at our second objective which is to introduce a new notion called sub sequential continuity in IFM-space by weaking the concept of reciprocal continuity

Definition 2.12 . Two self maps A and S on an IFM-space are called reciprocal continuous if $\lim_{n\to\infty} ASx_n = At$ and $\lim_{n\to\infty} SAx_n = St$ for some $t \in X$. whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \in X$.

Definition 2.13 . Two self maps A and S on a fuzzy metric space are said to be sub sequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$ and satisfy $\lim_{n\to\infty} ASx_n = At$ and $\lim_{n\to\infty} SAx_n = St$.

Remark 2.14. If A and S are both continuous or reciprocally continuous then they are obviously sub sequentially continuous.

The next example shows that there exist sub sequentlly continuous pairs of maps which are neither continuous nor reciprocally continuous.

Exampla 2.11 .Let $X = R$, endowed with usual metric d and define
$M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $x , y \in X$ and $t > 0$.
Define the maps $A, S: X \to X$ by setting
$Ax = \begin{cases} 2, & x < 3 \\ x, & x \geq 3 \end{cases}$, $Sx = \begin{cases} 2x - 4 , & x \leq 3 \\ 3 & x > 3 \end{cases}$
Define a sequence $x_n = 3 + \frac{1}{n}$, and $Ax_n = (3 + \frac{1}{n}) \to 3$, $Sx_n = 3$
$SAx_n = S\left(3 + \frac{1}{n}\right) = 3 \neq S(3) = 2$. Thus A and S are not reciprocally continuous but if we consider a sequence $x_n = 3 - \frac{1}{n}$, then $Ax_n = 2$, $Sx_n = 2\left(3 - \frac{1}{n}\right) - 4 = \left(2 - \frac{2}{n}\right) \to 2$

$ASx_n = A\left(2-\frac{2}{n}\right) = 2 = A(2)$, $SAx_n = S(2) = 0$
Therefore , A and S are sub sequentially continuous.

Lemma 2.1[1]. In intuitionistic fuzzy metric space X, $M(x, y, .)$ is non decreasing and $N(x, y, .)$ is non increasing for all $x, y \in X$

Lemma 2.2(1). Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space if there exist $k \in (0,1)$ such that

$$M(x, y, kt) \geq M(x, y, t)$$

and

$$N(x, y, kt) \leq N(x, y, t) \text{ for all } x, y \in X$$

Then $x = y$

Proof: $\because M(x, y, kt) \geq M(x, y, t)$

and

$$N(x, y, kt) \leq N(x, y, t)$$

Then we have

$$M(x, y, t) \geq M(x, y, \frac{t}{k})$$

and

$$N(x, y, t) \leq N(x, y, \frac{t}{k})$$

By repeated application of above inequality as we have

$$M(x, y, t) \geq M\left(x, y, \frac{t}{k}\right) \geq M\left(x, y, \frac{t}{k^2}\right) \geq ----M\left(x, y, \frac{t}{k^n}\right) \geq ----------$$

and

$$N(x, y, t) \leq N\left(x, y, \frac{t}{k}\right) \leq N\left(x, y, \frac{t}{k^2}\right) \leq -----------N\left(x, y, \frac{t}{k^n}\right) \leq ----$$

For $n \in N$ which tends to 1 and 0 as $n \to \infty$ respectively thus

$$M(x, y, t) = 1$$

and

$$N(x, y, t) = 0$$

For all $t > 0$ and we get $x = y$

Implicit relation let Φ_4 and Ψ_4 denote the set of all continuous functions from $[0,1]^4 \to R$ Satisfying the conditions
Φ_1: Φ is non-increasing in second and third argument and
Φ_2: $\Phi\ (u, v, v, v,) \geq 0\ u \geq v$ for $u, \in [0,1]$
and
Ψ_1: Ψ Is non-decreasing in second and third argument and
Ψ_2: $\Psi(u, v, v, v,) \geq 0, \Longrightarrow u \leq v$ For $u \in [0,1]$
Example $\Phi\ (t_1, t_2, t_3, t_4,) = t_1 - \max\{t_2, t_3, t_4, \}$
and
$\Psi(t_1, t_2, t_3, t_4,) = \min\ \{t_2, t_3, t_{4,}\} - t_1$

III. RESULTS AND DISCUSSION

Theorem 3.1. Let f, g, h and k be four self maps on an Intuitionistic fuzzy metric space$(X, M, N, *, \lozenge)$. If the pair (f, h) and (g, k) are sub compatible and sub sequentially continuous, then (3.1)f and h have a coincidence point, (3.2)g and k have a coincidence point.
Further, if

$$(3.3)\ \varphi\left(\begin{matrix}M(fx,gy,kt),\dfrac{M(hx,ky,t)+M(fx,hx,t)}{2},\dfrac{M(gy,ky,t)+M(hx,gy,t)}{2},\\ ,M(ky,fx,t)\end{matrix}\right)\geq 0$$

and

$$\Psi\left(\begin{matrix}N(fx,gy,kt),\dfrac{N(hx,ky,t)+N(fx,hx,t)}{2},\dfrac{N(gy,ky,t)+N(hx,gy,t)}{2},\\ ,N(ky,fx,t)\end{matrix}\right)\geq 0$$

For all $k\in(0,1)$ $x,y\in X, t>0,$ and $\varphi\in\Phi_4$ and $\Psi\in\Psi_4$. Then f,g,h and k have a unique common fixed point in X.

Proof. Since the pairs (f,h) and (g,k) are sub compatible and sub sequentially continuous, therefore, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} hx_n = u$ for some $u\in X$ and which satisfy

$$\lim_{n\to\infty}M(fhx_n,hfx_n,t) = M(fu,hu,t) = 1,$$

and

$$\lim_{n\to\infty}N(fhx_n,hfx_n,t) = N(fu,hu,t) = 0$$

$\lim_{n\to\infty} gy_n = \lim_{n\to\infty} ky_n = v$ for some $v\in X$ and which satisfy

$$\lim_{n\to\infty}M(gky_n,kgy_n,t) = M(gv,kv,t) = 1,$$

and

$$\lim_{n\to\infty}N(gky_n,kgy_n,t) = N(gv,kv,t) = 0$$

Therefore, $fu = hu$ and $gv = kv$. i.e., u is the coincidence point of f and h and v is a coincidence point of g and k. Now, using (3.3) for $x = x_n$ and $y = y_n$, we get

$$\varphi\left(\begin{matrix}M(fx_n,gy_n,kt),\dfrac{M(hx_n,ky_n,t)+M(fx_n,hx_n,t)}{2},\dfrac{M(gy_n,ky_n,t)+M(hx_n,gy_n,t)}{2},\\ ,M(ky_n,fx_n,t)\end{matrix}\right)\geq 0$$

and

$$\Psi\left(\begin{matrix}N(fx_n,gy_n,kt),\dfrac{N(hx_n,ky_n,t)+N(fx_n,hx_n,t)}{2},\dfrac{N(gy_n,ky_n,t)+N(hx_n,gy_n,t)}{2},\\ ,N(ky_n,fx_n,t)\end{matrix}\right)\geq 0$$

Letting $n\to\infty$,

$$\varphi\left(\begin{matrix}M(u,v,kt),\dfrac{M(u,v,t)+1}{2},\dfrac{1+M(u,v,t)}{2},\\ ,M(u,v,t)\end{matrix}\right)\geq 0$$

and

$$\Psi\left(\begin{matrix}N(u,v,kt),\dfrac{N(u,v,t)+0}{2},\dfrac{0+N(u,v,t)}{2},\\ ,N(u,v,t)\end{matrix}\right)\geq 0$$

Since φ in non-increasing in second and third argument, and Ψ is non-decreasing in second and third argument, therefore we have

$$\varphi\left(\begin{matrix}M(u,v,kt),M(u,v,t),M(u,v,t),\\ ,M(u,v,t)\end{matrix}\right)\geq 0$$

and

$$\Psi\left(\begin{matrix}N(u,v,kt),N(u,v,t),N(u,v,t),\\ ,N(u,v,t)\end{matrix}\right)\geq 0$$

Therefore by using the property of φ and Ψ we have

$$M(u, v, kt) \geq M(u, v, t) \text{ and } N(u, v, kt) \leq N(u, v, t)$$

Therefore by lemma 2.2 we have $u = v$.
Again using (3.3) for $x = u$, and $y = y_n$, we obtain

$$\varphi\left(M(fu, gy_n, kt), \frac{M(hu, ky_n, t) + M(fu, hu, t)}{2}, \frac{M(gy_n, ky_n, t) + M(hu, gy_n, t)}{2}, M(ky_n, fu, t)\right) \geq 0$$

and

$$\Psi\left(N(fu, gy_n, kt), \frac{N(hu, ky_n, t) + N(fu, hu, t)}{2}, \frac{N(gy_n, ky_n, t) + N(hu, gy_n, t)}{2}, N(ky_n, fu, t)\right) \geq 0$$

Letting $n \to \infty$,

$$\varphi\left(M(fu, v, kt), \frac{M(fu, v, t) + 1}{2}, \frac{1 + M(fu, v, t)}{2}, M(fu, v, t)\right) \geq 0$$

and

$$\Psi\left(N(fu, v, kt), \frac{N(fu, v, t) + 0}{2}, \frac{1 + N(fu, v, t)}{2}, N(fu, v, t)\right) \geq 0$$

Since φ in non-increasing in second and third argument, and Ψ is non-decreasing in second and third argument, therefore we have

$$\varphi\left(M(fu, v, kt), M(fu, v, t), M(fu, v, t), M(fu, v, t)\right) \geq 0$$

and

$$\Psi\left(N(fu, v, kt), N(fu, v, t), N(fu, v, t), N(fu, v, t)\right) \geq 0$$

Therefore by using the property of φ and Ψ we have

$$M(fu, v, kt) \geq M(fu, v, t) \text{ and } N(fu, v, kt) \leq N(fu, v, t)$$

Therefore by lemma 2.2 we have $fu = v = u$.
Therefore $u = v$ is a common fixed point of f, g, h and k.
For uniqueness, let $w \neq u$ be another fixed point of f, g, h and k.
Then from (3.3) we have

$$\varphi\left(M(fu, gw, kt), \frac{M(hu, kw, t) + M(fu, hu, t)}{2}, \frac{M(gw, kw, t) + M(hu, gw, t)}{2}, M(kw, fu, t)\right) \geq 0$$

and

$$\Psi\left(N(fu, gw, kt), \frac{N(hu, kw, t) + N(fu, hw, t)}{2}, \frac{N(gw, kw, t) + N(hu, gw, t)}{2}, N(kw, fu, t)\right) \geq 0$$

i.e.

$$\varphi\left(M(fu, gw, kt), \frac{M(fu, gw, t) + 1}{2}, \frac{1 + M(fu, gw, t)}{2}, M(gw, fu, t)\right) \geq 0$$

and

$$\Psi\begin{pmatrix} N(fu,gw,kt), \dfrac{N(fu,gw,t)+0}{2}, \dfrac{N(fu,gw,t)+0}{2}, \\ , N(gw,fu,t) \end{pmatrix} \geq 0$$

Since φ in non-increasing in second and third argument, and Ψ is non-decreasing in second and third argument, therefore we have

$$\varphi\begin{pmatrix} M(fu,gw,kt), M(gw,fu,t), M(gw,fu,t), \\ , M(gw,fu,t) \end{pmatrix} \geq 0$$

and

$$\Psi\begin{pmatrix} N(fu,gw,kt), N(gw,fu,t), N(gw,fu,t), \\ , N(gw,fu,t) \end{pmatrix} \geq 0$$

Therefore by using the property of φ and Ψ we have

$$M(fu,gw,kt) \geq M(gw,fu,t) \text{ and } N(fu,gw,kt) \leq N(gw,fu,t)$$

Therefore by lemma 2.2 we have $fu = gw$ and hence the theorem

Corollary 3.2. Let f and h be self maps on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$. Such that the pair (f, h) is sub compatible and sub sequentially continuous, then
(3.1)fand h have a coincidence point,
Further, if

$$(3.2)\ \varphi\begin{pmatrix} M(fx,fy,kt), \dfrac{M(hx,hy,t)+M(fx,hx,t)}{2}, \dfrac{M(fy,hy,t)+M(hx,fy,t)}{2}, \\ , M(hy,fx,t) \end{pmatrix} \geq 0$$

and

$$\Psi\begin{pmatrix} N(fx,fy,kt), \dfrac{N(hx,hy,t)+N(fx,hx,t)}{2}, \dfrac{N(fy,hy,t)+N(hx,fy,t)}{2}, \\ , N(hy,fx,t) \end{pmatrix} \geq 0$$

For all $k \in (0,1)$ $x, y \in X, t > 0$, and $\varphi \in \Phi_4$ and $\Psi \in \Psi_4$. Then f, and h have a unique common fixed point in X.

Corollary 3.3. Let f, g and h be self maps on an intuitionistic fuzzy metric
space $(X, M, N, *, \Diamond)$. Such that the pair (f, h) and (g, h) are sub compatible and sub sequentially continuous, then
(3.1)fand h have a coincidence point,
(3.2)gand h have a coincidence point,
Further, if

$$(3.3)\ \varphi\begin{pmatrix} M(fx,gy,kt), \dfrac{M(hx,hy,t)+M(fx,hx,t)}{2}, \dfrac{M(gy,hy,t)+M(hx,gy,t)}{2}, \\ , M(hy,fx,t) \end{pmatrix} \geq 0$$

and

$$\Psi\begin{pmatrix} N(fx,gy,kt), \dfrac{N(hx,hy,t)+N(fx,hx,t)}{2}, \dfrac{N(gy,hy,t)+N(hx,gy,t)}{2}, \\ , N(hy,fx,t) \end{pmatrix} \geq 0$$

For all $k \in (0,1)$ $x, y \in X, t > 0$, and $\varphi \in \Phi_4$ and $\Psi \in \Psi_4$. Then f, g and h have a unique common fixed point in X.

Exampla 2.11 .Let $X = R$, equipped with usual metric d and define $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $x, y \in X$ and $t > 0$.

Define the maps f, g, h and $k: X \to X$ by setting

$$f(x) = \begin{cases} x, & x \leq 1 \\ 3x + 1, & x > 1 \end{cases}, \quad h(x) = \begin{cases} 2x - 1, & x \leq 1 \\ 5x - 1, & x > 1 \end{cases},$$

$$g(x) = \begin{cases} 3 - 2x, & x \leq 1 \\ 3, & x > 1 \end{cases}, \quad k(x) = \begin{cases} 2, & x < 1 \\ 3x - 1, & x > 1 \end{cases}$$

Define a sequence $\{x_n\} = \{y_n\} = 1 - \frac{1}{n}$.

Then clearly fx_n, gx_n, hx_n and $kx_n \to 1$.

$fh(x_n) = f\left(1 - \frac{1}{n}\right) = \left(1 - \frac{1}{n}\right) \to 1 = f(1)$and $hf(x_n) = h\left(1 - \frac{1}{n}\right) = \left(1 - \frac{1}{n}\right) \to 1 = h(1)$

Thus (f, h) is sub compatible and sub sequentially continuous.

Again,

$$gk(x_n) = g\left(1 - \frac{1}{n}\right) = 3 - 2\left(1 - \frac{1}{n}\right) = \left(1 + \frac{2}{n}\right) \to 1 = g(1)$$

$$kg(x_n) = g\left(1 + \frac{1}{n}\right) = 3\left(1 + \frac{1}{n}\right) - 2 = \left(1 + \frac{3}{n}\right) \to 1 = k(1),$$

Which shows that (g, k) is sub compatible and sub sequentially continuous.

Also the condition (3.3) of our theorem satisfied and '1' is unique common fixed point of f, g, h and k.

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