

# Comparison of P-Delta Effect on a Slender RCC Chimney by using Beam-Column Theory and STAAD PRO

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## ABSTRACT

Chimneys are very tall and slender structures. As per ISIS 4998 (Part 1): 1992, major loads considered for analysis of chimneys are wind loads and earthquake load. Each of them is combined with dead load to get the most critical load for designing a chimney. This paper presents the analysis of a slender RCC chimney with a combination of dead load and wind load. Firstly beam column theory is applied on a tall slender chimney and general equations are developed for bending moment with and without the consideration of second order analysis. Validation of same is done by a typical problem of a tall chimney. Dead load of the structure is calculated by using frustum of a cone method. Wind load is calculated as per IS 875 (part3)-1987. Analysis is carried out by the combination of dead load and wind load. The dynamic effect is ignored. Bending moments are calculated manually by using beam column theory for first and second order analysis of chimney. These results are later compared with STAAD PRO results.

**Keywords:** RCC Chimney, Second order analysis, beam-column theory, P-delta effect.

## I. INTRODUCTION

A chimney is a system for venting hot flue gases or smoke from a boiler, stove, furnace or fireplace to the outside atmosphere. They are typically almost vertical to ensure that the hot gases flow smoothly, drawing air into the combustion through the chimney effect. Tall RC chimneys are commonly used to discharge pollutants at higher elevation. They are typically almost vertical to ensure that the hot gases flow smoothly, drawing air into the combustion through the chimney effect. Chimneys are tall to increase their draw of air for combustion and to disperse the pollutants in flue gases over a greater area in order to reduce the pollutant concentrations in compliance with regulatory or other limits. Chimneys with height exceeding 150 m are considered as tall chimneys. However it is not only a matter of height but also the aspect ratio when it comes to classifying a chimney as tall. Today, Reinforced Concrete is the dominant material used for the construction of tall chimneys and for short chimneys precast concrete with or without pre stressing, Modern industrial chimneys consists of a concrete windshield with a number of steel

stacks on the inside. Chimneys can be classified as one of those structures for which use of concrete is eminently used. Tall reinforced concrete (RC) chimneys form an important component of major industries and power plants.

## II. METHODS AND MATERIAL

### A. Beam Column Theory

Timoshenko (1961) described theory of beam-column. Using this theory of Beam column, general equation is developed for analysis of chimney subjected to vertical & lateral load and moment at top.

In the elementary theory of bending, it is found that stresses and deflections in beam are directly proportional to the applied loads. This condition requires that the change in shape of the beam due to bending must not affect the action of the applied loads. For example, the beam in fig (2.1a) is subjected only lateral loads, such as

Q1 and Q2, the presence of small deflections  $\delta_1$  and  $\delta_2$  and slight changes in the vertical lines of action of the loads  $W_1$  have only an insignificant effect on the moments and shear forces. Thus it is possible to make calculation for deflections, stresses, moments on the basis of initial configuration of the beam. Under this conditions, and also if Hook's law holds for the material, the deflections are proportional to the acting forces and the principle of superposition is valid i.e. the final deformations produced by the individual forces.

Conditions are entirely different when both uniaxial and lateral forces act simultaneously on the beam, as in fig (2.1b). The bending moments, shear forces, stresses and deflections in the beam will not be the proportional to the magnitude of the axial load. Furthermore there values will be dependent upon the magnitude of the deflection produced and will be sensitive to the even slight eccentricities in the application of the axial load. Beams subjected to the axial compression and simultaneously supporting lateral loads are known as beam-columns.

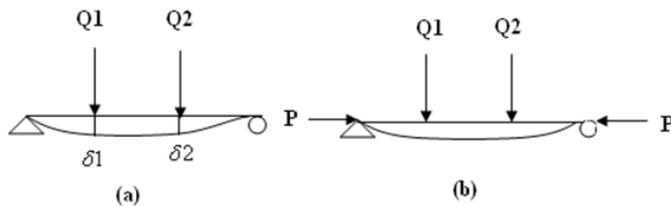
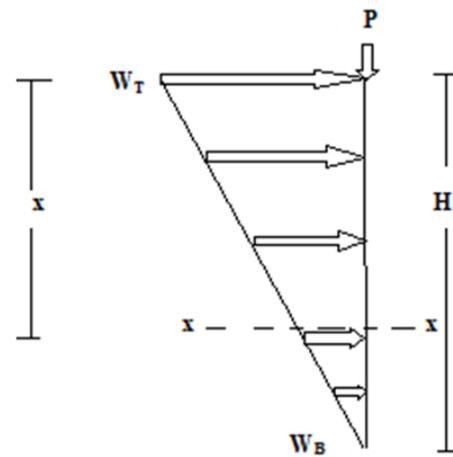


Figure 1: bending of Beam Column

The difference between the behaviour of short and slender columns is that, when slender columns are loaded even with axial loads, the lateral deflection (measured from the original centre line along its length) becomes appreciable where as in short columns this lateral deflection is very small and can be neglected. Hence slender columns, have to be designed for not only the external axial forces acting on them but also for the secondary moment produced by the lateral deflection.

## B. Evaluation of Equations using Beam-Column Theory

### First Order Analysis of Chimney:



$W_T$  = Lateral load intensity at top of chimney  
 $W_B$  = Lateral load intensity at bottom of chimney  
 $H$  = height of chimney  
 $W_x$  = Lateral load intensity at general section 'x' on chimney

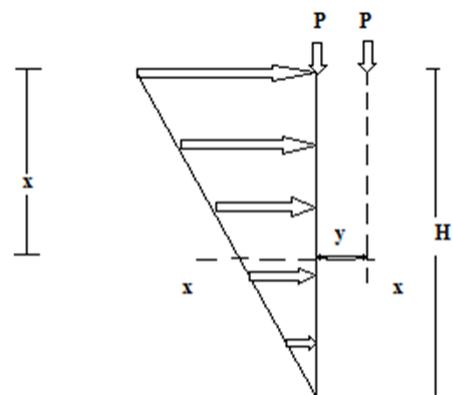
$$W_x = W_T - \left[ \frac{(W_T - W_B)x}{H} \right]$$

Let ' $M_x$ ' be the bending moment at a general section 'x' from top of chimney

$$\therefore M_x = -\frac{W_T x^2}{2} + \left( \frac{W_T - W_B x^3}{6H} \right) \dots \dots \dots 1.1$$

This is the equation for Bending Moment of Chimney at any height 'x'

### Second Order Analysis of Chimney:



$W_T$  = Lateral load intensity at top of chimney  
 $W_B$  = Lateral load intensity at bottom of chimney  
 $H$  = height of chimney  
 $W_x$  = Lateral load intensity at general section 'x' on chimney

$$W_x = W_T - \left[ \frac{(W_T - W_B)x}{H} \right]$$

$$\text{Put, } k_w = \frac{(W_T - W_B)}{H}$$

$$\therefore W_x = W_T - k_w x$$

$\therefore$  Bending moment at a general section 'x' is given by

$$M_x = Py - \frac{W_T x^2}{2} + \frac{k_w x^3}{6} \dots\dots 2.1$$

Now we have,  $EI_x \frac{\partial^2 y}{\partial x^2} = -M$

$$\therefore EI_x \frac{\partial^2 y}{\partial x^2} = -Py + \frac{W_T x^2}{2} - \frac{k_w x^3}{6}$$

$$\therefore EI_x \frac{\partial^2 y}{\partial x^2} + Py = \frac{W_T x^2}{2} - \frac{k_w x^3}{6} \dots\dots 2.2$$

For a circular hollow section the moment of inertia is given by,

$$I_x = \frac{\pi}{64} (d_1^4 - d_2^4)$$

And for a chimney the moment of inertia varies according to the height and is different for different section and depends upon cross section

On solving above equation we get,

$$y = \text{complementary solution} + \text{particular solution}$$

Where,

$$y_c = A \sin(\alpha x) + B \cos(\alpha x)$$

Where,

$$\alpha = \sqrt{\frac{P}{EI_x}}$$

$$\text{And, } y_p = ax^3 + bx^2 + cx = d$$

On solving above equation we get,

$$y_p = -\frac{k_w x^3}{6P} + \frac{W_T x^2}{2P} + 0x = ax^3 + bx^2 + cx$$

Now equating according to power of 'x', we get complete solution

$$y = y_c + y_p$$

$$\therefore y = A \sin(\alpha x) + B \cos(\alpha x) - \frac{k_w x^3}{6P} + \frac{W_T x^2}{2P}$$

...2.3

Taking derivative with respect to 'x', we get

$$\frac{\partial y}{\partial x} = A \alpha \cos \alpha x - B \alpha \sin \alpha x - \frac{k_w x^2}{2P} + \frac{W_T x}{P}$$

Taking derivative of equation 2.3 with respect to 'x', we get

$$\frac{\partial^2 y}{\partial x^2} = -A \alpha^2 \sin \alpha x - B \alpha^2 \cos \alpha x - \frac{k_w x}{P} + \frac{W_T}{P}$$

...2.5

Now using boundary condition, when  $x=0$ ,  $\frac{\partial^2 y}{\partial x^2} = 0$

Equation becomes

$$0 = -B \alpha^2 \cos \alpha x - \frac{k_w x}{P} + \frac{W_T}{P}$$

$$\therefore B = \frac{-\frac{k_w x}{P} + \frac{W_T}{P}}{\alpha^2 \cos \alpha x} \dots\dots\dots 2.6$$

Now using boundary condition, when  $x=H$ ,  $\frac{\partial y}{\partial x} = 0$

$$\text{We get, } 0 = A \alpha \cos \alpha x - B \alpha \sin \alpha H - \frac{k_w H^2}{2P} + \frac{W_T H}{P}$$

$$\therefore A = \frac{B \alpha \sin \alpha H + \frac{k_w H^2}{2P} - \frac{W_T H}{P}}{\alpha \cos \alpha H}$$

Now substitute the values of A and B in equation 2.3

$$\therefore \text{We get, } y = \left\{ \frac{B \alpha \sin \alpha H + \frac{k_w H^2}{2P} - \frac{W_T H}{P}}{\alpha \cos \alpha H} \sin \alpha x \right\} +$$

$$\left\{ \left[ \frac{-\frac{k_w x}{P} + \frac{W_T}{P}}{\alpha^2 \cos \alpha x} \right] \cos \alpha x \right\} - \frac{k_w x^3}{6P} + \frac{W_T x^2}{2P}$$

This is the equation of Deflection for Second Order Analysis of Chimney at any height 'x'

Now substituting the value of 'y' in equation 2.1

We get,

$$M_x = \left\{ P \left( \left\{ \frac{B \alpha \sin \alpha H + \frac{k_w H^2}{2P} - \frac{W_T H}{P}}{\alpha \cos \alpha H} \sin \alpha x \right\} + \left\{ \left[ \frac{-\frac{k_w x}{P} + \frac{W_T}{P}}{\alpha^2 \cos \alpha x} \right] \cos \alpha x \right\} - \frac{k_w x^3}{6P} + \frac{W_T x^2}{2P} \right) \right\} - \frac{W_T x^2}{2} - \frac{k_w x^3}{6}$$

This is the equation of Bending Moment for Second Order Analysis of Chimney at any height 'x'

### C. Problem Validation

#### Description of Loading

Details of the parameters are as follows

1. Height of the chimney - 270 m
2. Outer diameter of bottom - 36.6 m
3. Outer diameter at top - 19.6 m
4. Thickness of shell - 0.8 m
5. Grade of concrete - M25
6. Seismic zone - III

- 7. Basic wind speed - 39 m/sec (for Solapur)
- 8. Foundation type - RCC circular mat
- 9. Density of concrete - 1 kN/m<sup>3</sup>
- 10. Design life of structure - 100 yrs
- 11. Terrain category - 4
- 12. Probability factor - 1.06
- 13. Topography factor - 1
- 14. Class of structure - Class C

$z$  = Height of any section of the chimney in m measured from the top of foundation

$C_D$  = Drag coefficient of the chimney to be taken as 0.8

$d_z$  = Diameter of chimney at height  $z$  in m

∴ Wind load at top of chimney = 21.784kN/m

∴ Wind load at bottom of chimney = 0kN/m

### III. RESULTS AND DISCUSSION

#### A. Dead Load Calculations

#### Wind Analysis

Along wind loads are caused by the ‘drag’ component of the wind force on the chimney. This is accompanied by ‘gust buffeting’ causing a dynamic response in the direction of the mean flow. In the present analysis the across wind effect which is due to the dynamic response of chimney is neglected. Along wind effect is due to the direct buffeting action, when the wind acts in the face of the structure. For the purpose of estimation if this loads the chimney is modelled to act on the exposed face of the chimney causing predominant moments in the chimney.

Two methods of estimating of wind loads are given in IS 4998 (part 1)

1. Simplified Method
2. Random Response Method

Simplified method will be used for the estimation of wind loads

#### Simplified Method

The along wind load or the drag force per unit height of the Chimney at any level shall be calculated from the equation:

$$F_z = p_z \cdot C_D \cdot d_z$$

Where,

$p_z$  = Design wind pressure obtained in accordance with IS 875 (Part 3): 1987

Dead load calculations are carried out by Frustum of cone method. Dead loads for each 20 m interval of distance of chimney is calculated by developing a programme in excel. The results for the same are displayed below

TABLE 1

Height(m)	Dead Load (kN)
0	463046.2
20	418695.3
40	376097.1
60	335081.9
80	295649.8
100	257800.7
120	221629
140	186851.8
160	116035
180	122291.7
200	92345.4
220	63982.17
240	37202
260	12004.9
270	0

#### B. Results By Beam-Column Theory

TABLE 2

Height(m)	BM1(KN.m)	BM2(kN.m)	y(mm)
0	530250.3	530250.3	0
20	471441.4	472337.8	2.141
40	413279	414882.3	4.263
60	356409.7	358818.2	7.188
80	301480	304313.5	9.584

100	249136.6	252225	11.98
120	200025.8	203212	14.376
140	154794.4	157928.3	16.772
160	114088.8	116313	19.168
180	78555.6	81192.7	21.564
200	48841.32	51053.83	23.959
220	25592.53	27278.78	26.355
240	9455.766	10525.36	28.751
260	1077.58	1451.496	31.147
270	0	0	32.345

BM1- First order bending moment  
 BM2- Second order bending moment  
 y- deflection

### C. Validation In STAAD PRO

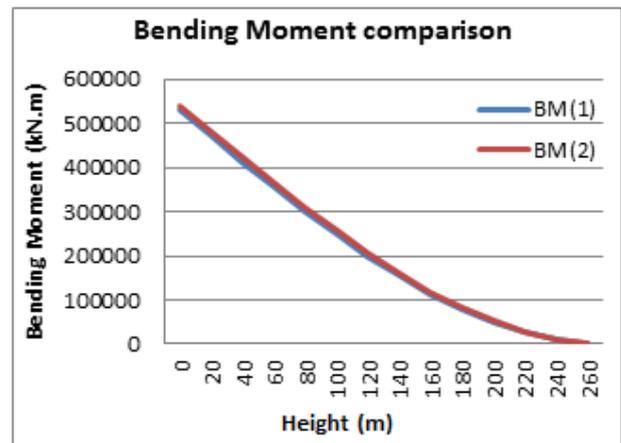
Same problem is now used for validation in software. A column of 270 m is modelled and dead load and wind load is applied. Following results are obtained.

TABLE 3

Height (m)	BM1(kN.m)	BM2(kN.m)	y(mm)
0	529351.2	540370.7	0
20	470642	480845.2	2.396
40	412578.2	421965.2	4.792
60	355805.3	364376.1	7.188
80	300968.8	308723.3	9.584
100	248714.1	255652.3	11.98
120	199686.6	205808.6	14.376
140	154531.9	159837.6	16.772
160	113895.3	118384.8	19.168
180	78422.37	82095.58	21.564
200	48758.48	51615.43	23.959
220	25549.11	27589.8	26.355
240	9439.703	10664.15	28.751
260	1075.722	1483.909	31.147
270	0	0	32.345

BM1- First order bending moment  
 BM2- Second order bending moment  
 y- Deflection

The results obtained manually by beam-column theory and by Staad pro match. The following graph shows the variation of bending moments manually and in software.



### D. Percentage Increase in Second Order Analysis

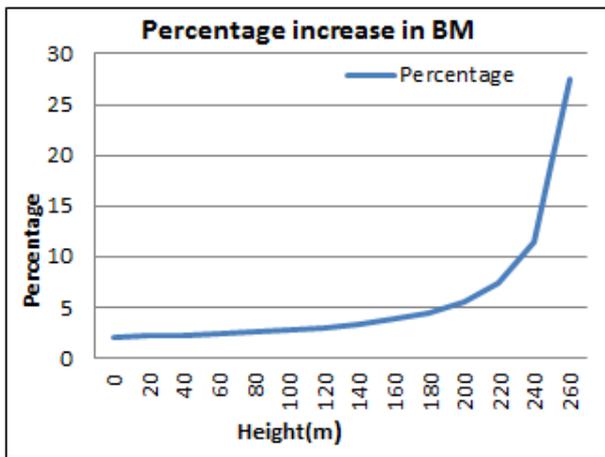
After getting the bending moments manually by beam-column theory and by Staad pro, now we will analyze the percentage increase in bending moment.

TABLE 4

Height (m)	BM1 (kN.m)	BM2 (kN.m)	Percentage increase
0	529351.188	540370.688	2.03924829
20	470641.961	480845.206	2.12193963
40	412578.186	421965.177	2.22458902
60	355805.316	364376.05	2.35216722
80	300968.8	308723.281	2.51179016
100	248714.093	255652.318	2.71393002
120	199686.644	205808.615	2.97459414
140	154531.907	159837.623	3.31944126
160	113895.332	118384.79	3.79225912
180	78422.373	82095.579	4.47430427
200	48758.481	51615.432	5.53507137
220	25549.107	27589.803	7.39655879
240	9439.703	10664.145	11.4818581
260	1075.722	1483.909	27.5075493

BM1- First order bending moment  
 BM2- Second order bending moment

The above results are now shown graphically below.



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#### IV. CONCLUSION

From the analysis carried out manually by using beam-column theory and later by Staad pro it can be concluded that, for the combination of wind load and dead load, the effect of second order increases with increase in height. The graph of percentage increase in bending moment clearly indicated that even when the bending moment went on decreasing with increasing height, the deflection goes on increasing and the percentage of second order effect also goes on increasing.

#### V. REFERENCES

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#### VI. BIOGRAPHIES



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