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Large displacement stability analysis of steel structure of nonlinearly varying section (convex) resting on elastic foundation

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ABSTRACT

In this research theoretical analysis is presented to estimate in a plan large displacement elastic stability behavior of frames having non prismatic number of non-linearly varying section resting on elastic foundation (Winkler type) using the non-prismatic segment .in the segmentation method. The stability and bowing function are estimated using the method of finite segment.

Keywords : Convex, Steel Structure, Winkler Type, Segmentation Method, Non-Prismatic Segment, Stiffness Factor, Moment Carry-Over Factor, Sway Moment Factor, Shear Factor

I. INTRODUCTION

In the present research the geometric non linearly of the framed structure is considered the effect of geometric nonlinear may be shown in large displacement problem this type of non-linearity comes from the interaction effect of the axial force bending moment the member and moments U_1, V_1 and V_2 of the prismatic member are expressed in terms of member and rotation Q1, Q2 and deflections Y1 and Y2 and relative axial displacement (U) as follows [(2)] for beam an elastic formulation.

$$M_1=\frac{EI}{L}(S\Theta_1+SC\Theta_2+\frac{Qy_1}{L}-\frac{qQy_2}{L})$$
$$M_1=\frac{EI}{L}(SC\Theta_1+S\Theta_2+\frac{qQy_1}{L}-\frac{Qy_2}{L})$$
$$V_1=\frac{EI}{L^2}(Q\Theta_1+qQ\Theta_2+\frac{Ty_1}{L}-\frac{tTy_2}{L})$$
$$V_2=\frac{EI}{L^2}(qQ\Theta_1+Q\Theta_2+\frac{tTy_1}{L}-\frac{Ty_2}{L})$$

Where:

S is the stiffness factor

SC moment carry-over factor

Q sway moment factor

qQ sway moment carry-over factor

T shear factor

tT shear carry-over factor

Finite segment method may be considered as physical inter pretention of the finite difference method that can be applied numerically to solve differential equations. The beam column problem can be formulation and solved approximately in terms behavior of these segments without recovers to complex differential equations.

The non-prismatic member on elastic foundation is divided into (n) a prismatic member as shown in fig (1-1) below.

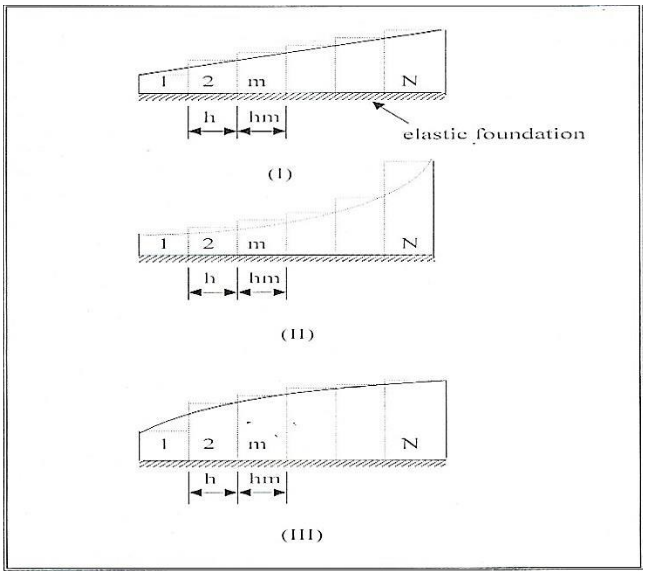

Fig.(1-1) Member segment for non-prismatic beam-column on the elastic foundation: (I) Tapered (II) Concave, and (III) Convex

II. OBJECTIVE OF THIS RESEARCH:

the objective of this study is to present theoretical basis for the large displacement elastic stability analysis of plane frame with non-prismatic members resting on elastic foundation, using the non-prismatic segment (tapered segments), in finding the stability functions.

III. REVIEW OF LITERATURE

In 1978, **Al-Sarraf** [2] derive modified stability functions by using modified slope-deflection equations for a uniform beam-column supported on or driven into continues Winkler foundation.

In 1997 **Al-Hachami**[1] presented a theatrical analysis for estimating in plane and in space large displacement elastic stability behavior structures subjected to either proportional or non-proportional increasing static loads , the analysis adopted the beam-column approach , large displacement analysis of beam-column resting on or driven into elastic foundation was presented.

In 2002, **Faris**[3] submitted a theatrical analysis for estimating the in-plane large displacement elastic-plastic stability behavior of steel frames having non-prismatic members of non-linearly varying sections. The stability and bowing functions were estimated using four methods, finite differences, finite element, finite segments and exact solution by using Bessel functions. Also, approximate results had been obtained by using approximate stability and bowing functions for nonlinearly tapered members.

In 2004, **Ahmed Tariq, Faris, H.A., N.Al-jumaily, Ibrahim**[4] , present a theoretical basis for the large displacement elastic stability analysis of plane frames with non-prismatic members resting on elastic foundation.

IV. Modeling of Subgrade Reaction

Large displacement analysis of beam-column on elastic foundation can be represented by two approaches[1].

In the first approach, the foundation is represented by isolated springs at the nodes of the beam-column.

In the second approach, foundation medium is assumed to be of Winkler type, i.e., the beam-column elements rest on distributed springs, Fig.(1-2) This study deals with the second approach in the solution of the geometric nonlinearity problems.

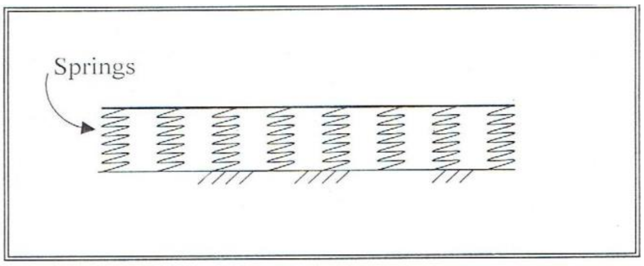

Fig (1-2) soil subgrade reaction of beam-column on elastic foundation

V. Stability and Bowing Functions

The relationship between the relative deformations Θ_1, Θ_2, y_1, and Θ_2, and the associated member end forces M_1, M_2, V_1, and V_2 can be written as follows:

$$M_1=\frac{EI}{L}\left(S\Theta_1 + SC\Theta_2 + \frac{Qy_1}{L} - \frac{qQy_2}{L}\right) \quad ....(1\text{-}1)$$

$$M_2 = \frac{EI}{L}\left(SC\Theta_1 + S\Theta_2 + \frac{qQy_1}{L} - \frac{Qy_2}{L}\right) \quad ....(1\text{-}2)$$

$$V_1 = \frac{EI}{L^2}\left(Q\Theta_1 + qQ\Theta_2 + \frac{Ty_1}{L} - \frac{tTy_2}{L}\right) \quad ....(1\text{-}3)$$

$$V_2 = \frac{EI}{L^2}\left(qQ\Theta_1 + Q\Theta_2 + \frac{tTy_1}{L} - \frac{Ty_2}{L}\right) \quad ....(1\text{-}4)$$

Where:

S is the stiffness factor

SC moment carry-over factor

Q sway moment factor

qQ sway moment carry-over factor

T shear factor

tT shear carry-over factor

and the relation between the axial deformation u and the axial force P can be expressed according to Oran[5] :

$$p= \frac{EA}{L}\left(u - C_b L\right) \quad ....(1\text{-}5)$$

where the C_b is the length correction factor due to bowing.

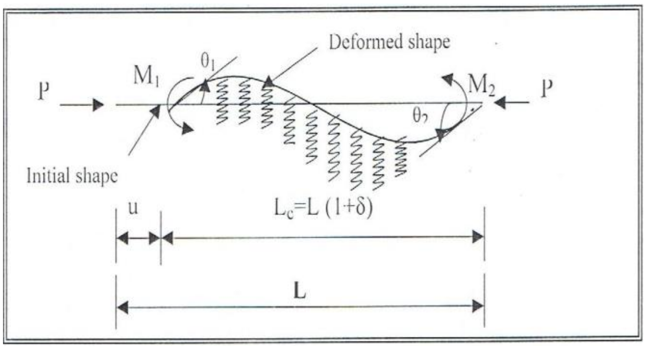

Fig (1-3) member forces and deformation in local coordinate

The stability function (S, SC, Q, qQ, T, and tT) and bowing function ($b_1 and\ b_2$) for the prismatic beam on elastic foundation may derived depending on the force affected and the parameters $[(\Psi^2/2)^2\text{-}4(\lambda L)^4]$[6] where $\Psi=\pi^2 p$.

(i) Case-1: compressive axial force

- Case-1 (a): $[(\Psi^2/2)^2\text{-}4(\lambda L)^4] >= 0$

$$S= (W^2\text{-}N^2)\frac{W\sin N\cos W - N\cos N\sin W}{(W^2+N^2)\sin N \sin W + 2NW(\cos W\cos N - 1)} \quad ....(1\text{-}6)$$

$$SC= (W^2\text{-}N^2)\frac{N\sin W - W\sin N}{(W^2+N^2)\sin N\sin W + 2NW(\cos W\cos N - 1)} \quad .....(1\text{-}7)$$

$$Q= \frac{2W^2N^2\sin N\sin W + WN\ (W2+N2)(\cos W\cos N - 1)}{(W^2+N^2)\sin N\sin W + 2NW(\cos W\cos N - 1)} \quad ....(1\text{-}8)$$

$$q\,Q= \frac{WN\left(W^2-N^2\right)(\cos W - \cos N)}{(W^2+N^2)\sin N\sin W + 2NW(\cos W\cos N - 1)} \quad ....(1\text{-}9)$$

$$T= \frac{WN\left(W^2-N^2\right)(W\sin W\cos N - N\sin N\cos W)}{(W^2+N^2)\sin N\sin W + 2NW(\cos W\cos N - 1)} \quad ....(1\text{-}10)$$

$$tT= \frac{WN\left(W^2-N^2\right)(N\sin N - W\sin W)}{(W^2+N^2)\sin N\sin W + 2NW(\cos W\cos N - 1)} \quad ....(1\text{-}11)$$

where

$$W^2=(\Psi^2/2)^2\text{-}[(\Psi^2/2)^2\text{-}4(\lambda L)^4]^{1/2} \quad ....(1\text{-}12)$$

$$N^2=(\Psi^2/2)^2\text{+}[(\Psi^2/2)^2\text{-}4(\lambda L)^4]^{1/2} \quad ....(1\text{-}13)$$

And the relationships of bowing functions b_1 and b_2 for a prismatic beam on elastic foundation also depend on the parameter $[(\Psi^2/2)^2\text{-}4(\lambda L)^4]$, which are

$$b_1=\left[\frac{S+SC}{2(W^2-N^2)}\right]^2 \left(\begin{array}{c} \frac{(W^2+W\sin W\cos W)}{\sin^2 W} + \frac{(N^2+N\sin N\cos N)}{\sin^2 N} \\ + \frac{4WN^2\cos W}{(W^2-N^2)\sin W} + \frac{4W^2N\cos N}{(W^2-N^2)\sin N} \\ + \frac{W(\cos W-1)}{2\sin^2 W}\left\{W - \sin W - \frac{4N^2\sin W}{(W^2-N^2)}\right\} \\ + \frac{N(\cos N-1)}{2\sin^2 N}\left\{N - \sin N - \frac{4W^2\sin N}{(W^2-N^2)}\right\} \end{array}\right) \quad ...(1\text{-}14)$$

$b_1=\left[\frac{S-SC}{2(W^2-N^2)}\right]^2$

$$\begin{pmatrix} \frac{W(\cos W-1)}{2sin^2W}\left\{W-\sin W-\frac{4N^2\sin W}{(W^2-N^2)}\right\} \\ +\frac{N(\cos N-1)}{2sin^2N}\left\{N-\sin N+\frac{4W^2\sin N}{(W^2-N^2)}\right\}\end{pmatrix} \quad ....(1\text{-}15)$$

- Case-1 (b): $[(\Psi^2/_2)^2 - 4(\lambda L)^4] < 0$

$S = -(\lambda L)\sin d$

$$\frac{\sin B\cos B\sin(d/2)-\sinh A cosh\,A\cos(d/2)}{sinh^2A\,cos^2(d/2)-sin^2\,B\;sin^2(d/2)} \quad ....(1\text{-}16)$$

$SC = \sqrt{2}\,(\lambda L)\text{sind}$

$$\frac{\sin B\cosh A\sin(d/2)-\sinh A cosB\cos(d/2)}{sinh^2A\,cos^2(d/2)-sin^2\,B\;sin^2(d/2)} \quad ....(1\text{-}17)$$

$Q = 2(\lambda L)^2\sin d$

$$\frac{sin^2\,B\;sin^2(d/2)+sinh^2A\,cos^2(d/2)}{sinh^2A\,cos^2(d/2)-sin^2\,B\;sin^2(d/2)} \quad .....(1\text{-}18)$$

$qQ = 2\,(\lambda L)^2\sin d$

$$\frac{\sin d\sinh B\sin A}{sinh^2A\,cos^2(d/2)-sin^2\,B\;sin^2(d/2)} \quad ....(1\text{-}19)$$

$T = 2\sqrt{2}(\lambda L)^3\sin(d/2)$

$$\frac{2\sinh A\cosh A\;cos^2(d/2)+\sin B\cos B\sin d}{sinh^2A\,cos^2(d/2)-sin^2\,B\;sin^2(d/2)} \quad ....(1\text{-}20)$$

$tT = 2\sqrt{2}(\lambda L)^3\sin d$

$$\frac{\sinh A\cos B\;cos(d/2)+\cosh A\sin B\sin(d/2)}{sinh^2A\,cos^2(d/2)-sin^2\,B\;sin^2(d/2)} \quad .....(1\text{-}21)$$

$$A=\sqrt{2}(\lambda L)\sin(d/2) \quad ....(1\text{-}22)$$

$$B=\sqrt{2}(\lambda L)\cos(d/2) \quad ......(1\text{-}23)$$

$$d= Cos^{-1}\{\Psi^2/[4\,(\lambda L)^2]\} \quad .....(1\text{-}24)$$

$$b_1=\frac{K^2(S+SC)^2}{4}(f_1+f_2+f_3) \quad .....(1\text{-}25)$$

$$b_2=\frac{K^2(S-SC)^2}{4}(f_1-f_2+f_3) \quad .....(1\text{-}26)$$

where

$f_1= f_0\,(f_4+f_5+f_6+f_7+f_8+f_9+f_{10}+f_{12}+f_{13})$

$f_2= f_0\,(f_{14}+f_{15}+f_{16}+f_{17}+f_{18}+f_{19}+f_{20})$

$f_3= f_0\,(f_{21}+f_{22}+f_{23})$

in which

$$f_0=\frac{1}{32K^2\,(AB)^2(sinh^2\,A+sin^2\,B)} \quad ....(1\text{-}27)$$

K=EI/L

$f_4= (1/4)\,(B\sinh 2A - A\sin 2B)^2$

$$\begin{bmatrix} 1+\frac{\sinh 2A}{2A}+\frac{\sin 2B}{2B} \\ +\frac{(A\sinh 2A\cos 2B+B\cosh 2A\sin 2B)}{2\,(A^2+B^2)}\end{bmatrix}$$

$f_5= (1/4)\,(A\sinh 2A + B\sin 2B)^2$

$$\begin{bmatrix} -1+\frac{\sinh 2A}{2A}+\frac{\sin 2B}{2B} \\ -\frac{(A\sinh 2A\cos 2B+B\cosh 2A\sin 2B)}{2\,(A^2+B^2)}\end{bmatrix}$$

$f_6=-$

$$B^2(\,sinh^2\,A + sin^2\,B\,)^2\begin{bmatrix} -1+\frac{\sinh 2A}{2A}+\frac{\sin 2B}{2B} \\ +\frac{(A\sinh 2A\cos 2B+B\cosh 2A\sin 2B)}{2\,(A^2+B^2)}\end{bmatrix}$$

$f_7=-$

$$A^2(\,sinh^2\,A + sin^2\,B\,)^2\begin{bmatrix} 1+\frac{\sinh 2A}{2A}-\frac{\sin 2B}{2B} \\ -\frac{(A\sinh 2A\cos 2B+B\cosh 2A\sin 2B)}{2\,(A^2+B^2)}\end{bmatrix}$$

$f_8= (1/2)(B\sinh 2A - A\sin 2B)(A\sinh 2A + B\sin 2B)(A\cosh 2A\sin 2B - B\sinh 2A\cos 2B) / 2(A^2 + B^2)$

$f_9= -B(\sinh^2 A + \sin^2 B)(B\sinh 2A - A\sin 2B)$

$$\left[\frac{\cosh 2A-1}{2A}+\frac{(A\cosh 2A\sin 2B+B\sinh 2A\cos 2B-A)}{2(A^2+B^2)}\right]$$

$f_{10}= -A(\sinh^2 A + \sin^2 B)(B\sinh 2A - A\sin 2B)$

$$\left[\frac{\cos 2B+1}{2B}+\frac{(A\sinh 2A\sin 2B-B\,\text{coch2A}\,cos2B+B)}{2(A^2+B^2)}\right]$$

$f_{11}= -B(\sinh^2 A + \sin^2 B)(A\sinh 2A + B\sin 2B)$

$$\left[\frac{\cos 2B-1}{2B}+\frac{(A\sinh 2A\sin 2B-B\,\text{coch2A}\,cos2B+B)}{2(A^2+B^2)}\right]$$

$f_{12}= -A(\sinh^2 A + \sin^2 B)(A\sinh 2A + B\sin 2B)$

$$\left[\frac{\cosh 2A-1}{2A}-\frac{(A\cosh 2A\cos 2B+B\sinh sin2B-A)}{2(A^2+B^2)}\right]$$

$f_{13}=-AB(\sinh^2A+\sin^2B)\left[\frac{(A\cosh 2A\sin 2B-B\,\text{sinh2A}\,cos2B)}{2(A^2+B^2)}\right]$

$f_{14}= (B\sinh 2A - A\sin 2B)(B\sinh A\cos B - A\sin B\cosh A)^*\left[1+\frac{\sinh 2A}{2A}+\frac{\sin 2B}{2B}+\frac{(A\sinh 2A\cos 2B+B\,\text{cosh2A}\;sin2B)}{2(A^2+B^2)}\right]$

$f_{15}= (A\sinh 2A + B\sin 2B)(A\sinh A\cos B + B\sin B\cosh A)^*$

$$\left[-1+\frac{\sinh 2A}{2A}+\frac{\sin 2B}{2B}-\frac{(A\sinh 2A\cos 2B+B\,\text{cosh2A}\;sin2B)}{2(A^2+B^2)}\right]$$

f_{16}= [2(A+B) sinh A cos B -2(A-B) sin B cosh A] $\left[\frac{(A\cosh 2A \sin 2B - B \sinh 2A\ \cos 2B)}{2(A^2+B^2)}\right]$

f_{17}= -2B($\sinh^2$ A + $\sin^2$ B)(B sinh A cos B - A sin B cosh A) $\left[\frac{\cos 2A-1}{2A} + \frac{(A\cosh 2A \sin 2B + B \sinh 2A \cos 2B - A)}{2(A^2+B^2)}\right]$

f_{18}= -2A($\sinh^2$ A + $\sin^2$ B)(B sinh A cos B - A sin B cosh A) $\left[\frac{\cos 2B+1}{2B} + \frac{(A\sinh 2A \sin 2B - B \cosh 2A \cos 2B + B)}{2(A^2+B^2)}\right]$

f_{19}= -2B($\sinh^2$ A + $\sin^2$ B)(A sinh A cos B + B sin B cosh A) $\left[\frac{\cos 2B-1}{2B} + \frac{(A\sinh 2A \sin 2B - B \cosh 2A \cos 2B + B)}{2(A^2+B^2)}\right]$

f_{20}= -2A($\sinh^2$ A + $\sin^2$ B)(A sinh A cos B + B sin B cosh A) $\left[\frac{\cosh 2A-1}{2A} - \frac{(A\cosh 2A \cos 2B + B \sinh 2A \sin 2B - A)}{2(A^2+B^2)}\right]$

$$f_{21} = (B \sinh A \cos B - A \sin B \cosh A)^{2*} \left[\begin{array}{c} 1 + \frac{\sinh 2A}{2A} + \frac{\sin 2B}{2B} \\ + \frac{(A \sinh 2A \cos 2B + B \cosh 2A \sin 2B)}{2(A^2+B^2)} \end{array}\right]$$

$$f_{22} = (A \sinh A \cos B + B \sin B \cosh A)^{2*} \left[\begin{array}{c} -1 + \frac{\sinh 2A}{2A} + \frac{\sin 2B}{2B} \\ - \frac{(A \sinh 2A \cos 2B + B \cosh 2A \sin 2B)}{2(A^2+B^2)} \end{array}\right]$$

f_{23}= 2(B sinh A cos B – A sin B cosh A)(A sinh A cos B + B sin B cosh A)* $\left[\frac{(A\cosh 2A \sin 2B - B \sinh 2A\ \cos 2B)}{2(A^2+B^2)}\right]$ .....(1-28)

(ii) case-2: tensile axial force

case-2 (a): if $[(\Psi^2/2)^2 - 4(\lambda L)^4] >= 0$

$S=(W^2-N^2)\frac{W \sinh N \cosh W - N \sinh W \cosh N}{(W^2+N^2)\sinh W \sinh N - 2NW(\cosh W \cosh N - 1)}$ ...(1-29)

$SC=(W^2-N^2)\frac{N \sinh W - W \sinh N}{(W^2+N^2)\sinh W \sinh N - 2NW(\cosh W \cosh N - 1)}$ .....(1-30)

$Q=\frac{2\,WN(W^2+N^2)[(\cosh W \cosh N - 1) - 2\,W^2N^2 \sinh W \sinh N}{(W^2+N^2)\sin N \sin W + 2NW(\cos W \cos N - 1)}$ .....(1-31)

$qQ=\frac{WN(W^2-N^2)(\cosh W - \cosh N)}{(W^2+N^2)\sin N \sin W + 2NW(\cos W \cos N - 1)}$ ....(1-32)

$T=\frac{WN(N^2-W^2)(N \sinh N \cosh W - W \sinh W \cosh N)}{(W^2+N^2)\sin N \sin W + 2NW(\cos W \cos N - 1)}$ .....(1-33)

$tT=\frac{WN(N^2-W^2)(N \sinh N - W \sinh W)}{(W^2+N^2)\sin N \sin W + 2NW(\cos W \cos N - 1)}$ .....(1-34)

the relation of bowing function b_1 and b_2 for a prismatic beam column on elastic foundation also depend on the axial force and the parameter $[(\Psi^2/2)^2 - 4(\lambda L)^4]$ which are

$$b_1 = \left[\frac{S+SC}{2(W^2-N^2)}\right]^2 \left(\begin{array}{c} \frac{(W^2 + W \sinh W \cosh W)}{\sinh^2 W} + \frac{(N^2 + N \sinh N \cosh N)}{\sinh^2 N} \\ + \frac{4WN^2 \cosh W}{(W^2-N^2)\sinh W} + \frac{4W^2 N \cosh N}{(W^2-N^2)\sinh N} \\ + \frac{W(\cosh W - 1)}{2\sin^2 W}\left\{W - \sinh W - \frac{4N^2 \sinh W}{(W^2-N^2)}\right\} \\ + \frac{N(\cosh N - 1)}{2\sinh^2 N}\left\{N - \sinh N + \frac{4W^2 \sinh N}{(W^2-N^2)}\right\} \end{array}\right)$$

....(1-35)

$$b_1 = -\left[\frac{S-SC}{2(W^2-N^2)}\right]^2 \left(\begin{array}{c} \frac{W(\cosh W - 1)}{2\sinh^2 W}\left\{W - \sinh W - \frac{4N^2 \sinh W}{(W^2-N^2)}\right\} \\ + \frac{N(\cosh N - 1)}{2\sinh^2 N}\left\{N - \sinh N + \frac{4W^2 \sinh N}{(W^2-N^2)}\right\} \end{array}\right) ..(1\text{-}36)$$

- Case-2 (b): $[(\Psi^2/2)^2 - 4(\lambda L)^4] < 0$

S = $-\sqrt{2}$ (λL) sin d $\frac{\sin A \cos A \cos(d/2) - \sinh B \cosh B \sin(d/2)}{\sinh^2 B \sin^2(d/2) - \sin^2 A\ \cos^2(d/2)}$ ...(1-37)

SC = $\sqrt{2}$ (λL) sin d $\frac{\cosh B \sin A \cos(d/2) - \sinh B \cos A \sin(d/2)}{\sinh^2 B \sin^2(d/2) - \sin^2 A\ \cos^2(d/2)}$ .....(1-38)

Q = $2(\lambda L)^2$ sin d $\frac{\sin^2 A\ \cos^2(d/2) + \sinh^2 B \sin^2(d/2)}{\sinh^2 B \sin^2(d/2) - \sin^2 A\ \cos^2(d/2)}$ ......(1-39)

qQ = 2 $(\lambda L)^2$ sin d $\frac{\sin d \sinh B \sin A}{\sinh^2 B \sin^2(d/2) - \sin^2 A\ \cos^2(d/2)}$ .....(1-40)

$T = 2\sqrt{2}(\lambda L)^3 \cos(d/2)$
$\frac{2\sinh B\cosh B\ \sin^2(d/2)+\sin A\cos A\sin d}{\sinh^2 B\sin^2(d/2)-\sin^2 A\ \cos^2(d/2)}$ …..(1-41)

$tT = 2\sqrt{2}(\lambda L)^3 \sin d$
$\frac{\sinh A\cos B\ \cos(d/2)+\cosh A\sin B\sin(d/2)}{\sinh^2 A\cos^2(d/2)-\sin^2 B\ \sin^2(d/2)}$ ….(1-42)

$b_1 = \frac{K^2(S+SC)^2}{4}(f_1+f_2+f_3)$ ……(1-43)

$b_2 = \frac{K^2(S-SC)^2}{4}(f_1-f_2+f_3)$ ….(1-44)

where

$f_1 = f_0(f_4+f_5+f_6+f_7+f_8+f_9+f_{10}+f_{12}+f_{13})$

$f_2 = f_0(f_{14}+f_{15}+f_{16}+f_{17}+f_{18}+f_{19}+f_{20})$

$f_3 = f_0(f_{21}+f_{22}+f_{23})$

in which

$f_0 = \frac{1}{32K^2(AB)^2(\sinh^2 B+\sin^2 A)}$ …..(1-45)

K=EI/L

$f_4 = (1/4)(A\sinh 2B - B\sin 2A)^2 \left[1+\frac{\sinh 2B}{2B}+\frac{\sin 2A}{2A}+\frac{(A\sinh 2B\cos 2A+A\cosh 2B\sin 2A)}{2(A^2+B^2)}\right]$

$f_5 = (1/4)(B\sinh 2B + A\sin 2A)^2 \left[-1+\frac{\sinh 2B}{2B}+\frac{\sin 2A}{2A}-\frac{(B\sinh 2B\cos 2A+A\cosh 2B\sin 2A)}{2(A^2+B^2)}\right]$

$f_6 = -A^2(\sinh^2 B + \sin^2 B)^2\left[-1+\frac{\sinh 2B}{2B}-\frac{\sin 2A}{2A}+\frac{(B\sinh 2B\cos 2A+A\cosh 2B\sin 2A)}{2(A^2+B^2)}\right]$

$f_7 = -B^2(\sinh^2 B + \sin^2 A)^2\left[1+\frac{\sinh 2B}{2B}-\frac{\sin 2A}{2A}-\frac{(B\sinh 2B\cos 2A+A\cosh 2B\sin 2A)}{2(A^2+B^2)}\right]$

$f_8 = (1/2)(A\sinh 2B - B\sin 2A)(B\sinh 2B + A\sin 2A)(B\cosh 2B\sin 2A - A\sinh 2B\cos 2A)/2(A^2+B^2)$

$f_9 = -A(\sinh^2 B + \sin^2 B)(A\sinh 2B - B\sin 2A)\left[\frac{\cosh 2B-1}{2B}+\frac{(B\cosh 2B\sin 2A+A\sinh 2B\cos 2A-B)}{2(A^2+B^2)}\right]$

$f_{10} = -B(\sinh^2 B + \sin^2 A)(A\sinh 2B - B\sin 2A)\left[\frac{\cos 2A+1}{2A}+\frac{(B\sinh 2B\sin 2A-A\,\mathrm{coch}2B\cos 2A+A)}{2(A^2+B^2)}\right]$

$f_{11} = -A(\sinh^2 B + \sin^2 A)(B\sinh 2B + A\sin 2A)\left[\frac{\cos 2A-1}{2A}+\frac{(B\sinh 2B\sin 2A-A\,\mathrm{coch}2B\cos 2A+A)}{2(A^2+B^2)}\right]$

$f_{12} = -B(\sinh^2 B + \sin^2 A)(B\sinh 2B + A\sin 2A)\left[\frac{\cosh 2B-1}{2B}-\frac{(B\cosh 2B\cos 2A+A\sinh 2B\sin 2A-B)}{2(A^2+B^2)}\right]$

$f_{13} = -AB(\sinh^2 B + \sin^2 A)\left[\frac{(B\cosh 2B\sin 2A-A\sinh 2B\cos 2A)}{2(A^2+B^2)}\right]$

$f_{14} = (A\sinh 2B - B\sin 2A)(A\sinh B\cos A - B\sin A\cosh B)^* \left[1+\frac{\sinh 2B}{2B}+\frac{\sin 2A}{2A}+\frac{(B\sinh 2B\cos 2A+A\cosh 2B\ \sin 2A)}{2(A^2+B^2)}\right]$

$f_{15} = (B\sinh 2B + A\sin 2A)(B\sinh B\cos A + A\sin A\cosh B)^* \left[-1+\frac{\sinh 2B}{2B}+\frac{\sin 2A}{2A}-\frac{(B\sinh 2B\cos 2A+A\cosh 2B\ \sin 2A)}{2(A^2+B^2)}\right]$

$f_{16} = [2(A+B)\sinh B\cos A - 2(B-A)\sin A\cosh B]\left[\frac{(B\cosh 2B\sin 2A-A\sin 2B\ \cos 2A)}{2(A^2+B^2)}\right]$

$f_{17} = -2A(\sinh^2 B + \sin^2 A)(A\sinh B\cos A - B\sin A\cosh B)\left[\frac{\cos 2B-1}{2B}+\frac{(B\cosh 2B\sin 2A+A\sinh 2B\cos 2A-B)}{2(A^2+B^2)}\right]$

$f_{18} = -2B(\sinh^2 B + \sin^2 A)(A\sinh B\cos A - B\sin A\cosh B)\left[\frac{\cos 2A+1}{2A}+\frac{(B\sinh 2B\sin 2A-A\cosh 2B\cos 2A+A)}{2(A^2+B^2)}\right]$

$f_{19} = -2A(\sinh^2 B + \sin^2 A)(B\sinh B\cos A + A\sin A\cosh B)\left[\frac{\cos 2A-1}{2A}+\frac{(B\sinh 2B\sin 2A-A\cosh 2B\cos 2A+A)}{2(A^2+B^2)}\right]$

$f_{20} = -2B(\sinh^2 B + \sin^2 A)(B\sinh B\cos A + A\sin A\cosh B)\left[\frac{\cosh 2B-1}{2B}-\frac{(B\cosh 2B\cos 2A+A\sinh 2B\sin 2A-B)}{2(A^2+B^2)}\right]$

$f_{21} = (A\sinh B\cos A - B\sin A\cosh B)^{2*}\left[1+\frac{\sinh 2B}{2B}+\frac{\sin 2A}{2A}+\frac{(B\sinh 2B\cos 2A+A\cosh 2B\sin 2A)}{2(A^2+B^2)}\right]$

$f_{22} = (B\sinh B\cos A + A\sin A\cosh B)^{2*}\left[-1+\frac{\sinh 2B}{2B}+\frac{\sin 2A}{2A}-\frac{(B\sinh 2B\cos 2A+A\cosh 2B\sin 2A)}{2(A^2+B^2)}\right]$

$f_{23} = 2(A\sinh B\cos A - B\sin A\cosh B)(B\sinh B\cos A + A\sin A\cosh B)^*\left[\frac{(B\cosh 2B\sin 2A-A\sinh 2B\ \cos 2A)}{2(A^2+B^2)}\right]$ …..(1-46)

(iii) Case-3: zero axial force (beam on elastic foundation)

This is a special case, in which Ψ= 0 ; d= π/2 ; A=B=(λL)

Thus:

$$S= - 2(\lambda L)\ \frac{\sin\lambda L\cos\lambda L\cos(d/2)-\sinh\lambda L\cosh\lambda L}{sinh^2\,\lambda L- sin^2\,\lambda L}$$
….(1-47)

$$SC= 2(\lambda L)\ \frac{\cosh\lambda L\sin\lambda L-\sinh\lambda L\cos\lambda L}{sinh^2\,\lambda L- sin^2\,\lambda L}$$
…..(1-48)

$$Q= 2(\lambda L)^2\ \frac{sin^2\,\lambda L+sinh^2\,\lambda L}{sinh^2\,\lambda L- sin^2\,\lambda L}$$
……(1-49)

$$qQ= 2(\lambda L)^2\ \frac{\sinh\lambda L\sin\lambda L}{sinh^2\,\lambda L- sin^2\,\lambda L}$$
…….(1-50)

$$T= 4(\lambda L)^3\ \frac{\cosh\lambda L\sinh\lambda L+\sin\lambda L\cos\lambda L}{sinh^2\,\lambda L- sin^2\,\lambda L}$$
……(1-51)

$$tT= 4(\lambda L)^3\ \frac{\cos\lambda L\sinh\lambda L+\sin\lambda L\cosh\lambda L}{sinh^2\,\lambda L- sin^2\,\lambda L}$$
……(1-52)

Equation (1-47) to (1-52) must be used in the first load increment.

The value of the stability and bowing functions are shown graphically in figures (1-4), (1-5), (1-6), (1-7), (1-8), (1-9), (1-10) and (1-11) for different values of soil subgrade parameter (λL) and different values of axial force parameters (P).

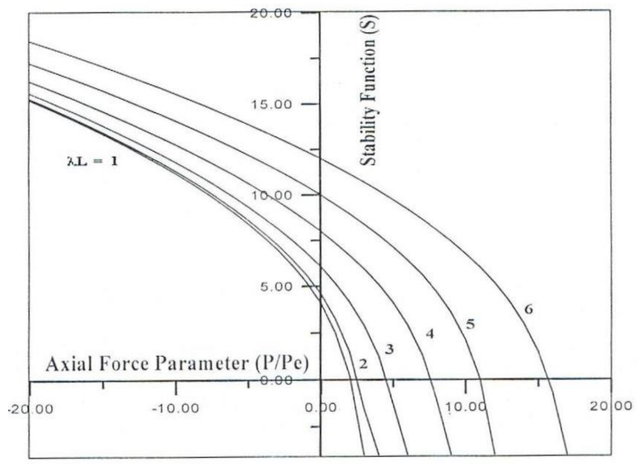

Fig.(1-4) Graphs of the function S

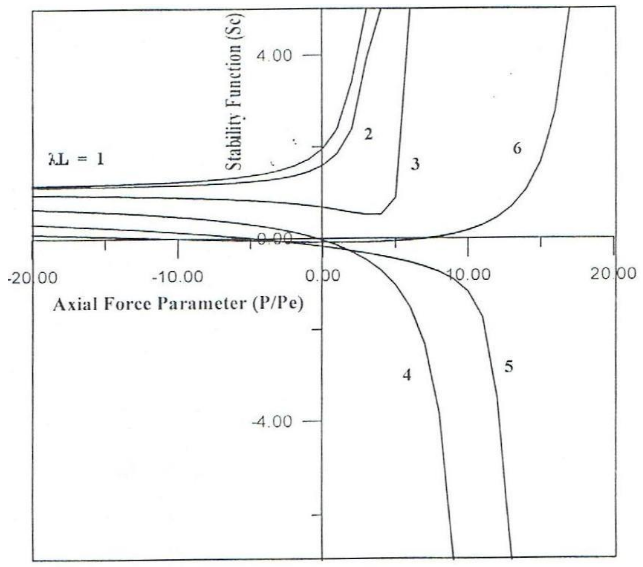

Fig.(1-5) Graphs of the function SC

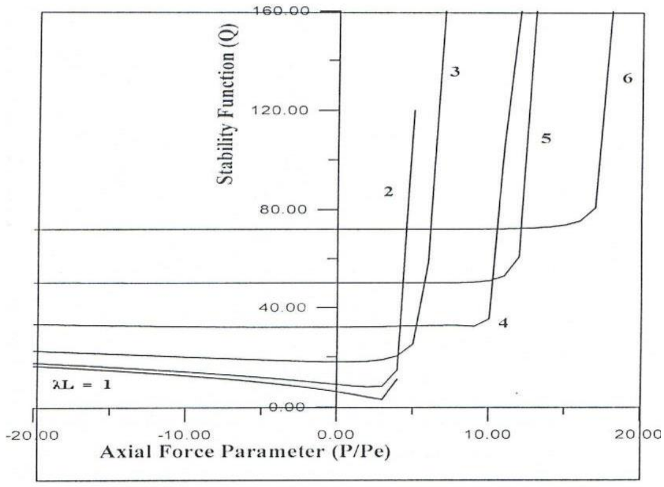

Fig.(1-6) Graphs of the function Q

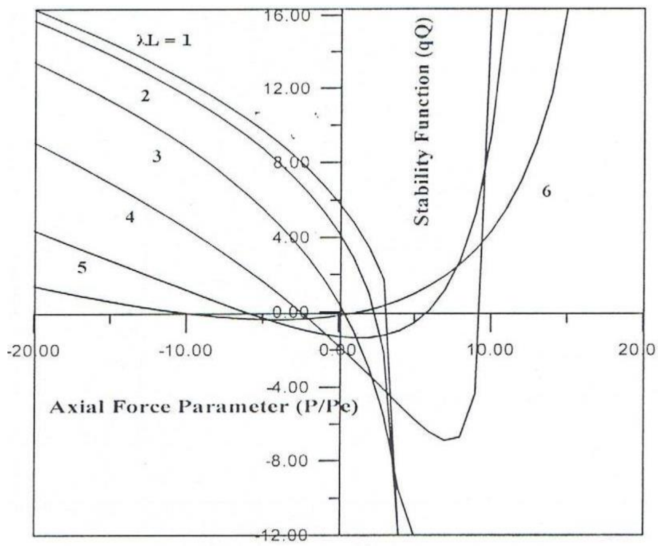

Fig.(1-7) Graphs of the function qQ

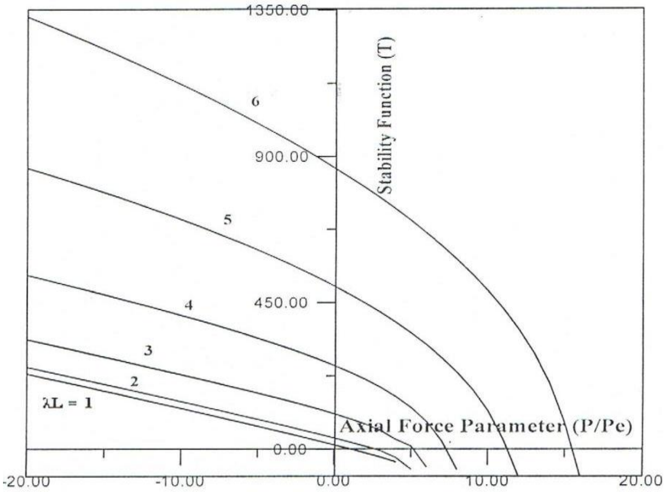

Fig.(1-8) Graphs of the function T

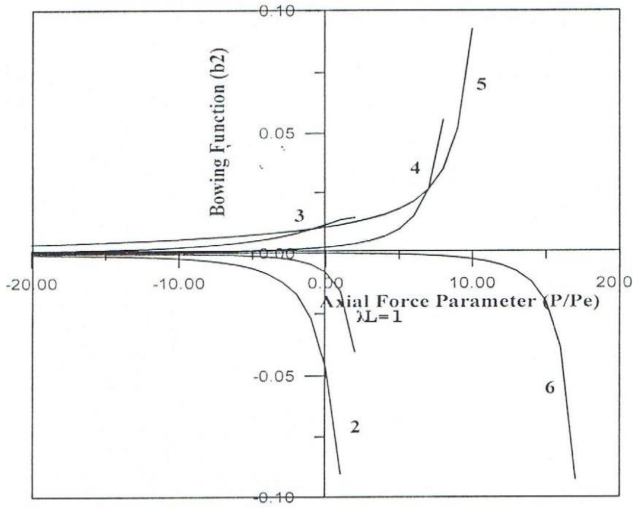

Fig.(1-11) Graphs of the function b_2

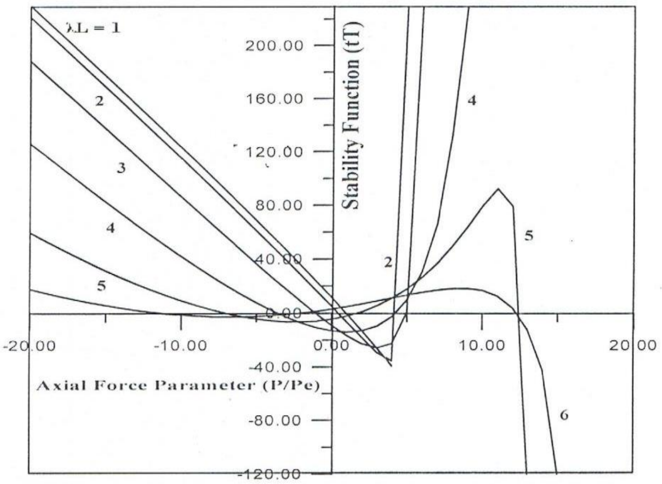

Fig.(1-9) Graphs of the function tT

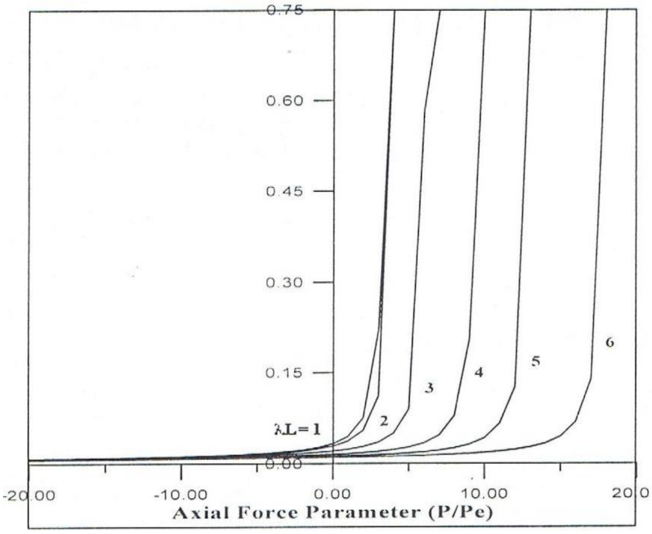

Fig.(1-10) Graphs of the function b_1

VI. Modified stability function

The basic differential equation for a non-prismatic beam on elastic foundation is[6]:

$$\frac{d}{dx^2}\left(EI_{(x)}\frac{d^2y}{dx^2}\right)+ky=0 \qquad ....(1\text{-}53)$$

If the effect of axial force (P) is considered, eq. (1-53) becomes

$$\frac{d}{dx^2}\left(EI_{(x)}\frac{d^2y}{dx^2}\right)+P\frac{d^2y}{dx^2}+ky=0 \qquad ....(1\text{-}54)$$

where y represents lateral deflection at distance x along the member, $El_{(x)}$ is the flexural stiffness of the member, P is axial force, and k represents the stiffness of the foundation.

For non-prismatic beam-column on elastic foundation, similar representation can be considered to that which is not resting on elastic foundation, which was presented by Al-Sarraf[2], as shown in Fig.(1-12). From Fig.(1-12), it is clear that a is considered as the distance of end 2 from the origin O, point of zero depth, and b = a + L.

All the members considered have uniform taper in either one or two directions. Therefore, the depth d_x may be expressed by:

$d_x = d_2\ (x/a)$ ......(1-55)

The moment of inertia of the cross-sectional area of the member about the axis of buckling may be expressed in the form

$I_{(x)} = I_2\ (x/a)^m$ .....(1-56)

where $I_{(x)}$ is the moment of inertia at distance x from the origin O, and m is the shape factor that depends on the cross-sectional shape and dimensions of the-member. The shape factor m may be evaluated by observing that Eq.(1-56) must give $I_{(x)} = I_1$ when x = b. This condition yields the relation:

$m = \log (I_1 / I_2) / \log U$ ....(1-57)

where U is (d_1 / d_2) end depth ratio.

So, the value of shape factor can be determined only when the dimensions of the cross-sections are known.

The values of the shape factor in are shown in Table (1-1) for different cross-sectional shapes(7).

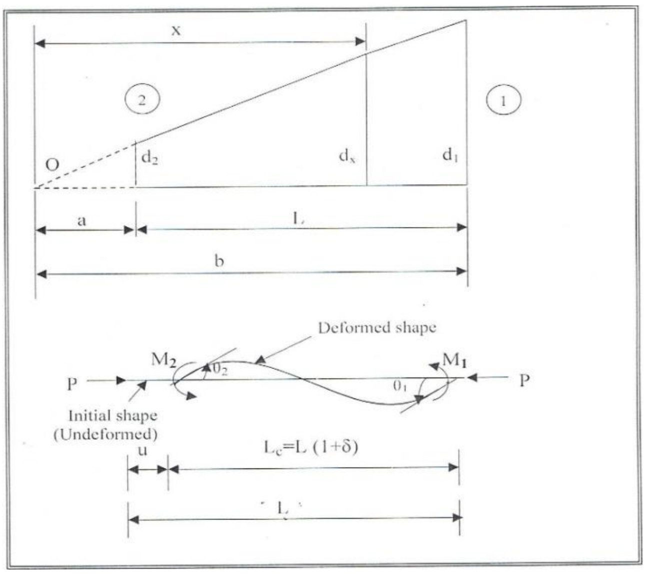

Fig (1-12) Tapered beam-column

By substituting Eq.(1-56) into Eq.(1-54) yields:

$$\frac{d}{dx^2}\left(EI_2\ (x/a)m\ \frac{d^2y}{dx^2}\right) + P\,\frac{d^2y}{dx^2} + ky = 0 \qquad ....(1\text{-}58)$$

And the above equation will be solved later by two approximate methods; finite differences and finite segments.

Table (1-1) Tapered beam cross-sectional shapes and shapes factors

Shape (1)	m (2)	n (3)
Wide-flange or I-section Constant dimensions b, t_w, t_f Varying depth d Bending about horizontal axis	2.1 to 2.6	varies
Closed box section Constant dimensions b, t_w, t_f Varying depth d Bending about horizontal axis	2.1 to 2.6	varies
Solid, rectangular section Constant width b Varying depth d Bending about horizontal axis	3	1
Solid, rectangular section Constant width b Varying depth d Bending about vertical axis	1	1
Open-web section Constant dimensions b, t_f Varying depth d Bending about horizontal axis	2	0
Tower section Constant areas concentrated near corners Varying dimension d	2	0
Solid, circular section Varying diameter d	4	2
Solid, square section Varying dimension d	4	2

6.1 nonlinearly tapered members:

For a member having nonlinear tapering in the either on or two directions as shown in fig.(1-13), the depth d_x may be expressed by

$d_x = d_2\ (x/a)^{\Psi}$ .....(1-59)

where a is the distance of end 2 from the origin O, point of zero depth, and d_2 is the depth at end 2, Ψ is the degree of variation.

From eq. (1-59) the depth of end 1 can be obtained as:

$d_1 = d_2\ (b/a)^{\Psi}$ ......(1-60)

where b is the distance of end 1 from the origin O, and:

$U^{1/\Psi} = b/a$ ...... (1-61)

Where $U = d_1/d_2$, eq.(1-60) can be written as:

$\overline{U}$ = b/a …… (1-62)

Where the $\overline{U}$ is the modified taper ratio and may be obtained as:

$\overline{U} = U^{1/\Psi}$ …. (1-63)

The moment of inertia of the cross-sectional area of the member about the axis of the bending may be expressed in the form:

$I_{(x)} = I_2\,(x/a)^{\Psi m}$ …..(1-64)

Where the $I_{(x)}$ is the moment of inertia of a section at distance x from the origin for the nonlinearly tapered member eq. (1-64) can be written as:

$I_{(x)} = I_2\,(x/a)^{\overline{m}}$ ……(1-65)

Where $\overline{m}$ is the modified shape factor and may be determined as:

$\overline{m} = \Psi m$ …. (1-66)

Where m= log (I_1/I_2) / log U …… (1-67)

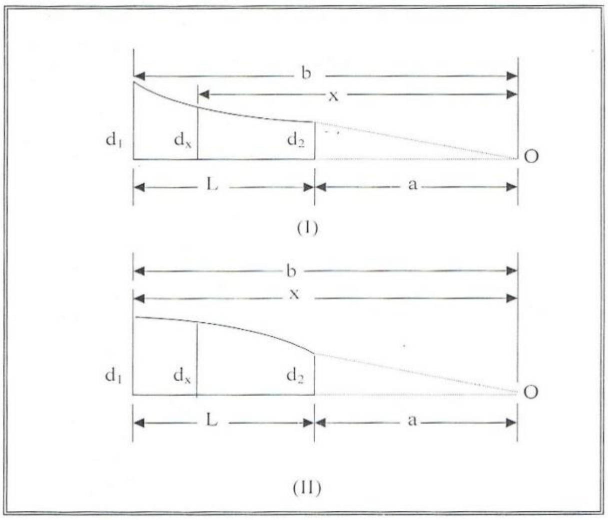

Fig.(1-13) nonlinear Tapered beam-column: (I) concave Ψ>1

(II) concave Ψ<1

6.2 Estimating of modified stability and bowing functions using finite segment method:

Finite segment method may be considered as a physical interpretation of the finite differences method that can be applied numerically to solve differential equations.

Now, the beam-column problem can be formulated and solved approximately in terms of the behavior or these segments without recourse to complex differential equations. In this method, the non-prismatic member on elastic foundation is divided into (n) prismatic members, as shown in Fig.(1-14)

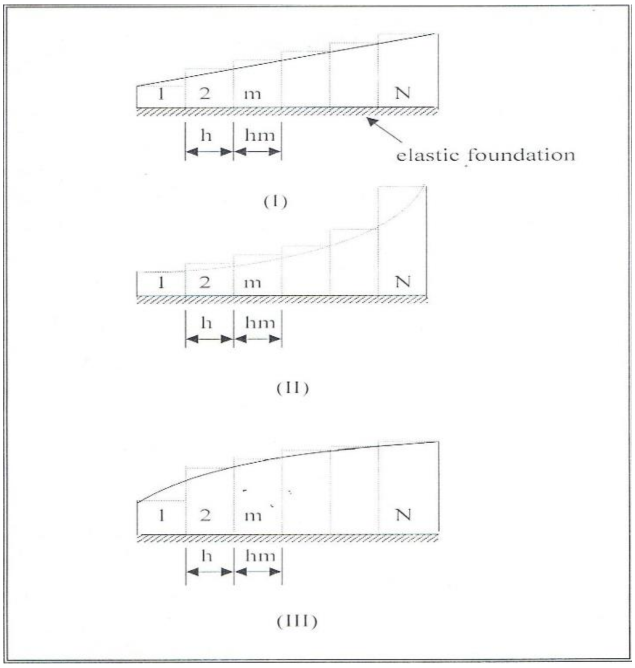

Fig.(1-14) member segment for non-prismatic beam – column on elastic foundation: (I) Tapered (II) Concave and (III) convex

6.2.1 estimation of modified stability functions:

The exact stability function derived by Al-Hachami[1] , are used to calculate the modified stability functions For the segment m, the local end force-deformation relationships

Are:

$$M_i = \frac{EI_i}{h_m}\left[C_{1m}\varphi_i + C_{2m}\varphi_j + (C_{1m}+C_{2m})\frac{y_i}{h_m} - (C_{1m}+C_{2m})\frac{y_j}{h_m}\right] \text{…..(1-68)}$$

$$M_j = \frac{EI_i}{h_m}\left[C_{2m}\varphi_i + C_{1m}\varphi_j + (C_{1m}+C_{2m})\frac{y_i}{h_m} - (C_{1m}+C_{2m})\frac{y_j}{h_m}\right] \text{…..(1-69)}$$

$$V_i = \frac{EI_i}{h^2{}_m}\left[(C_{1m}+C_{2m})\varphi_i - (C_{1m}+C_{2m})\varphi_j + A_m\frac{y_i}{h_m} - A_m\frac{y_j}{h_m}\right] \text{…….(1-70)}$$

$$V_i = \frac{EI_i}{h^2{}_m}\left[-(C_{1m}+C_{2m})\varphi_i - (C_{1m}+C_{2m})\varphi_j - A_m\frac{y_i}{h_m} - A_m\frac{y_j}{h_m}\right] \text{…..(1-71)}$$

equation (1-63) to (1-71) can be written in matrix form as:

$$\{f\}_m = [K]_m\{V\}_m \qquad \text{.... (1-72)}$$

$$\{f\}_m = \begin{bmatrix} ViL \\ Mi \\ VjL \\ Mj \end{bmatrix} \qquad \text{.... (1-73)}$$

$$\{K\}_m = \begin{bmatrix} y_i/L \\ \varphi_i \\ y_j/L \\ \varphi_j \end{bmatrix} \qquad \text{.....(1-74)}$$

And the stiffness matrix $[K]_m$ can be written as:

$$[K]_m = \frac{EI_m}{L} \begin{bmatrix} \frac{A_m}{f_{rm}^3} & \frac{(C_{1m}+C_{2m})}{f_{rm}^2} & -\frac{A_m}{f_{rm}^3} & \frac{(C_{1m}+C_{2m})}{f_{rm}^2} \\ & \frac{C_{1m}}{f_{rm}} & -\frac{(C_{1m}+C_{2m})}{f_{rm}^2} & \frac{C_{2m}}{f_{rm}} \\ & & \frac{A_m}{f_{rm}^3} & -\frac{(C_{1m}+C_{2m})}{f_{rm}^2} \\ & & & \frac{C_{1m}}{f_{rm}} \end{bmatrix} \qquad \text{......(1-75)}$$

In which:

$A_m = 2(C_{1m}+C_{2m}) - \pi^2 q_m$ ...... (1-76)

$q_m = q_e \cdot f_{rm}^2$ ....(1-77)

where:

q_m is the segment m axial force parameter, while q_e is the total element axial force parameter

$f_{rn} = h_m / L$ ......(1-78)

C_{1m} and C_{2m} : stability function of a prismatic segment, which are function of q_m.

y_i and y_2 : are sways of end i and j of segment m.

φ_i and φ_j : are angle of rotations of end i and j of segment m.

h_m : is the length of segment m.

I_m : is the moment of inertia for segment m.

For the case of beam-column resting of elastic foundation (Winkler model), where the soil subgrade reaction is assumed to be uniformly distributed along the beam-column, the segment stiffness matrix $[K]_m$ in eq.(1-76) must be rewritten as:

$$[K]_m = \frac{EI_m}{L} \begin{bmatrix} \frac{J_{3m}}{f_{rm}^3} & \frac{J_{1m}}{f_{rm}^2} & -\frac{J_{4m}}{f_{rm}^3} & \frac{J_{2m}}{f_{rm}^2} \\ & \frac{C_{1m}}{f_{rm}} & -\frac{J_{2m}}{f_{rm}^2} & \frac{C_{2m}}{f_{rm}} \\ & & \frac{J_{3m}}{f_{rm}^3} & -\frac{J_{1m}}{f_{rm}^2} \\ & & & \frac{C_{1m}}{f_{rm}} \end{bmatrix} \qquad \text{.....(1-79)}$$

Where C_{1m}, C_{2m}, J_{1m}, J_{2m}, J_{3m}, and J_{4m} denote the stability function for a prismatic beam-column resting on elastic foundation.

$(\lambda L)_m = f_m \cdot (\lambda L)_e$ ......(1-80)

Where the $(\lambda L)_m$ is the segment m axial force parameter , while $(\lambda L)_e$ is the total element axial force parameter.

Now, each stability function will be derived depending on the applied boundary conditions, as follows:

1- Determination of S_1, $\overline{SC}$:

The boundary conditions: ($\varphi_i = 1, y_1 = y_{n+1} = \varphi_{n+1} = 0$), are seen in fig.(1-15).

Using the equation of non-prismatic member resting on elastic foundation:

$M_1 = \frac{EI_1}{L} S_1$ .....(1-81)

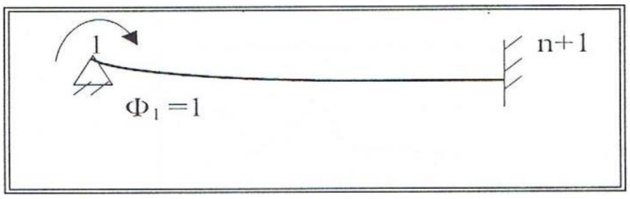

Fig.(1-15) boundary conditions

Using eq.(1-68) for the first segment gives:

$M_1 = \frac{EI_1}{Lf_{r1}}\left[C_{11} + C_{21}\varphi_2 + J_{21}\frac{y_2}{LF_{r1}}\right]$ .....(1-82)

Equating eqs.(1-82)and (1-81) yields:

$S_1 = n[C_{11} + C_{21}\varphi_2 - nJ_{21}y_2]$ .....(1-83)

Where the n is the number of segments.

To find the $\overline{SC}$ the same boundary conditions are used:

$M_{n+1} = \frac{EI_1}{L}\overline{SC}$ ....(1-84)

Using the eq.(1-69) for segment n:

$M_{n+1} = \frac{EI_1}{Lf_{r1}}\left[C_{2n}\varphi_n + J_{1n}\frac{y_n}{LF_{r1}}\right]$ .....(1-85)

$\overline{SC} = n(I_n/I_1)[C_{2n}\varphi_n - nJ_{1n}y_n]$ .....(1-86)

Where

C_{11}, C_{21}, J_{11}, and J_{21}: are stability functions of the first prismatic segment.

C_{1n}, C_{2n}, J_{1n}, and J_{2n}: are the stability functions of prismatic segment n

I_1, I_n, : is the moment of inertia at end 1 and n respectively.

2- Determination of S_2, $\overline{SC}$:

The boundary conditions: ($\varphi_{n+1} = -1, y_1 = y_{n+1} = \varphi_1 = 0$), are seen in fig.(1-16). Using the equation:

$M_1 = -\frac{EI_1}{L} S_2$ .....(1-87)

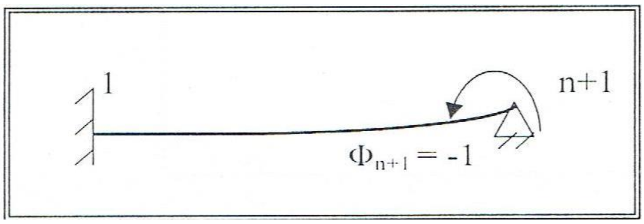

Fig.(1-16) boundary conditions

Using eq.(1-69) for the segment n gives:

$S_2 = -n[-C_{1n} + C_{2n} + nJ_{2n}y_n]$ .....(1-88)

To find the $\overline{SC}$ the same boundary conditions are used:

$\overline{SC} = -n[C_{21}\varphi_2 - nJ_{21}y_2]$ .....(1-89)

3- determination of Q_1 and qQ_1:

the boundary conditions: ($y_1 = 1, y_{n+1} = \varphi_1 = \varphi_{n+1} = 0$).

$M_1 = -\frac{EI_1}{L^2} Q_1$ .....(1-90)

Using eq.(1-69) for 1st segment, the following is obtained :

$Q_1 = -n[C_{21}\varphi_2 L + J_{11} - nJ_{21}y_2]$ .....(1-91)

Using the same boundary conditions, for segment n:

$qQ_1 = n(^{I_n}/_{I_1})[C_{2n}\varphi_n L - nJ_{11}y_n]$ .....(1-92)

4- determination of Q_2 and qQ_2:

the boundary condition: ($y_{n+1} = 1, y_1 = \varphi_1 = \varphi_{n+1} = 0$).

$Q_2 = -n(^{I_n}/_{I_1})[C_{2n}\varphi_n L + nJ_{1n}y_n - nJ_{2n}]$ .....(1-93)

$qQ_2 = -n[C_{21}\varphi_2 L - nJ_{21}y_2]$ .....(1-94)

5- determination of T_1 and tT_1:

the boundary conditions: ($y_1 = 1, y_{n+1} = \varphi_1 = \varphi_{n+1} = 0$).

Using the equation:

$V_1 = \frac{EI_1}{L^3} T_1$ .....(1-95)

For segment one, eq(1-70) is used:

$V_1 = \frac{EI_1}{f_{r1}L}\left[(C_{11} + C_{21})\varphi_2 + \frac{J_{31}}{f_{r1}L} - J_{41}\frac{y_2}{f_{r1}L}\right]$ ....(1-96)

$T_1 = n^2[(C_{11} + C_{21})\varphi_2 L + nJ_{31} - nJ_{41}y_2]$ .....(1-97)

$tT_1 = n^2(^{I_n}/_{I_1})[-J_{2n}\varphi_n L - nJ_{3n}y_n]$ .....(1-98)

6- determination of T_2 and tT_2:

The boundary conditions: ($y_{n+1} = 1, y_1 = \varphi_1 = \varphi_{n+1} = 0$).

$T_2 = -n^2(^{I_n}/_{I_1})[-J_{2n}\varphi_n L - nJ_{4n}y_n + nJ_{3n}]$ ....(1-99)

$tT_2 = -n^2[(C_{11} + C_{21})\varphi_2 L - nJ_{41}y_2]$ .....(1-100)

7. estimation of modified stability and bowing functions using finite non-prismatic (tapered) segments method.

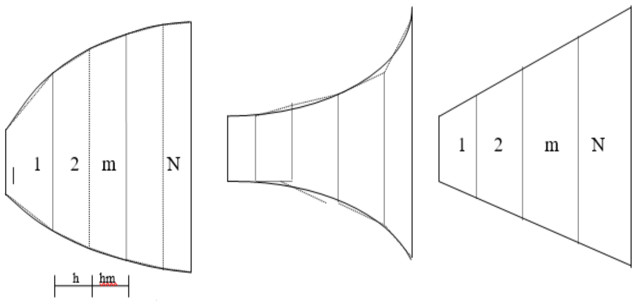

Figure (1-17) non prismatic segment

$M_1 = \frac{EI_1}{h_m}(S\Theta_1 + SC_1\Theta_2 + Qy_1 - \frac{qQy_2}{h_m})$ .... (1-101)

$M_2 = \frac{EI_N}{h_m}(S\ C_N\Theta_{1N} + S_{2N}\Theta_{2N} + \frac{qQy_{1N}}{h_m} - \frac{Qy_{2N}}{h_m})$ ....(1-102)

$V_1 = \frac{EI_1}{h_m{}^2}(Q_1\Theta_1 + qQ\Theta_2 + \frac{T_1y_1}{h_m} - \frac{tT_1y_2}{h_m})$ ....(1-103)

$V_2 = \frac{EI_N}{h_m{}^2}(qQ_N\ \Theta_{1N} + Q_N\Theta_{2N} + \frac{tT_Ny_{1N}}{h_m} - \frac{T_Ny_{2N}}{h_m})$ ....(1-104)

1. **Determination of S_{11} : ($\varphi_1 = 1, \varphi_2 = y_1 = y_2 = 0$)**
   $S_{11} = N\left[(S_1 + SC_1\varphi_2) - N\frac{qQy_2}{L}\right]$
2. **Determination of SC_1 : ($\varphi_2 = 1, y_1 = y_2 = \varphi_1 = 0$)**
   $SC_1 = N\frac{I_N}{I_N}\left[(SC_1\varphi_{2N}) - N\frac{qQ_N y_N}{L}\right]$
3. **Determination of S_{22} : ($\varphi_2 = 1, y_1 = y_2 = \varphi_1 = 0$)**
   $S_{22} = N\frac{I_N}{I_N}\left[(SC_N\varphi_{1N}) + qQy_{1N} - \frac{N}{L}\right]$
4. **Determination of Q_{11} : ($y_1 = 1, y_2 = \varphi_1 = \varphi_2 = 0$)**
   $Q_{11} = N\left[(SC_1\varphi_2 + Q_N) - N\frac{qQy_2}{L}\right]$
5. **Determination of qQ_{11} : ($y_1 = y_2 = \varphi_1 = 0, \varphi_2 = 1$)**
   $qQ_{11} = N^2\left[qQ\varphi_2 - N\frac{tTy_2}{L}\right]$
6. **Determination of T_{11} : ($\varphi_1 = \varphi_2 = y_2 = 0, y_1 = 1$)**
   $T_{11} = N^2\left[qQ_1\varphi_2 + \frac{N}{L}(T_1 - tT_1y_2)\right]$
7. **Determination of tT_{11} : ($\varphi_1 = \varphi_2 = y_1 = 0, y_2 = 1$)**
   $tT_{11} = N^2\left[qQ\varphi_2 - \frac{N}{L}tTy_2\right]$

where :

S_{11}, SC_1, S_{22}, Q_{11}, qQ_{11}, T_{11}, tT_{11} : are the stability functions of the non-prismatic beam of nonlinearly varying section .

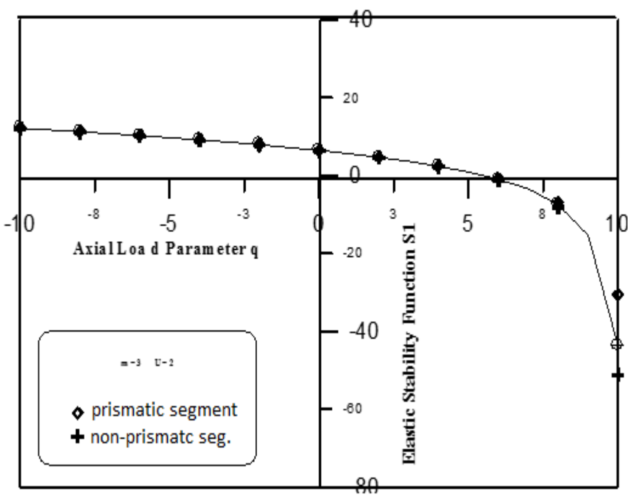

Fig. (1-18) Graphs of S1 for non-linear tapered member using finite element method.

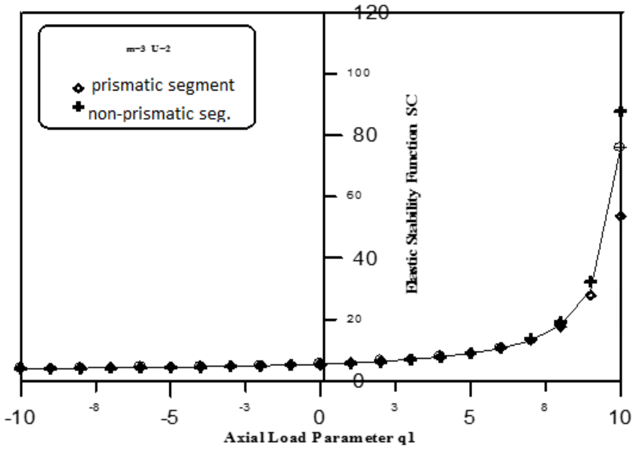

Fig. (1-19) Graphs of SC for non-linear tapered member using finite element method.

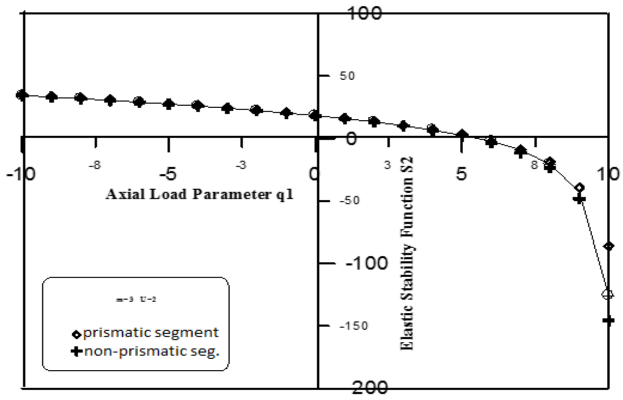

Fig. (1-20) Graphs of S2 for non-linear tapered member using finite element method.

Example (1-1):

Simply supported beam with uniformly distributed load

Figure (1-20) shows the geometry and loading conditions for example (1-1) finite difference, finite segment, and approximate methods are used to solve this problem. Two elements and five load increments are considered. Figure (1-21) shows the load mid-span deflection for this beam. Good agreement exists between the results obtained by above three methods.

Example (1-2):

Nonlinearly tapered (convex) cantilever beam with two-concentrated loads

this application with nonlinearly convex varying section, figure (1-23). Same methods of analysis are used here too. Results are listed in table (1-2).

Example (1-3):

Nonlinearly tapered (convex) column

This example above is repeated here with convex variation in section in order to illustrate the difference between the use of various type of nonlinearly in sections. The beam is shown in figure (1-24). Consistency can be seen between results obtained by methods of analysis as explained in figure (1-25).

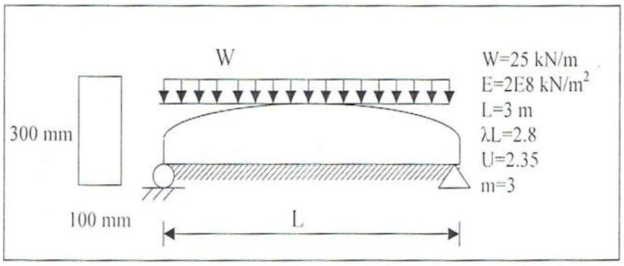

Figure (1-21) Geometry and loading of Ex. (1-1)

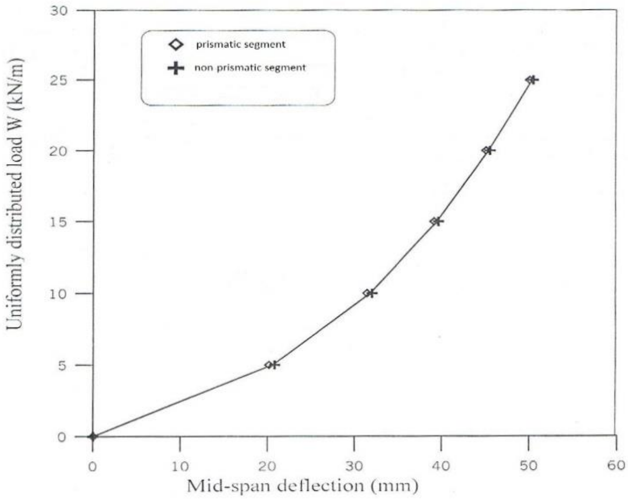

Figure (1-22) load – mid – span deflection for Ex. (1-1)

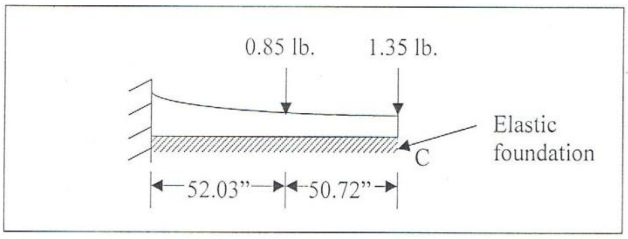

Figure (1-23) loading of Ex. (1-2)

Table (1-2) results of Ex. (1-2)

	δHc/L	δVc/L
Finite non-prismatic segment	0.0157	0.1665
Finite prismatic segment	0.0158	0.1670

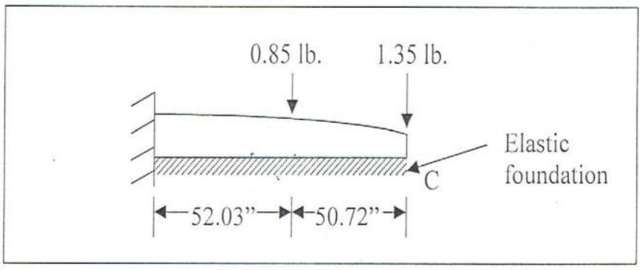

Figure (1-24) loading of Ex. (1-2)

Table (1-3) results of Ex. (1-2)

	δHc/L	δVc/L
Finite non-prismatic segment	0.0153	0.160
Finite prismatic segment	0.0154	0.1649

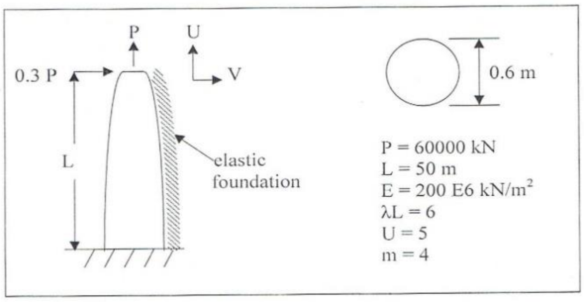

Figure (1-25) Geometry and load of Ex. (1-3)

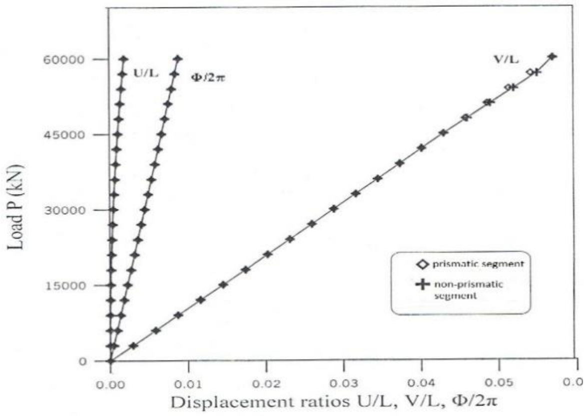

Figure (1-26) load – displacement curve for Ex. (1-3)

VII. CONCLUSIONS:

The following conclusions can be drawing depending on the results obtained from the present work:

1. This study shows that the large displacement elastic behavior of plan frames having linearly and nonlinearly tapered members resting on elastic foundation (winkler model) can be accurately predicted by using the beam-column approach.
2. The stability and bowing functions can be derived using finite segment method.
3. For linearly and nonlinearly tapered members resting on elastic foundation, the stability and bowing functions can be estimated approximately by using the stability and bowing functions for prismatic and non-prismatic members using different factors depending on the tapering ratio, shape factor, axial force parameter and sometimes non-dimensional soil parameter.
4. In the segmentation method, the non-prismatic segment gives more accurate results than these of prismatic segment.

VIII. REFERENCES

[1]. Al-Hachami, E.K.,"Large Displacement Analysis of Structures with Applications to Piles and Submarine Pipelines" ,ph.d. thesis, University of Technology-Iraq, 1997.

[2]. Al-Saraf, S.,"Elastic Instability of Struts on, or Driven into, Elastic Foundations", The Structural Engineering, Vol.56B, No.1, March, 1978,pp.13-19.

[3]. Faris, H.A."Large Displacement Elastic-Plastic Analysis of Plane Frames with Non-Prismatic Members of Non-linearly Varying sections", Ph.D. thesis, University of Technology-Iraq, 2002.

[4]. Ahmed Tariq, Faris, H.A., N.Al-jumaily, Ibrahim," Large Displacement Elastic Stability Analysis of Plane Frames With Non-Prismatic Members Resting on Elastic Foundation", M.Sc. thesis, University of Technology, 2004.

[5]. Davis, R.O. and Selvaduri, A.P.S.,"Elasticity and Geomechanics" , Cambridge University Press, 1st Published, 1996,PP.113-115.

[6]. Ghali, A. and Neville, A.M.," Structural Analysis " E and FN Spon, London, 4TH Edition, 1997, PP.437-446.

[7]. Gupta, A.K.," Vibration of Tapered Beams ", Journal Of Structural Engineering, ASCE, Vol. 117, No. 4, Jan., 1991, PP.111-127.