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A Study on Perfect 3 Coloring of Cubic Graph of Order 10

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ABSTRACT

Perfect coloring is a generalization of the notion of completely regular code. This thesis entitled " A STUDY OF PERFECT 3 COLORING OF CUBIC GRAPH OF ORDER 10" examines the idea identified with perfectly coloring a cubic graph of order 10. A perfect m coloring of a graph G with m colors is a partition of vertex set of G into m parts A_1, A_2, A_3 .....A_m such that, for all i,j ∈ $\{1,2,\dots.m\}$ every vertex of A_i is adjacent to the same number of vertices, namely, aij vertices of Aj and we classify the realizable coloring of parameter matrix (symmetric)of perfect 3-colorings for the cubic graph of order 10.

Keywords : Perfect Colouring, Parameter Matrix, Peterson Graphs.

I. INTRODUCTION

The concept of a perfect m-coloring plays an important role in graph theory, algebraic combinatorics, and coding theory(completely regular codes). There is another term for this concept in the literature as 'equitable partition'.

The existence of completely regular codes in graphs is a historical problem in mathematics. Completely regular codes are a generalization of perfect codes. In 1973, Delsarte conjectured the non-existence of perfect codes in Johnson graphs. Therefore (Some effort has been done on enumerating the parameter matrices of some Johnson graphs, including J(6,3), J(7,3), J(8,3), J(8,4) and J(v,3) (v odd).

Fon-Der-Flass enumerated the parameter matrices of n-dimensional hypercube Q_n for n<24. He also obtained some constructions and a necessary condition for the existence of perfect 2-coloring of the n-dimensional hypercube with a given parameter matrix. In this place enumerate the parameter matrices of all perfect 2-colorings of GP (n,2).

A cubic graph is a 3-regular graph.it is shown that the number of connected cubic graphs with vertices is 19.

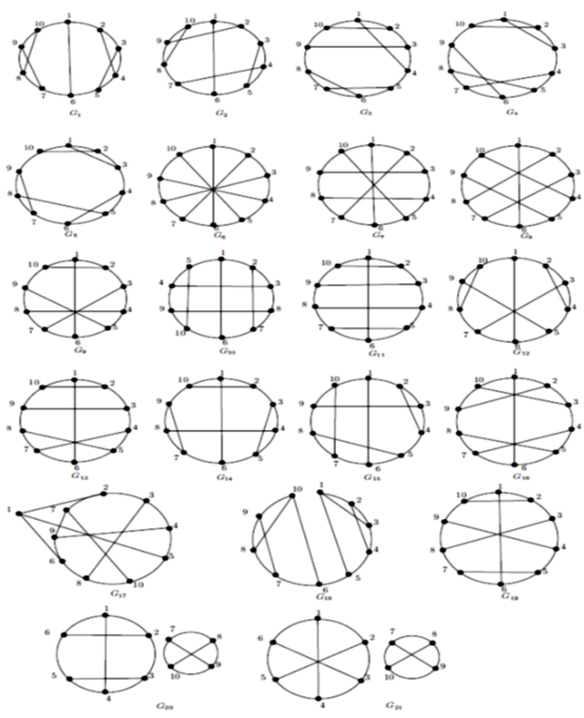

Figure 1. Cubic graphs of order 10.

II. METHODOLOGY

For a graph G and positive integer *m*, a mapping T : V (G) → (1, …m) is called a perfect m – coloring with matrix $A^T = (a_{ij})_{i,j} \in (1, \dots m)$, if it is surjective, and for all i, j for every vertex of colour i, the number of its neighbours of colour j is equal to a_{ij}. The matrix A^T is called the *parameter matrix* of a perfect coloring. In the case $m = 3$, we call the first colour white, the second color black, and the third colour red. In this paper, we generally show a parameter matrix by

$$A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

1. In this paper, we consider all perfect 3 –colouring up to remaining the colours, i.e. we identify the perfect 3 coloring with the matrices.

$$\begin{bmatrix} a & g & d \\ c & i & f \\ b & h & e \end{bmatrix} \quad \begin{bmatrix} e & b & h \\ d & a & g \\ f & c & i \end{bmatrix} \quad \begin{bmatrix} e & h & b \\ f & i & c \\ d & g & a \end{bmatrix}$$

$$\begin{bmatrix} i & f & c \\ h & e & b \\ g & d & a \end{bmatrix} \quad \begin{bmatrix} i & c & f \\ g & a & d \\ h & b & e \end{bmatrix}$$

obtained by switching the colours with the original coloring.

2. Here, we present some results concerning necessary conditions for the existence of perfect 3 coloring of a cubic connected graph of order 10 with a given parameter matrix

$$A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

The simplest necessary condition for the existence of perfect 3 – coloring of a cubic connected graph with the matrix $\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$ is :

$$a + d + g = b + e + h = c + f + i = 3$$

Also, it is clear that we cannot have $d = g = 0, b = h = 0, or c = f = 0$, since the graph is connected. In addition, $d = 0, g = 0, h = 0$ $if b = 0, \; c = 0, f = 0$, respectively.

The number θ is called an eigenvalue of a graph G, if θ is an eigenvalue of the adjacency matrix of this graph. The number θ is called an eigenvalue of a perfect coloring T into three colors with the matrix A^T. The following lemma demonstrates the connection between the introduced notions.

Lemma 3.2.1

If T is a perfect coloring of a graph G in m colours, then any eigenvalue of T is an eigenvalue of G.

Now, without lost of generally, we can assume that $|W| \leq |B| \leq |R|$. The following proposition gives us the size of each class of color.

Proposition 3.1

Let T be perfect 3 – coloring of a graph G with the matrix $A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

1. If $d, g, h \neq 0$, then

$$|W| = \frac{|V(G)|}{\frac{d}{a} + 1 + \frac{g}{a}}, \; |B| = \frac{|V(G)|}{\frac{b}{d} + 1 + \frac{h}{f}}, \quad |R| = \frac{|V(G)|}{\frac{f}{h} + 1 + \frac{c}{g}}$$

If $d = 0$, then

$$|W| = \frac{|V(G)|}{\frac{g}{c} + 1 + \frac{gf}{ch}}, |B| = \frac{|V(G)|}{\frac{h}{f} + 1 + \frac{cf}{fg}}, \quad |R| = \frac{|V(G)|}{\frac{f}{h} + 1 + \frac{c}{g}}$$

If *g= o,* then

$$|W| = \frac{|V(G)|}{\frac{d}{b} + 1 + \frac{dh}{bf}}, |B| = \frac{|V(G)|}{\frac{b}{d} + 1 + \frac{h}{f}}, \quad |R| = \frac{|V(G)|}{\frac{f}{h} + 1 + \frac{bf}{dh}}$$

1. If *h = 0*, then

$$|W| = \frac{|V(G)|}{\frac{d}{b} + 1 + \frac{g}{c}}, |B| = \frac{|V(G)|}{\frac{b}{d} + 1 + \frac{bg}{cd}}, \quad |R| = \frac{|V(G)|}{\frac{c}{g} + 1 + \frac{dc}{bg}}$$

Proof (1):

Consider the 3 – partite graph obtained by removing the edges uu such that u and u are the same color. By counting the number of edges between parts, we can easily obtain $|W|d = |B|b$, $|W|g = |R|$, and $|B|h = |R|f$, Now we can conclude the desire result from $|W| + |B| + |R| = |V(G)|$.

In the next lemma, under the condition |W| = 1, we enumerate all matrices that can be parameter matrix for a cubic connected graph.

Lemma 3.2.2

Let G be a cubic connected graph of order 10. If T is a perfect 3 – coloring with the matrix, A, and $|W| = 1$, then A should be the following matrix.

$$A^T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Proof. Let $A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$ be a parameter matrix with |W| = 1, Consider the white vertex, It is clear that none of its adjacent vertices are white : i.e. $a = 0$ Therefore, we have two cases below.

(1) The adjacent vertices of the white vertex are the same color. If they are black, then $d = 3$ *and* $g = 0$. *From* $g = 0$, *we get* $c = 0$. Also since the graph is connected, we have $f, h \neq 0$. Hence we obtain the following matrix. $\begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

If the adjacent vertices of the white vertex are red, then $g = 3$ $d = 0$. *From* $d = 0$. we get Also, since the graph is connected, we have $f, h \neq 0$. Hence we obtain the following matrix.

$$\begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

Fully, by using Remark 3.1 and the fact, that $|W| \leq |B| \leq |R|$ it is obvious that there are in (1), as shown A_1, A_2, A_3

$$A_1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix},\ A_3 = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

(2) The adjacent vertices of the white vertex are different color. If immediately gives that $d, g \neq 0$. Also, it can be seen that $b = c = 1$. An easy computation as in (1), shows that there are only two matrices that can be parameter matrix in this case, as shown

$A_4 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, A_5 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ proposition 3.1, It is obvious that just the matrix $A_1 = A_2$ can be a parameter.

Lemma 3.2.3

Let G be a cubic connected graph of order 10. If T is a perfect 3 – coloring with the matrix A^T and $|W| = |B| = 2, |R| = 6$, then A^T should be the following matrix. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

Proof. First, suppose that $d, g \neq 0$ As $|W| = 2$, by Proposition, 3.1 It is follows that $\frac{d}{b} + \frac{g}{c} = 4$. Therefore $d = g = 2, c = 1$ and we get a contradiction with $d + g \leq 3$.

Second, suppose that $d = 0$, and then $b = 0$, As $|R| = 4$, by proposition 3.1 we have $\frac{c}{g} + \frac{f}{g} = \frac{2}{3}$. Therefore $g = h = 3, c = f = 1$, and we get a contradiction g + h ≤ 3

Finally, suppose that $c = 0$, and then $c = 0$, As $|B| = 2$, by proposition 3.1 we have $\frac{b}{d} + \frac{h}{f} = 4$ Therefore $d = h = 2, b = f = 1, or\ d = 3, b = f = h = 1\ or\ d = 3, b = 1, f = h = 2$. Hence,By using the Proposition 3.1 It can be seen that no matrix can be a parameter.

Lemma 3.2.4

Let G be a cubic connected graph of order 10. Then G has no perfect 3 – coloring T with the matrix $|W| = 2, |B| = 3, |R| = 5$

Proof. If T It is perfect 3 – coloring with the similar proving Lemma 3.2.3 A should be one of the following matrices.

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

By using the Proposition 3.1 It can be seen that no matrix can be a parameter.

Lemma 3.2.5 Let G be a cubic connected graph of order 10. If T is a perfect 3 – coloring with the matrix. A^T and $|W| = 2, |B| = 4, |R| = 4$, then A should be one of the following matrix.

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

Proof. If T It is perfect 3 – coloring with the similar proving Lemma. 3.2.1 A should be one of the following matrices.

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

By using the Proposition 3.1 It can be seen that only two matrix can be a parameter.

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

Lemma 3. 2. 6 Let G be a cubic connected graph of order 10. Then G has no perfect 3 – coloring T with the matrix $|W| = 3, |B| = 3, |R| = 4$

Proof. If T It is perfect 3 – coloring with the similar proving Lemma. 3.2.3 A should be one of the following matrices.

$$\begin{bmatrix}1&0&2\\0&3&0\\0&1&2\end{bmatrix}, \begin{bmatrix}0&3&0\\3&0&0\\0&0&3\end{bmatrix}, \begin{bmatrix}0&0&3\\0&3&0\\3&0&0\end{bmatrix}$$
$$\begin{bmatrix}2&0&1\\0&1&2\\1&2&1\end{bmatrix}, \begin{bmatrix}2&1&0\\1&0&2\\0&2&1\end{bmatrix}, \begin{bmatrix}1&0&2\\0&2&1\\2&1&0\end{bmatrix}, \begin{bmatrix}1&1&1\\1&0&2\\1&2&0\end{bmatrix}$$

By using the Proposition 3.1 It can be seen that no matrix can be a parameter.

By using the lemma 3.2.2, 3.2.3 and 3.2.5 It can be seen that only the following matrix can be a parameter ones.

$$\begin{bmatrix}1&0&2\\0&3&0\\0&1&2\end{bmatrix}, \begin{bmatrix}0&3&0\\3&0&0\\0&0&3\end{bmatrix}, \begin{bmatrix}0&0&3\\0&3&0\\3&0&0\end{bmatrix}$$
$$\begin{bmatrix}2&0&1\\0&1&2\\1&2&1\end{bmatrix}, \begin{bmatrix}2&1&0\\1&0&2\\0&2&1\end{bmatrix}, \begin{bmatrix}1&0&2\\0&2&1\\2&1&0\end{bmatrix}, \begin{bmatrix}1&1&1\\1&0&2\\1&2&0\end{bmatrix}$$

By Remark 3.1 It is clear that the matrix $\begin{bmatrix}2&0&1\\0&1&2\\1&2&1\end{bmatrix}$, is the same as the matrix $\begin{bmatrix}2&1&0\\1&0&2\\0&2&1\end{bmatrix}$ and the matrix is $\begin{bmatrix}0&3&0\\3&0&0\\0&0&3\end{bmatrix}$ the same as the matrix $\begin{bmatrix}0&0&3\\0&3&0\\3&0&0\end{bmatrix}$ up to remaining the colors. Therefore, if T is a perfect 3-coloring with the matrix A^T should be one of the following matrices.

$$A_1 = \begin{bmatrix}2&1&0\\1&0&2\\0&2&1\end{bmatrix}, A_2 = \begin{bmatrix}1&1&1\\1&0&2\\1&2&0\end{bmatrix}, A_3 = \begin{bmatrix}0&0&3\\0&3&0\\3&0&0\end{bmatrix}$$
$$A_4 = \begin{bmatrix}1&0&2\\0&2&1\\2&1&0\end{bmatrix}$$

The next theorem can be useful to find the eigenvalues of a parameter matrix.

Theorem 3.1 Let $A^T = \begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}$ be a parameter matrix of a k- regular graph. Then the eigen value of A^T are $A^T{}_{1,2} = \frac{tr\,(A^T)-k}{2} \pm \sqrt{(\frac{tr\,(A^T)-k}{2})^2 - \frac{\det(A^T)}{k}}$

$$\lambda_3 = k.$$

Proof. By using the condition $a+d+g = b+e+h = c+f+i = k$, it is clear that one of the eigenvalues is k. Therefore $\det(A^T) = k\lambda_1\lambda_2$. From $\lambda_2 = tr\,(A^T) - \lambda_1 - k$, we get $\det(A^T) = k\lambda_1\,(tr\,(A^T) - \lambda_1 - k) = -k\lambda_1^2 + k(tr(A^T) - k)\lambda_1$

By solving the equation $\lambda^2 + k - tr(A^T)\lambda + \frac{\det(A^T)}{k} = 0$, we obtain

$$\lambda_{1,2} = \frac{tr\,(A^T)-k}{2} \pm \sqrt{(\frac{tr\,(A^T)-k}{2})^2 - \frac{\det(A^T)}{k}}$$

III. PERFECT 3 – COLORINGS OF THE CUBIC CONNECTED GRAPHS OF ORDER 10

As it been shown in Section 3, only matrices A_1, A_2, A_3 and A_4 can be parameter matrices. With consideration of cubic graphs eigen values and using Theorem 3.1, it can be seen that the connected cubic graphs with 10 vertices can have perfect 3-coloring with matrices A_1, A_2, A_3 and A_4 which is represented by Table.

Graphs	Matrix A_1	Matrix A_2	Matrix A_3	Matrix A_4
1	√	×	√	√
2	√	√	√	√
4	√	√	×	√
5	√	√	×	√
6	√	√	√	√
9	×	√	×	√
10	√	×	√	√
13	×	√	√	√
14	×	×	√	√
18	√	√	×	√
19	×	×	√	√

$T_{1(a_1)} = T_{1(a_8)} = 1$, $T_{1(a_4)} = T_{1(a_7)} = 2$,
$T_{1(a_2)} = T_{1(a_4)} = 1$, $T_{1(a_3)} = T_{1(a_5)} = T_{1(a_8)} = T_{1(a_9)} = 3$,
$T_{2(a_4)} = T_{2(a_8)} = 1$, $T_{2(a_1)} = T_{2(a_2)} = T_{2(a_6)} = T_{2(a_7)} = 2$,
$T_{2(a_6)} = T_{2(a_8)} = T_{2(a_2)} = T_{2(a_3)} = 3$.

It is clear that T_1 and T_2 are perfect 3- coloring with the matrices A_1 and A_3 respectively. The graph 2 has perfect 3-colorings with the matrices A_1, A_3 and A_4 Consider three mappings T_1, T_2 and T_3 as follows.

$T_{1(a_4)}$ = $T_{1(a_5)}$= 1, $T_{1(a_3)}$ = $T_{1(a_5)}$ = 2,
$T_{1(a_1)}$= $T_{1(a_8)}$= 1, $T_{1(a_6)}$= $T_{1(a_4)}$= $T_{1(a_7)}$=$T_{1(a_3)}$ =3,
$T_{2(a_1)}$= $T_{2(a_4)}$= 1, $T_{2(a_3)}$= $T_{2(a_9)}$= $T_{2(a_6)}$= $T_{2(a_5)}$ =2,
$T_{2(a_3)}$ = $T_{2(a_7)}$ = $T_{2(a_2)}$ = $T_{2(a_{10})}$ = 3.
$T_{3(a_1)}$ = 1, $T_{3(a_3)}$ = $T_{3(a_4)}$ = $T_{3(a_8)}$=2,
$T_{3(a_2)}$ = $T_{3(a_6)}$ =$T_{3(a_7)}$ = $T_{3(a_9)}$ = $T_{3(a_5)}$ = $T_{3(a_1)}$= 3

It is clear that T_1, T_2 and T_3 are perfect 3-coloring with the matrices A_1, A_3 and A_4 respectively.

The graph 4 has perfect 3-colorings with the matrix A_1, consider the mapping T_1 as follows.

$T_{1(a_4)}$ = $T_{1(a_{10})}$ = 1 $T_{1(a_3)}$ = $T_{1(a_5)}$ = 2,
$T_{1(a_4)}$ = $T_{1(a_2)}$ = $T_{1(a_6)}$ = $T_{1(a_9)}$ = $T_{1(a_7)}$ = 3.

It is clear that T_1 is a perfect 3-coloring with the matrix A_1

The graph G has perfect 3-colorings with the matrices A_1 and A_3. Consider two mappings T_1 and T_2 as follows.

$T_{1(a_4)}$ = $T_{1(a_5)}$ = 1 $T_{1(a_2)}$ = $T_{1(a_7)}$ = 2,
$T_{1(a_2)}$ = $T_{1(a_5)}$ = $T_{1(a_9)}$= $T_{1(a_7)}$ = $T_{1(a_1)}$= $T_{1(a_6)}$= 3.
$T_{2(a_3)}$ = $T_{2(a_9)}$ =1, $T_{2(a_7)}$ = $T_{2(a_5)}$ = 2= $T_{2(a_4)}$ = $T_{2(a_1)}$ =2,
$T_{2(a_5)}$ = $T_{2(a_2)}$ =$T_{2(a_6)}$ = $T_{2(a_8)}$ = 3.

It is clear that T_1 and T_2 is a perfect 3-coloring with the matrix A_1 and A_3 respectively.

The graph 13 has perfect 3-colorings with the matrices A_4. Consider two mappings T_1 as follow

$T_{1(a_4)}$ =1, $T_{1(a_1)}$ =$T_{1(a_7)}$ = $T_{1(a_4)}$ = 2
$T_{1(a_3)}$ = $T_{1(a_1)}$ =$T_{1(a_8)}$ = $T_{1(a_9)}$= $T_{1(a_6)}$= $T_{1(a_{10})}$= 3.

It is clear that T_1 is a perfect 3-coloring with the matrix A_4

The graph 18 has perfect 3-colorings with the matrices A_3 *and* A_4. Consider two mappings T_1 and T_2 as follows

$T_{1(a_5)}$= $T_{1(a_3)}$= 1, $T_{1(a_8)}$ =$T_{1(a_9)}$= $T_{1(a_7)}$= $T_{1(a_4)}$= 2.
$T_{1(a_3)}$ = $T_{1(a_9)}$ = $T_{1(a_{10})}$ = $T_{1(a_{18})}$ = 3.
$T_{2(a_4)}$ =1, $T_{2(a_8)}$ = $T_{2(a_4)}$ =2 = $T_{2(a_8)}$ = 2.
$T_{2(a_5)}$ =$T_{2(a_9)}$= $T_{2(a_7)}$= $T_{2(a_4)}$ = $T_{2(a_1)}$ = $T_{2(a_2)}$= 3.

It is clear that T_1 and T_2 is a perfect 3-coloring with the matrix A_3 and A_4 respectively.

It is clear that T_1 and T_2 is a perfect 3-coloring with the matrix A_2 and A_6 respectively. There are no perfect 3-colorings with the matrices A3 of graph 1.

Contrary to our claim, suppose that T is a perfect 3-coloring with the matrix A_1 for graph 1. According to the matrix A_1 each vertex with white color has a neighbour with white color, so the two vertices with white color are adjacent. In the case that $a_2 \leftrightarrow a_4, a_1 \leftrightarrow a_3, a_1 \leftrightarrow a_2, a_3 \leftrightarrow a_4,$ by symmetry $a_7 \leftrightarrow a_8, a_7 \leftrightarrow a_9, a_8 \leftrightarrow a_{10},$ hey have less than four adjacent vertices. These vertices are red color, which is a contradiction. So $a_4 \leftrightarrow a_6, a_5 \leftrightarrow a_6,$ and its symmetric $a_6 \leftrightarrow a_7$ will be remain that are white color. In the case that $a_4 \leftrightarrow a_5$, the neighbours of a_4 and a_5 are red color and vertex a_1 that is their neighbours is also red color has two neighbours with red color which it is not possible. If a_5 are red color and vertex a_1 that is their neighbours is also red color has two neighbours with red color which it is not possible. If a_5 and a_6 are white color, adjacent black color vertex, which is a contradiction. So we conclude the graph 1 has no perfect 3-coloring with matrix A_1.

Contrary to our claim, suppose that T is a perfect 3-coloring with the matrix A_4 for graph 1. According to the matrix A_4 each vertex with white color has three adjacent with black color. If a_1 is white color, the a_2, a_3, a_5 are black color, which is a contradiction with the second row of matrix A_4, the vertices a_1, a_3, a_4 are of matrix A_4. If a_3 is white color, then according to the matrix A_4. the vertices a_1, a_2, a_4 are black color, which is a contradiction with the second row of matrix A_4. If a_4 is white color, then according to the matrix A_4. the vertices a_2, a_3, a_5 are black color, which is a contradiction with the second row of matrix A_4. If a_5 is white color, then a_3 is a vertex that is black color which is a contradiction with the second row of matrix A_4. According to the symmetric, the vertices $a_6, a_7, a_8, a_9, a_{10}$ can not be white color. Therefore the graph 1 has no perfect 3-coloring with matrix A_4.

As it is stated in the before paragraph, the graph 1 has no perfect 3-coloring with matrices A_2.

About other graphs in Figure 2.1, Similarly we can get same result.

IV. CONCLUSION

In this thesis entitled “A study on perfect 3-coloring of cubic graph of order 10” the concept related to the perfect m coloring notion of regular codes is one of the important role in graph thoery. Here we have introduced symmetric parametric matrix Further, an overview of some perfect m coloring is also done.

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